



Occurrence of Digit 0 in First 10 Billion Digits of e after Decimal Point

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Abstract: e is one of the well-recognized irrational numbers. It is famous for its transcendence. In this work, amongst the random-like digits in e after the decimal point, the occurrences of digit 0 in first 10 billion digits after the decimal point are analyzed for both successive and non-successive modes.

Keywords: Digit 0, digits after decimal point, e

Mathematics Subject Classification 2010: 11Y35, 11Y60, 11Y99

I. Introduction

All real numbers are called real as they can be really perceived with common sense and logic. But still amongst them, two types of numbers are odd-man out candidates, viz., rational numbers with infinite recurring non-zero digits after the decimal point and all irrational numbers. Both have infinite number of non-zero digits after the decimal point, and cannot be written down with full precision in that form. Of course, candidates of first category are easy to understand by writing them in their naïve form $\frac{p}{q}$, but irrationals can never be written down with perfection.

Recently, one such irrational number, the famous candidate from geometry π [1], has been analyzed for occurrence of 0 in its digits [2]. We take on e here for similar analysis. The mathematical constant e is favourite more amongst analysts. John Napier seems to have used it first while he was introducing logarithms [3].

II. Digit 0 in e

As e is irrational number, no surprise that there has been lot of work on the digits it contains. Here we have taken up occurrence of 0 in its digits after decimal point. Availability of modern fast electronic computers and powerful high level programming languages like Java [4] have helped make available as many as 10 billion digits of e after the decimal point. As the base of decimal system is 10, we chose the discrete ranges of $1-10^n$, for $1 \leq n \leq 10$. All further values are for digits after the decimal point.

Table 1: Occurrences of 0 in Digits of e in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1-x$	Range as Ten Power 10^x	Number of Occurrences of 0 in e	First Occurrence of 0 in e at Digit Number	Last Occurrence of 0 in e at Digit Number
1.	1–10	10^1	0	-	-
2.	1–100	10^2	5	13	72
3.	1–1,000	10^3	100	13	997
4.	1–10,000	10^4	974	13	9,988
5.	1–100,000	10^5	9,885	13	99,993
6.	1–1,000,000	10^6	99,425	13	999,980
7.	1–10,000,000	10^7	998,678	13	9,999,996
8.	1–100,000,000	10^8	9,999,138	13	99,999,999
9.	1–1,000,000,000	10^9	100,004,425	13	999,999,997
10.	1–10,000,000,000	10^{10}	1,000,024,802	13	9,999,999,993

In the very first 10 power block 1 – 10, the digit 0 just doesn't occur in e . It appears first at digit number 13.

If all 10 digits from 0 to 9 had evenly occurred in all ranges, the expected average of occurrence of each digit would have been $1/10^{\text{th}}$ of the range-limit. Taking this average as base, the deviation from the average for occurrence of 0 is as shown below in figure 1.

In the above 10 discrete digit range-values, 0 switches below and above average. Its no specific dominance throughout is seen, but all higher ranges need not show same pattern.

As mentioned earlier, first 10 digits of e do not contain 0. It comes first at digit number 13 and this is 0's first occurrence for all higher ranges.

Due to reason stated above, ignoring the first block of 1 – 10, the last appearance of digit 0 falls short to catch the range-end by following amounts.

