

Optimization of Bus Route using Floyd-Warshal's Algorithm

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Abstract: This paper presents the bus routing system which consists of finding satisfied routes for public transportation within the Upper East region in Ghana. This is because urban scenarios get more complex as times go on, which makes the design of routes within the region an overwhelming task whose results are unsatisfactory with high cost and long distances. This problem is being formulated as a network of distances and solution is presented based on Floyd-Warshal's algorithm to determine the shortest path within the region. Data on distances are obtained from the Metro Mass Transit (Upper East). The total distance in the original network has been reduced drastically compared to the new network.

Keywords: Graph Model, Travel Length Matrix, Optimal Route, Nearest Neighbour Algorithm

I. Introduction

Optimal route problem has been studied by many researchers. It is a well-known problem, and can be used to solve many real-world problems. The goal of this study is to determine traveler's shortest route that stops at given towns. The optimization of routing and packing decisions plays a crucial role in logistics, since the final cost of goods is strongly affected by an efficient distribution. This paper investigates theoretical properties of the double traveling salesman problem with multiple stacks [10]. Other works proposed the issue of asymmetry in the distance matrix from road transportation networks [3].

Figure 1: Map of Upper East Region (Source: maps.google.com.gh)



In literature, nearest neighbor algorithm is praised for route optimization. An example is studying rich vehicle routing problem, incorporating various complexities found in real life applications [1]. The second example uses variable neighborhood search to solve the multi depot vehicle routing problem with time windows [11]. The third example is solved using a two-phase solution framework based upon a hybridized search, within a new reactive variable neighborhood search meta-heuristic algorithm [6]. The fourth example is a mixed load algorithm for the school bus routing problem that seeks to find an optimal set of routes for a fleet of vehicles, linear crack surfaces in cutting and fracture simulations. The algorithm combines a mesh less discretization of the deformation field with explicit surface tracking using a triangle mesh [7]. Embedded systems become a prevailing element in our daily lives. They dominate mobile communication and perform control tasks in most automation domains such as in

buildings and automotive. The complexity and diversity of implementations steadily increase with new features added to the already complex control tasks like anti-lock braking system or electronic stability control [14]. Typically, the single-source shortest-path problem was discovered by [8], while the all-pair shortest-path algorithm is due to [12], who obtained the result based on a theorem by [13].

In this paper, analysis of algorithms and procedures of determining the optimal route among given towns are presented. In an optimal route modelling with graph, the model of the problem is presented as no directional graph $G=(N, E)$, where nodes are towns and crossings outside towns: $N=(1, \dots, n)$, and edges are roads among towns and crossings $E=(1, \dots, m)$. Each node has some information attached to it (name and size). All towns are connected by roads. These roads are shown as graph edges. Each edge also has information sketched in type of road (main, country, and district), length of road, allowable speed on the road and other useful information. Floyd-Warshall's algorithm is selected for finding shortest way from one graph node to another selected node.

Results of the algorithm is two symmetric and quadratic matrices n measurements; distance matrix $[DM]$, which keeps graph's shortest distances among nodes, and path matrix $[PM]$, which keeps the found shortest way intermediate node numbers. Matrix $[DM]$ is used for finding route distance. Matrix $[PM]$ is used for finding shortest path from start node until end node [4].

In this section, literature review in the area of shortest path algorithms for determining the optimal bus route system in order to solve the problem of the shortest path in the Upper East region is presented. The review will focus on Shortest Path Algorithms, particularly the Floyd-Warshall's algorithm.

A. Shortest Path Problem

The study reviews pertinent literature in the field of shortest path algorithms. The shortest path problem is concerned with finding the shortest path from a specified origin to a specified destination in a given network while minimizing the total cost associated with the path [2].

Furthermore, the shortest path problem also has numerous variations including the minimum weight problem, quickest path problem, and the most reliable path problem amongst others. A system for visualizing correctness proofs of graph algorithms exist [9]. In graph theory, the shortest path problem is the problem of finding a path between two vertices such that the sum of the weights of its constituent edges is minimized. An example is finding the quickest way from one location to another on a road map. In this case, the vertices represent location and edges represent segments of road and are weighted by the time needed to travel that segment.

