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Effect of q -nonextensive velocity distribution of electrons on the solitary waves in multispecies plasma

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Abstract: In the present investigation, authors have studied ion-acoustic waves in multispecies plasma consisting of cold positive and negative ions with q -nonextensive velocity distribution of electrons. Using standard Reductive Perturbation Theory, a nonlinear KdV equation is derived and analysis is made for different plasma system viz. H^+H^- , Ar^+F^- and $H^+O_2^-$. Results of numerical computation for such plasma system have been shown in the form of graphs in different parameters regimes. It is observed that both rarefactive and compressive type of solitons exist for H^+H^- and Ar^+F^- plasma systems while only compressive type of solitons exist for $H^+O_2^-$ for the given parameter regime.

Keywords: Solitary waves; multispecies plasma; reductive perturbation technique; nonextensive distribution.

I. Introduction

Pioneer work of Zabusky and Kruskal [1] on one-dimensional soliton solution of the Korteweg-de Vries (KdV) equation in the plasma physics opened new vistas in the study of nonlinear phenomena in various branches of science such as biology, particle physics, condensed matter physics, plasma physics, nonlinear optics, neuro physics and in Bose-Einstein condensates. The study of nonlinear waves, their excitation, propagation and interaction is an important issue in theoretical physics. Washimi and Taninuti [2] were the first to use Reductive Perturbation Technique (RPT) to derive KdV equation for ion-acoustic solitons (IASs) in plasma. These are the most studied nonlinear structures arise out of delicate balance between properties like nonlinearity and dispersion. KdV model used to study IASs is applicable only for small amplitude solitary waves [3]-[5]. However, another method known as Sagdeev potential method is also used to study ion-acoustic solitary waves (IASWs) which makes use of full nonlinearity of equations of plasma dynamics [6]. IASWs in magnetized as well as unmagnetized plasmas were reported using both RPT as well as Sagdeev potential method as witnessed by a large number of publications. The earlier experimental observations confirmed the existence of various solitons in laboratory [7]-[10] and are reported in an excellent review [11].

Space plasmas contain several ions having different charge states are usually of multispecies type. Various satellite missions provide abundant information about stationary nonlinear ES/EM structures occurring in space plasma where both positive as well as negative ions exist. Further, the response of plasma to disturbance in the presence of negative ions is significantly modified [12]. Moreover, it was well known that negative ions have considerable effect on the characterization of ion waves [13]-[17]. Functional relationship of relative density and idea of critical density of negative ions have been introduced theoretically [18, 19] and confirmed experimentally [20]-[23]. Negative ions are found in the D -region of Earth's ionosphere [24]. As an approximation, plasma can be treated collisionless in the upper part of the D -region of Earth's ionosphere where collision frequency is expected to be small and neglected [25]. Two ion species having different masses, concentration, charge states and temperature [3, 5, 26] are very common in space and laboratory e.g. $Ar^+SF_6^-$, Ar^+F^- , $H^+O_2^-$ and H^+H^- plasma systems.

Recent observations of space and laboratory plasma have shown that both linear and nonlinear properties are influenced by velocity distribution of the particle constituents of the plasma. In practice, it is observed that particle distributions are better modeled by velocity distribution having flat top with high energy tails. The nonthermal and nonisothermal particle distributions are two most commonly used non-Maxwellian type distributions. Over the last few years, a great deal of attention has been paid to nonextensive static mechanics based on the deviation of Boltzmann-Gibbs-Shanon (BGS) entropic measure. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was first recognized by Renyi [27] and subsequently proposed by Tsallis [28]. They extended the standard additivity of the entropies to the non-linear, nonextensive case where one particular parameter, the entropic index q , characterizes the degree of nonextensivity of the system ($q=1$ corresponds to standard extensive BGS statistics). A wide range of phenomena have employed the Tsallis non additive q -entropy and the ensuing generalized statistics [29]-[41]. Akhtar et al [41] have studied the double layers in multicomponent plasma containing negative ions and reported both rarefactive as well as compressive double layers. Problem of q -nonextensive distribution of electrons in electrons-positrons-ion plasma with high relativistic ions has been

addressed by Pakzad and Javidan [40]. It is well known that the Maxwellian distribution in Boltzmann-Gibbs statistics believed to valid universally for the macroscopic ergodic equilibrium systems. However, in the systems with long range interactions such as plasma and gravitational systems, where the nonequilibrium stationary states exist, Maxwellian distribution might be inadequate for the description of the system. A growing body of evidence suggests that the *q-entropy* may provide a convenient frame for the analysis of many astrophysical scenarios such as stellar prototypes, solar neutrino problem and peculiar velocity distribution of galaxy clusters.