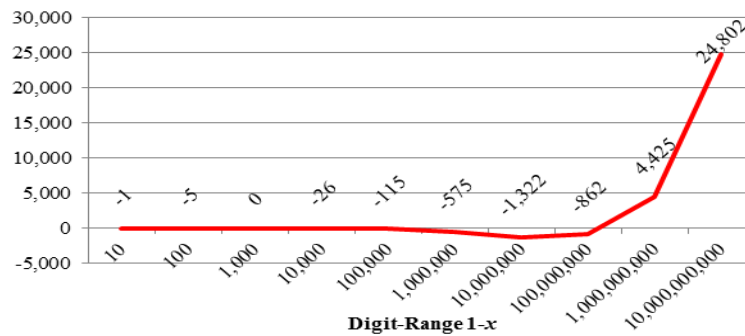


Figure 1: Deviation from Average Number of Occurrences of Digit 0 in e

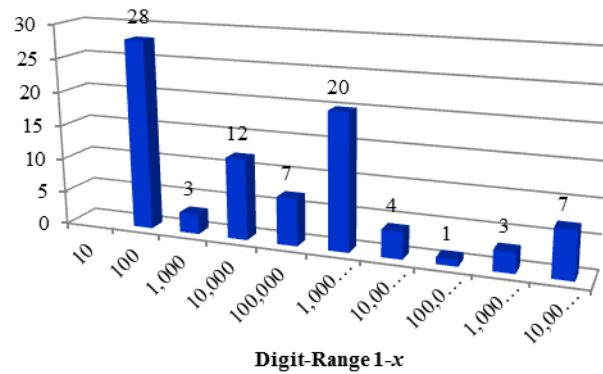


Figure 2: The Extents by which the Last Occurrences of Digit 0 in e are Away from Ends

Quantitatively, 0 is farthest from end in block of $1 - 10^2$ and nearest in block of $1 - 10^8$. But for most of the initial blocks of 10 powers it is quite near end.

III. Successive Occurrence of Digit 0 in e

The successive occurrences of digit 0 in e have also been determined.

Table 2: Successive Occurrences of 0 in Digits of e in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range 1 - x	Range as Ten Power 10^x	Number of Successive Occurrences of 0 in e	First Successive Occurrence of 0 in e at Digit Number	Last Successive Occurrence of 0 in e at Digit Number
1.	1 - 10	10^1	0	-	-
2.	1 - 100	10^2	0	-	-
3.	1 - 1,000	10^3	8	112	829
4.	1 - 10,000	10^4	97	112	9,987
5.	1 - 100,000	10^5	971	112	99,992
6.	1 - 1,000,000	10^6	9,855	112	999,979
7.	1 - 10,000,000	10^7	100,145	112	9,999,879
8.	1 - 100,000,000	10^8	1,000,454	112	99,999,886
9.	1 - 1,000,000,000	10^9	10,001,625	112	999,999,996
10.	1 - 10,000,000,000	10^{10}	100,004,381	112	9,999,999,950

In the first two 10 power blocks 1 - 10 and 1 - 100, digit 0 doesn't occur consecutively. It starts occurring so from digit number 112.

The percentage of occurrence of successive 0's in higher blocks rises and settles to 1 in our ranges.

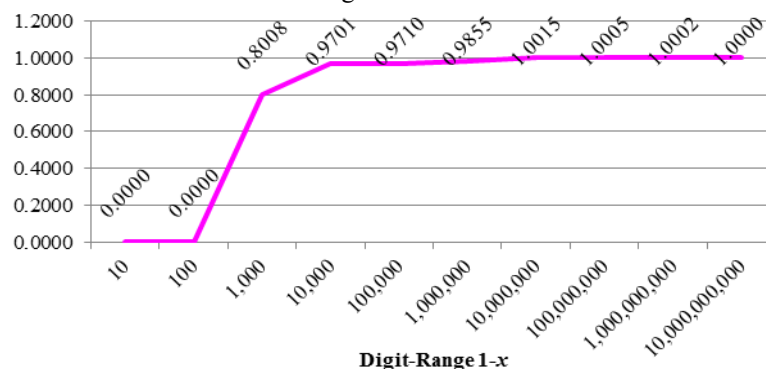


Figure 3: Percentages of Successive Occurrences of 0 in Digits of e after Decimal Point.

The first successive occurrence of 0 comes in after more than 8 times of solo occurrence.

This time barring the first two blocks of $1 - 10$ & $1 - 100$, the last successive occurrences of digit 0's stop before last digit in range by keeping following gaps.

