



Estimation of Variance using Linear Combination of Mid Range and Quartiles

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Abstract: In this paper, we have proposed a class of modified ratio type variance estimator for estimation of population variance of the study variable, when Mid Range and Quartiles of the auxiliary variable are known. The bias and mean square error (MSE) of the proposed estimator are obtained. From the numerical study it is observed that the proposed estimator performs better than the existing estimators in the literature.

Key words: Simple random sampling, bias, mean square error, Tri-mean, Quartiles, auxiliary variable.

I. Introduction

Here we consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on U_i , $i=1,2,3,\dots,N$ giving a vector $Y = \{y_1, y_2, \dots, y_N\}$. The goal is to

estimate the population means $\bar{Y} = \frac{1}{N} \sum_{I=1}^N y_I$ or its variance $S_Y^2 = \frac{1}{N-1} \sum_{I=1}^N (y_i - \bar{y})^2$ on the basis of random

sample selected from a population U . Sometimes in sample surveys along with the study variable Y , information on auxiliary variable X , which is positively correlated with Y , is also available. The information on auxiliary variable X , may be utilized to obtain a more efficient estimator of the population. In this paper, our aim is to estimate the population variance on the basis of a random sample of size n selected from the population U . The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as Isaki (1983), who proposed ratio and regression estimators. Latter various authors such as Kadilar & Cingi (2006), Subramani & Kumarapandiyan (2015) improved the already existing estimators.

Notations: N = Population size. n = sample size. $\gamma = \frac{1}{n}$, Y = study variable. X = Auxiliary variable.

$\bar{X}, \bar{Y}$ = Population means. $\bar{x}, \bar{y}$ = Sample means. S_Y^2, S_x^2 = population variances. s_y^2, s_x^2 = sample variances. C_x, C_y = Coefficient of variation. ρ = Correlation coefficient. $\beta_{1(x)}$ = Skewness of the auxiliary variable. $\beta_{2(x)}$ = Kurtosis of the auxiliary variable. $\beta_{2(y)}$ = Kurtosis of the study variable. M_d = Median of the auxiliary variable. $B(.)$ = Bias of the estimator.

$MSE(.)$ = Mean square error. $\hat{S}_R^2$ = Ratio type variance estimator. $\hat{S}_{Kc1}^2, \hat{S}_{jG}^2$ = Existing modified ratio estimators. $Q_a = (Q_3 + Q_1)/2$, Population semi-quartile average of the auxiliary variable x . MR = Mid Range. QD = quartile deviation, $Q_{M.A}$ = quartile mean average, Q_1 = first quartile, Q_2 = second quartile, Q_3 = third quartile, Q_r = quartile range

II. Ratio type Variance estimator proposed by Isaki (1983)

Isaki suggested a ratio type variance estimator for the population variance S_Y^2 when the population variance S_x^2 of the auxiliary variable X is known. Its bias and mean square error are given by

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \quad (1)$$

$$\text{Bias}(\hat{S}_R^2) = \gamma S_y^2 [(\beta_{2(x)} - 1) - (\lambda_{22} - 1)]$$

$$\text{MSE}(\hat{S}_R^2) = \gamma S_y^4 [(\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)]$$

Kadilar and Cingi (2006) Estimators:

The authors suggested four ratio type variance estimators using known values of C.V and coefficient of kurtosis of an auxiliary variable X.

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{\beta_2(x) S_x^2 + C_x}{\beta_2(x) s_x^2 + C_x} \right] \quad (2)$$

$$\text{Bias}(\hat{S}_{kc1}^2) = \gamma S_y^2 A_1 \left[A_1 (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right]$$

$$\text{MSE}(\hat{S}_{kc1}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_1^2 (\beta_{2(x)} - 1) - 2A_1 (\lambda_{22} - 1) \right]$$

Recent Developments:

Subramani and Kumarapandiyan (2015) estimators where the authors used median, quartiles and Deciles of an auxiliary variable.

$$\hat{S}_{jG}^2 = s_y^2 \left[\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i} \right] \quad (3)$$

$$\text{Bias}(\hat{S}_{jG}^2) = \gamma S_y^2 A_{jG} \left[A_{jG} (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right]$$

$$\text{MSE}(\hat{S}_{jG}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_{jG}^2 (\beta_{2(x)} - 1) - 2A_{jG} (\lambda_{22} - 1) \right]$$

When $\alpha=0$ in equation (3), the above estimator reduces to Isaki (1983) estimator.

When $\alpha=1$ in equation (3), the above estimator reduces to Kadilar and Cingi (2006) estimator.

