



Common Fixed Point Theorems in Complete Intuitionistic

Fuzzy Metric Space via Occasionally Weakly Compatible Mappings

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Abstract: In this paper we used the concept of occasionally weakly compatible maps in intuitionistic fuzzy metric space to prove common fixed point theorem. Our results are the generalization of the result of Sharma.

Keywords: Common fixed points, Fuzzy metric space, Intuitionistic fuzzy metric space, Compatible maps, Weak compatible maps, Weak compatible mapping and occasionally weakly compatible mapping.

I. Introduction

Fuzzy set theory was first introduced by L.A. Zadeh [11] in 1965 to describe the situation in which data are imprecise or vague or uncertain. Thereafter the concept of fuzzy set was generalized as intuitionistic fuzzy set by K. Atanassov [10] in 1986. All results which hold for fuzzy sets can be transformed into intuitionistic fuzzy sets but converse need not be true. Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [4] proved the well-known fixed point theorems of Banach [15] in the setting of intuitionistic fuzzy metric spaces. Later on, Turkoglu et al. [6] proved Jungck's [8] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu et al. [6] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [14]. Saadati and Park [13] studied the concept of intuitionistic fuzzy metric space and its applications. No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has discovered new possibilities for fixed point theorists. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric space. In 2012, Sharma A. et al. [2], proved various fixed point theorems using concept of semi compatible mappings, property (E.A.) and absorbing mappings. For the sake of completeness we are giving some definitions and results in fuzzy and intuitionistic fuzzy metric space.

II. Preliminaries

Definition-2.1: A binary operation $\star: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is *continuous t-norm* if " $\star$ " satisfies the following conditions:

- (i) $\star$ is commutative and associative
- (ii) $\star$ is continuous
- (iii) $a \star 1 = a$ for all $a \in [0, 1]$
- (iv) $a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$

Definition-2.2: A binary operation $\diamond: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is *continuous t-conorm* if " $\diamond$ " satisfies the following

conditions:

- (i) $\diamond$ is commutative and associative
- (ii) $\diamond$ is continuous
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Definition 2.3: The 3-tuple $(X, M, \star)$ is called a *fuzzy metric space* (shortly FM-space) if X is an arbitrary set, “ $\star$ ” is a continuous t-norm and M is a fuzzy set in $X^2 \in [0, 1]$ satisfying the following conditions for all x, y, z in X and $t, s > 0$,

- (FM-1) $M(x, y, 0) = 0$
- (FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$
- (FM-3) $M(x, y, t) = M(y, x, t)$,
- (FM-4) $M(x, y, t) \star M(y, z, s) \rightarrow M(x, z, t+s)$,
- (FM-5) $M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$ is left continuous.

In what follows, $(X, M, \star)$ will denote a fuzzy metric space. Note that $M(x, y, t)$ can be thought as the degree of nearness between x and y with respect to t . We identify $x = y$ with $M(x, y, t) = 1$ and $t > 0$ with $\varepsilon > 0$..

Example 2.1: Let (X, d) be a metric space. Define $a \star b = ab$ or $a \star b = \min \{a, b\}$ and for all x, y in X and $t > 0$. Then $(X, M, \star)$ is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Remark 2.1: Every metric induces a fuzzy metric

Definition-2.4: A 5-tuple $(X, M, N, \star, \diamond)$ is said to be an *intuitionistic fuzzy metric space* if X is an arbitrary set, $\star$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, and $s, t > 0$,

- (IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$
- (IFM-2) $M(x, y, t) > 0$
- (IFM-3) $M(x, y, t) = 1$ if and only if $x = y$
- (IFM-4) $M(x, y, t) = M(y, x, t)$
- (IFM-5) $M(x, y, t) \star M(y, z, s) \leq M(x, z, t + s)$
- (IFM-6) $M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$ is left continuous
- (IFM-7) $N(x, y, t) > 0$
- (IFM-8) $N(x, y, t) = N(y, x, t)$
- (IFM-9) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$
- (IFM-10) $N(x, y, \cdot): (0, \infty) \rightarrow [0, 1]$ is continuous

Then $(X, M, N, \star, \diamond)$ is called an *intuitionistic fuzzy metric* on X .