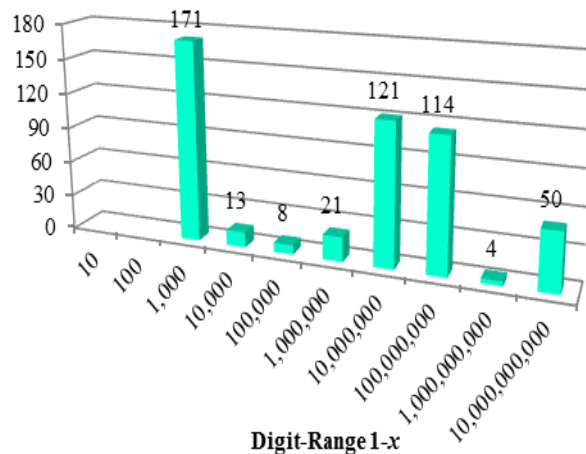


Figure 4: Extents by which the Last Successive Occurrences of 0 in Digit of e are Away from Ends

For initial blocks of $1-10^3$, $1-10^7$ and $1-10^8$, it lags behind more.

The aforementioned analysis is for 2 consecutive 0's. As many as 9 consecutive occurrences of 0's are found in these digit ranges.

Table 3: Multiple Successive Occurrences of 0 in Digits of e in Blocks of 10 Powers

Digit Range	Number of Successive 0's in Digits of e after Decimal Point.									
	1	2	3	4	5	6	7	8	9	10
$1-10^1$	0	0	0	0	0	0	0	0	0	0
$1-10^2$	5	0	0	0	0	0	0	0	0	0
$1-10^3$	100	8	1	0	0	0	0	0	0	0
$1-10^4$	974	97	13	1	0	0	0	0	0	0
$1-10^5$	9,885	971	105	12	2	1	0	0	0	0
$1-10^6$	99,425	9,855	983	92	16	4	0	0	0	0
$1-10^7$	998,678	100,145	10,073	1,016	121	17	3	1	0	0
$1-10^8$	9,999,138	1,000,454	99,785	9,941	1,022	111	12	2	0	0
$1-10^9$	100,004,425	10,001,624	999,732	99,158	9,747	956	112	13	1	0
$1-10^{10}$	1,000,024,802	100,004,380	10,000,358	999,749	99,691	9,935	1,006	99	11	0

In any particular digit block of $1 - 10^x$, during existence, the rate of recurrence of increasing number of successive 0's diminishes roughly by order of magnitude 10.

The first occurrences of these successive patterns of 0's get approximated by $y = 1.6712e^{1.9898x}$.

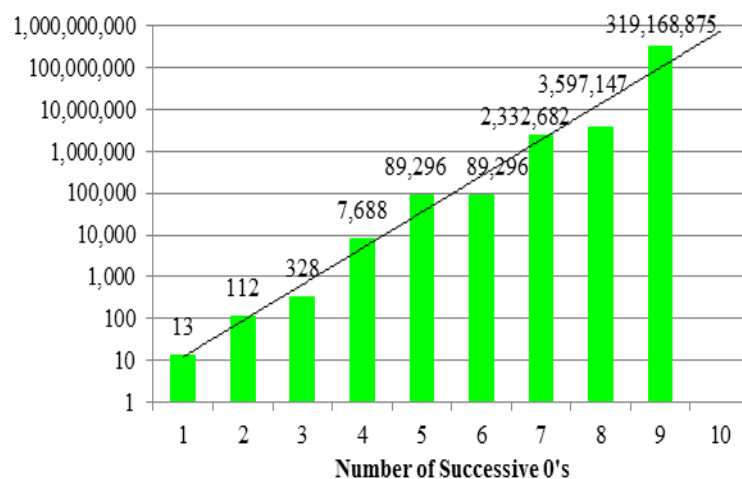


Figure 5: First Occurrences of Multiple Successive 0's in Digits of e .

The last occurrences of successive 0's in the blocks of 10^n are determined to be following.

Table 4: Last Multiple Successive Occurrences of 0 in Digits of e in Blocks of 10 Powers

0's ↓	Digit Range and Last Occurrence of Successive 0's in Digits of e after Decimal Point.									
	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
1	-	72	997	9,988	99,993	999,980	9,999,996	99,999,999	999,999,997	9,999,999,993
2	-	-	829	9,987	99,992	999,979	9,999,879	99,999,886	999,999,996	9,999,999,950
3	-	-	328	8,860	99,606	998,440	9,997,462	99,998,448	999,999,628	9,999,999,623
4	-	-	-	7,688	94,428	962,677	9,993,806	99,967,786	999,997,193	9,999,996,761
5	-	-	-	-	89,297	957,955	9,893,761	99,786,354	999,668,845	9,999,966,243
6	-	-	-	-	89,296	601,113	8,696,075	96,651,493	998,543,094	9,998,916,557
7	-	-	-	-	-	-	3,597,148	95,744,113	984,180,846	9,966,165,482
8	-	-	-	-	-	-	3,597,147	36,455,726	818,244,412	9,966,165,481
9	-	-	-	-	-	-	-	-	319,168,875	9,842,906,267
10	-	-	-	-	-	-	-	-	-	-