Proposed estimator:

We have proposed a new modified ratio type variance estimator of the auxiliary variable by using linear combination of measure based on the median of the Walsh averages which is defined as Mid range and population quartiles of the auxiliary variable. It is highly insensitive to outliers and is robust against outliers.

$$\begin{aligned} \hat{S}_{MS1}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_1)}{s_x^2 + (MR + Q_1)} \right] & \hat{S}_{MS2}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_2)}{s_x^2 + (MR + Q_2)} \right] & \hat{S}_{MS3}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_3)}{s_x^2 + (MR + Q_3)} \right] \\ \hat{S}_{MS4}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_d)}{s_x^2 + (MR + Q_d)} \right] & \hat{S}_{MS5}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_a)}{s_x^2 + (MR + Q_a)} \right] & \hat{S}_{MS6}^2 &= s_y^2 \left[\frac{S_x^2 + (MR + Q_r)}{s_x^2 + (MR + Q_r)} \right] \end{aligned} \quad (4)$$

The bias and mean square error of the above estimator after simplification is given as

$$\text{Bias}(\hat{S}_{MSi}^2) = \gamma S_y^2 A_{MSi} \left[A_{MSi} (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right]$$

$$\text{MSE}(\hat{S}_{MSi}^2) = \gamma S_y^4 [(\beta_{2(y)} - 1) + A_{MSi}^2 (\beta_{2(x)} - 1) - 2A_{MSi} (\lambda_{22} - 1)]$$

$$\text{Where } A_{MS1} = \frac{S_x^2}{S_x^2 + (MR + Q_1)} \quad A_{MS2} = \frac{S_x^2}{S_x^2 + (MR + Q_2)} \quad A_{MS3} = \frac{S_x^2}{S_x^2 + (MR + Q_3)}$$

$$A_{MS4} = \frac{S_x^2}{S_x^2 + (MR + Q_d)} \quad A_{MS5} = \frac{S_x^2}{S_x^2 + (MR + Q_a)} \quad A_{MS6} = \frac{S_x^2}{S_x^2 + (MR + Q_r)}$$

Numerical Illustration

We use the data of Murthy (1967) page 228 in which fixed capital is denoted by X(auxiliary variable) and output of 80 factories are denoted by Y(study variable).we apply the proposed and existing estimators to this data set and the data statistics is given below:

$$N=80, S_x=8.4542, n=20, C_x=0.7507, \bar{X} = 11.2624, \beta_{2(x)} = 2.8664, \bar{Y} = 51.8264, \beta_{2(y)} = 2.2667, \rho = 0.9413, \beta_{1(x)} = 1.05, \lambda_{22} = 2.2209, S_y=18.3569, Q_1=9.318, C_y=0.3542, Q_2=7.5750, Q_3=16.975, QD=5.9125, Qa=11.0625, Qr=11.82, MR=17.955$$

Bias and Mean Square Error of the existing and the proposed estimators

Estimators	Bias	Mean Square Error
Isaki (1983)	10.8762	3925.1622
Kadilar&Cingi(2006) 1	10.7222	3898.5560
Subramani&Kumarapandiyan(2015)	6.1235	3180.7740
Proposed Estimator (MS1)	1.5553	2708.2175

Proposed Estimator (MS2)	1.9096	2728.6569
Proposed Estimator (MS3)	0.3719	2660.5251
Proposed Estimator (MS4)	2.2496	2753.6384
Proposed Estimator (MS5)	1.2761	2691.7525
Proposed Estimator (MS6)	1.1464	2684.9394

Percent relative efficiency of proposed estimators with existing estimators

Estimators	Isaki (1983)	Kadilar&Cingi(2006) 1	Subramani&Kumarapandiyan(2015)
P1	144.9352	143.9528	117.4497
P2	143.8496	142.8745	116.5699
P3	147.5333	146.5333	119.5551
P4	142.5445	141.5783	115.5124
P5	145.8218	144.8333	118.1681
P6	146.1918	145.2008	118.4679

III. Conclusion

The paper proposes a new ratio type variance estimator using known values of an auxiliary variable. Results show that the bias and mean square error are less than the already existing estimators in the literature and also by the percentage relative efficiency criteria are more efficient than the existing estimators. Hence, the proposed estimator may be preferred over existing estimators for use in practical applications.

References:

- Cochran , W. G. (194). *Sampling Techniques. Third Edition, Wiley Eastern limited.*
- Isaki, C.T. (1983). Variance estimation using auxiliary information. *Journal of the American Statistical Association*,78,117-123.
- Kadilar, C. & Cingi, H. (2006). Improvement in Variance estimation using auxiliary information. *Hacettepe Journal of mathematics and Statistics*, 35(1),117-115.
- Murthy, M. N. (1967). *Sampling theory and methods. Calcutta Statistical Publishing House, India.*
- Sumramani, J. and Kumarapandiyan, G. (2015).Generalized modified ratio type estimator for estimation of population variance. *Sri-Lankan journal of applied Statistics*,vol16-1,69-90.
- Wolter, K. M. (1985). *Introduction to variance estimation. Springer- Verlag*