Note: $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to ‘ t ’ respectively.

Remark -2.2: Every fuzzy metric space $(X, M, \star)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, \star, \diamond)$ such that t-norm “ $\star$ ” and t-conorm “ $\diamond$ ” are associated.

i.e. $x \diamond y = 1 - ((1 - x) \star (1 - y))$ for any $x, y \in [0, 1]$.

Example-2.2: Let (X, d) be a metric space. Denote $a \star b = ab$ and $a \diamond b = \min \{1, a + b\}$ for all $a, b \in [0, 1]$. Let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M(x, y, t) = \frac{t}{t + md(x, y)}, \quad N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

in which $m > 1$. Then $(X, M, N, \star, \diamond)$ is an intuitionistic fuzzy metric space.

Example- 2.3: Let $X = N$. Define $a \star b = \max \{0, a + b - 1\}$ and $a \diamond b = a + b - ab$ for all $a, b \in [0, 1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows:

$$M(x, y, t) = \begin{cases} x/y & ; x \leq y \\ y/x & ; y \leq x \end{cases}$$

$$N(x, y, t) = \begin{cases} (x-y)/y & ; x \leq y \\ (x-y)/x & ; y \leq x \end{cases}$$

and

Remark-2.3: Note that, in the above example, the t -norm “ $\star$ ” and the t -conorm “ $\diamond$ ” are not associated. And there exists no metric d on X satisfying

$$M(x, y, t) = \frac{t}{t + md(x, y)}, \quad N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Where $M(x, y, t)$ and $N(x, y, t)$ are as defined in above example. Also note the above functions (M, N) is not an intuitionistic fuzzy metric with the t -norm and t -conorm defined as

$$a \star b = \min \{a, b\} \text{ and } a \diamond b = \max \{a, b\}.$$

Definition-2.5: Let $(X, M, N, \star, \diamond)$ be an intuitionistic fuzzy metric space. Then

- (i) A sequence $\{x_n\}$ in X is said to be *Cauchy sequence* if for each $n > 0$ and $t > 0$, there exist $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ and $N(x_n, x_m, t) < \varepsilon$ for all $n, m = n_0$.
- (ii) A sequence $\{x_n\}$ in X is said to be *converges* to x if for each $\varepsilon > 0$ and $t > 0$, there exist $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ and $N(x_n, x, t) < \varepsilon$ for all $n = n_0$.
- (iii) $(X, M, N, \star, \diamond)$ is called *complete intuitionistic fuzzy metric space* if every Cauchy sequence is convergent in it.

Definition-2.6: Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ into itself. The maps A and B are said to be *commutative*, if $ABx_n = BAx_n$ whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X.$$

The following definition was introduced by Mishra et al. [17].

Definition-2.7: Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ into itself. The maps A and B are said to be *compatible* if, for all $t > 0$, if

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1 \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that}$$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X$$

Definition 2.8: Two self-maps A and B of an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ are said to be *weakcompatible*, if they commute at their coincidence points,

i.e. $Ax = Bx$ implies $ABx = BAx$.

Definition 2.9: Self maps A and S of an Intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ are said to be occasionally weakly compatible if and only if there is a point x in X , which is coincidence point of A and S at which A and S commute.

Proposition 2.1: In an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ limit of a sequence is unique.

Proposition 2.2: Let S and T be compatible self-maps of an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ and let $\{x_n\}$ be a sequence in X such that $Sx_n, Tx_n \rightarrow u$ for some u in X .

Then $STx_n \rightarrow Tu$ provided T is continuous.

Proposition 2.3: Let S and T be compatible self-maps of an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ and $Su = Tu$ for some u in X then $STu = TSu = SSu = TTu$.

Lemma 2.1: Let $(X, M, N, \star, \diamond)$ be an Intuitionistic fuzzy metric space, Then for all $x, y \in X$, $M(x, y, \cdot)$ is a non-

decreasing function and $N(x, y, \cdot)$ is non-increasing function.