IV. Non-successive Occurrence of Digit 0 in e

The appearance of digit 0 with other digit(s) in between it and next 0 is also determined. In this determination, such an occurrence is counted only if the next 0 comes in the same block otherwise it is not credited for that block.

Table 5: Non-successive Occurrences of 0 in Digits of e in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1-x$	Range as Ten Power 10^x	Number of Non-Consecutive Occurrences of 0 in Digits of e	First Non-Consecutive Occurrence of 0 in Digits of e at Digit Number	Last Non-Consecutive Occurrence of 0 in Digits of e at Digit Number
1.	1 – 10	10^1	0	-	-
2.	1 – 100	10^2	4	13	67
3.	1 – 1,000	10^3	91	13	994
4.	1 – 10,000	10^4	876	13	9,983
5.	1 – 100,000	10^5	8,913	13	99,986
6.	1 – 1,000,000	10^6	89,569	13	999,962
7.	1 – 10,000,000	10^7	898,532	13	9,999,978
8.	1 – 100,000,000	10^8	8,998,683	13	99,999,985
9.	1 – 1,000,000,000	10^9	90,002,799	13	999,999,991
10.	1 – 10,000,000,000	10^{10}	900,020,420	13	9,999,999,954

In the first 10 power block 1 – 10, digit 0 doesn't come non-consecutively. It starts occurring such from digit number 13.

The occurrences of non-consecutive 0's dominate those of consecutive ones.

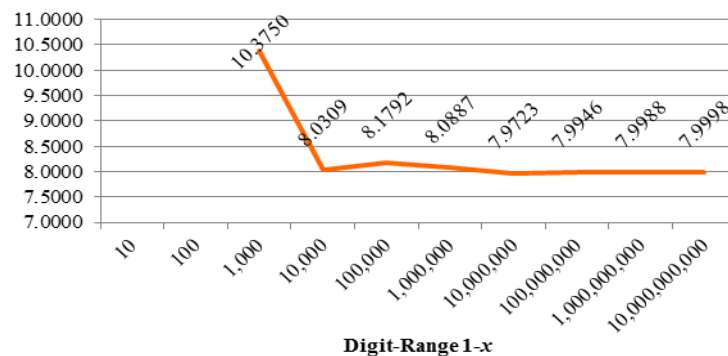


Figure 6: Number of Times Non-successive 0's are more than Successive Ones in Digits of e .

The very first non-consecutive occurrence of digit 0 matches with that of the first occurrence itself.

Except first block of 1 – 10 of non-occurrence, the last non-successive occurrence of digit 0 comes before last digit in range by following amounts.

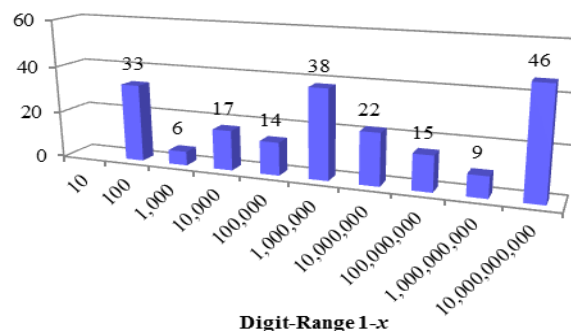


Figure 7: Extents by which the Last Non-successive Occurrence of 0 in Digits of e is Away from End

As much as 7 times dominance of non-consecutive occurrences of digit 0 brings down the heights of these bars compared to their counterparts for consecutive occurrences.

This work is an attempt to recognize patterns and regularity, if any, in decimal digits of e through occurrence of digit 0 in them. Other digits also deserve exploratory analysis like this.

V. References

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