Formally, given a weighted graph (a set V of vertices, a set E of edges, and a real valued weight function $f: E \rightarrow R$) and one element v of V , find the path p from v to v' of V so that

$\sum_{p=1}^n f(p)$ is minimal among all paths connecting v to v' . Shortest path problems are among the most studied network flow optimization problems with interesting applications in a wide range of fields [5].

The shortest path between the two nodes might not be a direct edge between them, but instead involve a detour through other nodes. The all-pairs shortest path problem requires that shortest path distances between every pair of nodes in the network. The Floyd-Warshall's algorithm obtains a matrix of shortest path distances with $O\{n^3\}$ computations.

The Floyd-Warshall's algorithm, also known as Floyd's algorithm, is an algorithm for efficiently and simultaneously finding the shortest paths between every pair of vertices in a weighted and potentially directed graph [12].

Floyd-Warshall's algorithm is a graph algorithm for finding shortest paths in a weighted graph (with positive or negative edge weights). It is also for finding transitive closure of a single execution of the algorithm which will find the lengths (summed weights) of the shortest paths between all pairs of vertices through it which does not return details of the paths themselves. The algorithm is an example of dynamic programming. The Floyd's algorithm compares all possible paths through the graph between each pair of vertices. It enables to do this with only $(|V|^3)$ comparisons in a graph. This is remarkable considering that there may be up to $\omega \in (|V|^2)$ edges in the graph, and every combination of edges is tested. It does so by incrementally improving an estimate on the shortest path between two vertices until the estimate is optimal [12].

II. Materials and Methods

Bus route in Upper East region is a problem connecting to various towns and cities. The main problem involves the insufficient route to the various towns and cities in the region. Shortest path problems are the most fundamental and most commonly encountered problem in the study of transportation. This study is to determine the shortest paths of the various routes in the region.

A. Some Forms of Solving Shortest Path Algorithm

There are numerous well-known algorithms for solving the shortest path problem, some of these includes; Floyd-Warshall's algorithm (solves all pair's shortest paths), Dijkstras algorithm (solves the singular-pair, single-source, and single destination shortest path problems), Bellman-Ford algorithm (solves the single source problem if edge

weights may be negative), and A search algorithm (solves for single pair shortest path using heuristics to try to speed up the search) amongst others [2],[4].

1. All-Pairs Shortest Path Problem

The All pairs shortest path problem (APSP) finds out the shortest path for all source destination pairs in a weight belonging to the most fundamental problems in graph theory. The problem occurs in many algorithms in communication, network, and circuit.

The All-pairs shortest path problem can be considered the mother of all routing problems. It computes the shortest path from each vertex v to every other u . Using standard single-source algorithms, you can expect to get naive implementation of $O(n^3)$ if you use Dijkstra for example. All-pairs shortest distance problems, which aim to find just the distance from each node to another. The result of this operation is an $n \times n$ matrix, which estimated distances to each node. This has many problems when the matrix gets too big, as the algorithm will scale very poorly [4],[5],[8].

2. Prim's Algorithm

Prim's algorithm solves the shortest path problem for a directed graph with non-negative edge weights. For example, if the vertices of the graph represent routes or roads connecting the towns and edge weights represent distances between pair of towns, this algorithm can be used to find the shortest route between two towns. The algorithm deals with procedures. Here are as follows [20],[21]:

Step 1: Start with any vertex.

Step 2: Join this vertex to the nearest vertex.

Step 3: join on the vertex which is nearest to either of those already connected.

Step 4: repeat until all vertices are connected.

Step 5: add the lengths of all the edges included in the minimum length spanning tree.