In this research work, we consider the effect of nonextensive velocity distribution of electrons on the propagation of IASWs in collisionless plasma consisting of cold positive and negative ions. Regarding the organization of the manuscript, section II is devoted to formulation of basic equations governing the dynamic of given plasma system and derivation of KdV equation using RPT. In section III, we present discussion of numerical computation of different plasma system.

II. Basic Equations and Derivation of KdV Equation

Consider a collisionless unmagnetized plasma consisting of cold positive and negative ions with nonextensive electron distribution. we assume that low frequency electrostatic waves propagate in the plasma. The nonlinear behavior of the ion-acoustic waves may be described by the following set of normalized fluid equations [3, 5]:

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j v_j)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} = \frac{1}{s_j} \frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - \frac{n_1}{1 - \alpha \epsilon_z} + \frac{\alpha \epsilon_z}{1 - \alpha \epsilon_z} n_2 \quad (3)$$

where, $j=1,2$, where 1,2 stand for positive and negative ions and $s_1 = -\delta$, $s_2 = \eta \delta / \epsilon_z$. Here n_1 and n_2 are the densities of positive and negative ion species normalized by their equilibrium densities $n_1^{(0)}$ and $n_2^{(0)}$. Fluid velocities of two ion components v_1 and v_2 , have been normalized with respect to the ion-acoustic speed in the mixture, $C_s = (T_e \delta Z_1 / m_1)^{1/2}$. Further, the electrostatic potential (ϕ), time (t) and space coordinate (x) are normalized with respect to thermal potential T_e/e , inverse of ion plasma frequency in the mixture $\omega_{pi}^{-1} = (m_1 / 4\pi n_{e0} \delta Z_1)^{1/2}$ and Debye length $\lambda_D = (T_e / 4\pi n_{e0} e^2)^{1/2}$ respectively. $\eta = m_2 / m_1$ is the mass ratio of the negative ion species to the positive ion species, α is the equilibrium density ratio of the negative ion to positive ion species, ϵ_z is the charge multiplicity ratio of the negative ion to positive ion species. Here Z_1 and Z_2 are the charge on positive and negative ions respectively and $\delta = (\eta + \alpha \epsilon_z) / \eta(1 - \alpha \epsilon_z)$. Here

$$n_e = [1 + (q-1)\phi]^{(q+1)/(2(q-1))}$$

The parameter q stands for the strength of nonextensivity. It may useful to note that for $q < -1$, the nonextensive electron distribution is unnormalizable and in the extensive limiting case ($q=1$), the electrons distribution reduces to well known Maxwell-Boltzmann velocity distribution. On linearizing equations (1)-(3) and assuming plane waves, i.e. perturbation of the form $\exp(i(kx - \omega t))$, we get the dispersion relation as $\omega = k / \sqrt{c_1^2 + k^2}$. It may be noted that for Maxwellian electrons i.e. $q=1$, the dispersion relation reduces to that obtained by Mishra and Chhabra [3] for $\sigma_1 = \sigma_2 = 0$. In Fig. 1, a plot dispersion curve (ω vs k) has been shown for Ar^+F^- plasma system for the range $-1 < q < 0$. As observed in earlier investigation [3], dispersion is weak in long wavelength regime. However, dispersion decreases with nonextensivity in this case. A similar kind of behavior has been observed for other ranges of q viz. $0 < q < 1$ and $q > 1$ (not shown here). It is further reported that this trend remains the

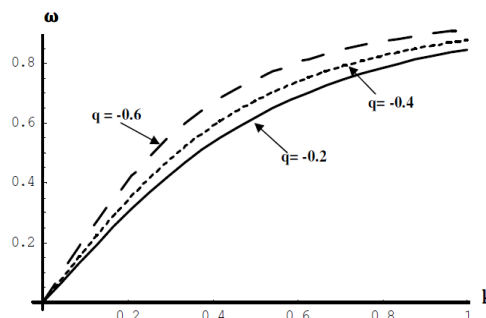


Figure 1 For the range $-1 < q < 0$, dispersion curve for Ar^+F^- plasma system with $\epsilon_z = Z_1 = Z_2 = 1$, same for all plasma systems taken here.

In order to derive a nonlinear dynamical equation for the ion-acoustic waves from equations (1)-(3), we must use the stretched co-ordinates as $\xi = \varepsilon^{1/2}(x - \lambda t)$ and $\tau = \varepsilon^{3/2}t$ where λ is the phase velocity of IAWs and ε measures the size of perturbation amplitude. The dependent variables are expanded as power series in ε about their equilibrium values as: $P_j = P_0 + \sum_{r=1}^{\infty} \varepsilon^r P_j^{(r)}$, $P_j = n_j, v_j, \phi$ and $P_0 = 1, 0, 0$. Substituting the stretched coordinates and the perturbation relations in the basic set of equations (1)-(3) and following the usual RPT [2], we get the relation for phase velocity as $\lambda = (2/(q+1)\eta)^{1/2}$. Apparently the phase velocity is function of several parameters including mass ratio η and nonextensive parameter q . It is found that phase velocity decreases with increase in nonextensive behavior of electrons. This is shown in Fig. 2, where plot of phase velocity λ as a function of q has been given for three different plasma systems. It is also observed that phase velocity decreases with increase mass ratio of negative ions. This occurs because for $q < 1$, there are more energetic electrons in the tail of electrons distribution which increases the thermal pressure of the electrons that required shielding out the positive ions for the propagation of electrostatic wave [41].

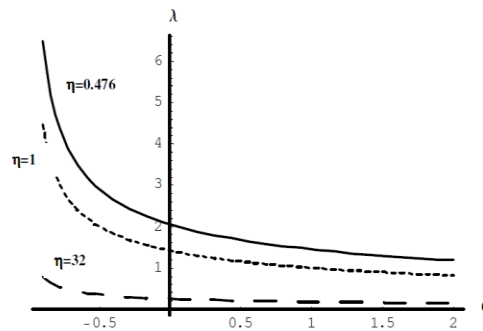


Figure 2 Phase velocity λ as a function of nonextensive parameter q for three different plasma systems with taken as $\varepsilon_z = Z_1 = Z_2 = 1$.

To the next order in ε , we obtain a system of equations in the second order perturbed quantities. On solving we obtain Korteweg-de-Vries (KdV) equation as

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (4)$$

Here $A = 2Q/P$ and $B = 1/P$, P and Q are given by

$$P = \frac{2}{\lambda^3}, \quad Q = \frac{1 - 2\lambda^2}{2\lambda^4} + \frac{3}{2\lambda^4} \frac{(\eta - \alpha \varepsilon_z^3)}{\eta^2 \delta^2 (1 - \alpha \varepsilon_z)} \quad (5)$$

The nonextensive parameter appears in nonlinearity and dispersion coefficients A and B through dispersion relation. It is observed that for $q=1$, the KdV equation (4) becomes same as that of reference [3,5] in the limit $\sigma_1 = \sigma_2 = 0$. To strike a delicate balance between nonlinearity and dispersion that leads to the formation of KdV soliton, the soliton solution for equation (4) is given by

$$\phi = \phi_0 \operatorname{sech}^2(\zeta/d) \quad (6)$$

where $\zeta = \xi - u\tau$ where u is a normalized constant velocity and the amplitude ϕ_0 and width d are given by

$$\phi_0 = \frac{3u}{A}, \quad d = \sqrt{\frac{4B}{u}} \quad (7)$$

As the peak amplitude ϕ_0 and width d depends on coefficients A and B . It is clear that these are further the functions of various parameters including the nonextensive parameter q . It may be mentioned that equation (4) permits compressive as well as rarefactive type of solutions. These results are consistent with those of Gill *et al.* [5]. We have taken three different ranges of q to explain the IAWs in three plasma system viz. Ar^+F^- , H^+H^- and $H^+O_2^-$.