III. Results and Discussions

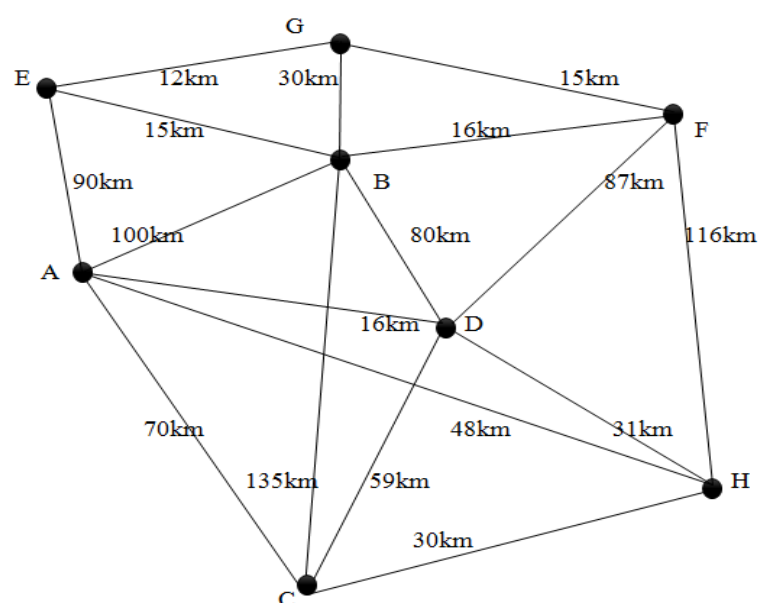
Floyd-Warshal's algorithm can be used to find the shortest path between every pair of vertices in an undirected graph. An advantage of Floyd's algorithm is that, the weights of the connections between the vertices can be negative [8]. The algorithm is a dynamic programming technique, which means it is similar to the divide and conquer methods but with the important difference that it doesn't recalculate "things" which were calculated. The algorithm is recommended for graphs which are dense, for sparse graphs other methods are preferred [2],[4]-[5],[8].

The Floyd-Warshal's algorithm will be applied to the distance matrix to calculate the shortest path between every pair of nodes. This is a graph of nodes or vertices consisting of eight towns and cities found in the Upper East region with its respective distances in kilometers (km). For easy computation, A, B, C, D, E, F, G and H represent Bongo, Bawku, Sandema, Bolgatanga, Pusiga, Binduri, Garu, and Navrongo respectively.

A. Computational Procedure and Analysis of Data

The nodes, represented by characters depicts the various destinations, while the edges represents the distance between destinations in kilometers respectively as shown in the figure 2 below.

Figure 2: Distance Network for 8 Nodes Problem (Source: Metro Mass Transit Ghana Limited).



The matrix below represents the distance (Km) of direct direction between operation towns.

Table 1: Matrix Representation of Distances or Routes

Route	Node/ Site	A	B	C	D	E	F	G	H	Total	
BONGO	A	1	∞	100	70	16	90	105	110	48	536
BAWKU	B	2	100	∞	135	80	15	16	30	132	508
SANDEMA	C	3	70	135	∞	59	160	135	110	30	699
BOLGATANGA	D	4	16	80	59	∞	106	87	110	31	489
PUSIGA	E	5	90	15	160	106	∞	31	12	140	554
BINDURI	F	6	105	16	135	87	31	∞	15	120	509
GARU	G	7	110	30	169	110	12	15	∞	141	587
NAVRONGO	H	8	44	132	30	31	140	116	141	∞	634

The algorithm can be stated as follows:

1. Choose a starting vertex
2. Join this vertex to the nearest vertex directly connected to it
3. Join the nearest vertex, not already in the solution to any vertex in the solution, provided it does not form a cycle
4. Find the shortest path by tracing through the network
5. Repeat until all the vertices have been included

Analysis

From the nodes A

1. From A to B: (The direct distance is ∞).
 $A \rightarrow E \rightarrow B = 90 + 15 = 105$
 $A \rightarrow D \rightarrow B = 16 + 80 = 96$
 Minimum (105, 96) = 96
2. From A to C: (The direct distance is 70).
 $A \rightarrow D \rightarrow C = 16 + 59 = 75$
 $A \rightarrow H \rightarrow C = 45 + 30 = 75$
 Minimum (75)

The value is not less than 70. So we retain 70.

Having done that, to all the various nodes, the table below gives the results.