III. Numerical Results and Discussion

For Ar^+F^- ($\eta=0.476$), a plot of peak amplitude (ϕ_0) as a function of q has been given in Fig. 3 for three different values of α with other parameters as $\varepsilon_z = Z_1 = Z_2 = 1$, $u=0.1$ and $\alpha=0.01$ (solid) $\alpha=0.05$ (dotted) and $\alpha=0.1$ (dashed). Rarefactive solitons are obtained for lower values of q for which the peak amplitude increases with q . At some critical value q_c of q transition occurs and compressive type of solitons are obtained for which the peak amplitude decreases with increase in q . It may be mentioned that the critical value of q depends on the density ratio α , and increases with increase in α . A similar kind of behavior is observed for H^+H^- plasma system (not shown). However, only compressive solitons have been observed for $H^+O_2^-$ plasma system for all the values of q (not shown). In this way, entropy index plays an important role in characterizing the behaviour of solitons.

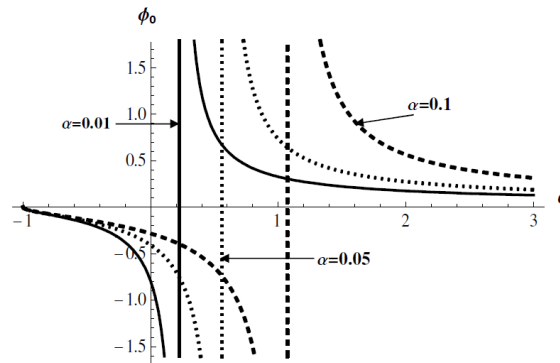


Figure 3 For Ar^+F^- plasma, plot of peak amplitude ϕ_0 as a function of nonextensive parameter q for three different values of density ratio α with other parameters of Fig. 1.

It is further observed that the width of solitons have been found to decrease with increase in nonextensivity (not shown here). Further, the heavier mass of negative ions favours the formation of compressive solitons. However, this behaviour is entirely different from the one predicted by Mishra and Chhabra [3] where both compressive and rarefactive solitons were obtained for this plasma system.

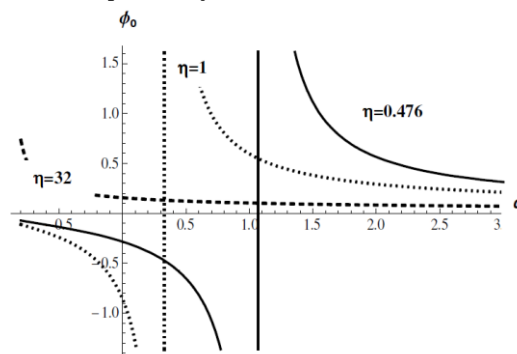


Figure 4 Peak amplitude ϕ_0 as a function of nonextensive parameter q for three different plasma with other parameters of Fig. 1.

In order to investigate the effect of mass ratio η , a plot of peak amplitude as a function of nonextensive parameter q is depicted in Fig. 4 for three different plasma systems. While a transition from rarefactive to compressive solitons is observed at some critical value of q for $\eta=0.476$ and $\eta=1$, only compressive solitons exist for $\eta=32$.

To conclude, we say that the solitary type solution of equations (4) having sech^2 profile, contains peak amplitude which is a function of several parameters such as relative density of negative ions α , soliton velocity u and nonextensive parameter q . While both rarefactive and compressive type of solitons have been observed for Ar^+F^- and H^+H^- plasma systems, only rarefactive solitons are reported for H^+O_2^- plasma system for all the ranges of q . It may be mentioned that the findings of the numerical study of present work may be useful for understanding of nonlinear electrostatic structures propagating in space and laboratory plasma where nonextensivity of electrons is considered.

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