Table 2: Results of the Shortest Distance of all Pairs of Nodes

Nodes		A	B	C	D	E	F	G	H	Total
A	1	∞	96	70	16	90	103	102	44	521
B	2	96	∞	135	80	15	16	27	111	480
C	3	70	135	∞	59	150	146	165	30	599
D	4	16	80	59	∞	95	87	102	31	470
E	5	90	15	150	95	∞	27	12	126	515
F	6	103	16	144	87	27	∞	15	116	508
G	7	102	27	165	102	12	15	∞	131	554
H	8	47	111	30	31	126	118	131	∞	539

Table 2 gives the total results of the various operational nodes within the region. Having done the calculation using the improved Floyd's algorithm, notice that the total distances to the various nodes has decreased drastically as compared to the traditional algorithm. This means, the shortest path of the various routes has been minimized.

IV. Conclusions

Route selection plays an important role in bus route system, and there are a large number of nodes in urban road network. If the traditional Floyd algorithm is used to calculate the shortest path, each node in the network must be involved in order to increase computation.

For the above problem, the traditional Floyd algorithm was improved and optimized further by constructing iterative matrix and serial number matrix, and the improved algorithm was successfully used to solve route selection problem in bus route system. Those inserted nodes which are not directly connected to the node pair cannot make the length of path shorter. There are also some nodes directly connected to the node pair, but the length of path after inserting them is longer than before.

Floyd algorithm is used to calculate the shortest path between any two points in urban road network in order to conveniently choose the best route for the driver. The objective stemmed from the fact that more transportation network should be constructed in order to reduce traffic jam in the region. The major towns were captured in a network and Floyd algorithm was used to find the optimal route required to connect the cities.

V. Future Research

In future, the proposed Floyd-Warshall's algorithm framework will be extended and improved in finding the shortest path or distance between places in the region that represents any type of network.

VI. References

- [1] A. Goel and V. Gruhn. "A General Vehicle Routing Problem". *European Journal of Operational Research* 191: 650–660. 2008.
- [2] A. Goldberg. "Shortest Path Algorithms". *Engineering Aspects*, Springer-Verlag London, UK, 6. 2001.
- [3] A. Rodriguez and R. Ruiz. "The effect of asymmetry on traveling salesman problems". *European Journal of Operational Research*. 2010.
- [4] A. Sokas. "Algorithms and procedures of determining the shortest route in the graph". *Proceedings of the 7th international scientific conference Transbaltica 2011*. Vilnius: Technika, Pp. 211–215. 2011.
- [5] C. Mouli, S. M. Kumari, and N. Geethanjali. "Shortest Path Algorithms in Transportation Networks". *Global Journal of Computer Science and Technology*, 10(1), pp.36–40. 2010.
- [6] D. C. Paraskevopoulos, P. P. Repoussis, G. Ioannou and G. P. Prastacos. "A Reactive Variable Neighborhood Tabu-Search for the Heterogeneous Fleet Vehicle Routing Problem with Time Windows". *Journal of Heuristics*, 14(5), pp. 425–455. 2008.
- [7] D. Steinemann, M. A. Otaduy and M. Gross. "Splitting meshless deforming objects with explicit surface tracking". *Journal Graphical Models*, 71(6), pp.209–220. 2009.
- [8] E. Dijkstra. "A note two problems in connection with graphs", *Numerical Mathematics* 1: 269–271. 1959.
- [9] Gloor. "Systems of visualizing correctness proofs of graph algorithms". 1993.
- [10] M. Casazza, A. Ceselli and M. Nunkesser. "Efficient algorithms for the double traveling salesman problem with multiple stacks". *Computers & Operations Research* 39: 1044–1053. 2012.
- [11] M. Polacek, S. Benkner, K. K. Doerner and R. F. Hartl. "A Cooperative and Adaptive Variable Neighborhood Search for the multi Depot Vehicle Routing Problem with time windows". *Business Research*, Volume 1(Issue 2), pp 207–218. 2008.
- [12] R. W. Floyd. "Algorithm 97: shortest path". *Communications of the Association for Computing Machinery* 5(6): 345. 1962.
- [13] S. Warshall. "A theorem on boolean matrices". *Journal of the Association for Computing Machinery* 9(1): 11–12. 1962.
- [14] U. Ryssel, J. Ploennigs and K. Kabitzsch. "Automatic library migration for the generation of hardware-in-the-loop models".