Lemma 2.2: Let $(X, M, N, \star, \diamond)$ be an Intuitionistic fuzzy metric space, if there exist $k \in (0, 1)$ such that for all $x, y \in X$

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ for all $t > 0$ then $x = y$.

Lemma 2.3: Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, \star, \diamond)$ if there exist a number $k \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, kt) \leq N(x_{n+1}, x_n, t) \forall t > 0$ and $n \in \mathbb{N}$.

Then $\{x_n\}$ is a Cauchy sequence in X .

Lemma 2.4: The only t-norm $\star$ satisfying $r \star r \geq r$ for all $r \in [0, 1]$ is the minimum t-norm, that is

$$a \star b = \min \{a, b\} \text{ for all } a, b \in [0, 1].$$

III. Main Result

We prove the following results:

Theorem 3.1: Let $(X, M, N, \star, \diamond)$ be a complete intuitionistic fuzzy metric space with, $t \star t \geq t$ for all $t \in [0, 1]$.

Let A, B, S, T, P and Q be mappings from X into itself satisfying

(1) $P(X) \subset AB(X), Q(X) \subset ST(X)$;

(2) $AB = BA, ST = TS, PB = BP, SQ = QS, QT = TQ$;

(3) Pairs (P, AB) and (Q, ST) are occasionally weakly compatible mappings

(4) There exists a number $k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq M(ABx, Px, t) \star M(STy, Qy, t) \star M(STy, Px, \beta t) \star M(ABx, Qy, (2 - \beta)t) \star M(ABx, STy, t).$$

For all $x, y \in X, \beta \in (0, 2)$ and $t > 0$.

If the range of the subspaces $P(X)$ or $AB(X)$ or $Q(X)$ or $ST(X)$ is complete, then A, B, S, T, P and Q have a unique common fixed point in X .

Proof. Since $\{y_n\}$ is a Cauchy sequence in X and X is complete, so $\{y_n\}$ converges to a point $z \in X$

Since $\{Px_{2n}\}, \{Qx_{2n+1}\}, \{ABx_{2n+1}\}$ and $\{STx_{2n+2}\}$ are subsequences of $\{y_n\}$, they also converge to the same point z .

Since $P(X) \subset AB(X)$, there exists a point $u \in X$ such that $ABu = z$. Then, using (4)

$$\begin{aligned} M(Pu, z, kt) &\geq M(Pu, Qx_{2n+1}, kt) \\ &\geq M(ABu, Pu, t) \star M(STx_{2n+1}, Qx_{2n+1}, t) \star M(STx_{2n+1}, Pu, \beta t) \star M(ABu, Qx_{2n+1}, (2 - \beta)t) \star M(ABu, STx_{2n+1}, t). \end{aligned}$$

And

$$\begin{aligned} N(Pu, z, kt) &\leq N(Pu, Qx_{2n+1}, kt) \\ &\leq N(ABu, Pu, t) \diamond N(STx_{2n+1}, Qx_{2n+1}, t) \diamond N(STx_{2n+1}, Pu, \beta t) \diamond N(ABu, Qx_{2n+1}, (2 - \beta)t) \diamond N(ABu, STx_{2n+1}, t). \end{aligned}$$

Proceeding limit as $n \rightarrow \infty$ and setting $\beta = 1$,

$$\begin{aligned} M(Pu, z, kt) &\geq M(Pu, z, t) \star M(z, z, t) \star M(z, Pu, \beta t) \star M(z, z, t) \star M(z, z, t) \\ &= M(Pu, z, t) \star 1 \star M(Pu, z, t) \star 1 \star 1; \\ &\geq M(Pu, z, t). \end{aligned}$$

$$\begin{aligned} \text{And } N(Pu, z, kt) &\leq N(Pu, z, t) \diamond N(z, z, t) \diamond N(z, Pu, \beta t) \diamond N(z, z, t) \diamond N(z, z, t) \\ &= N(Pu, z, t) \diamond 1 \diamond N(Pu, z, t) \diamond 1 \diamond 1 \\ &\leq N(Pu, z, t). \end{aligned}$$

By Lemma (2.2),

$Pu = z$. Therefore, $ABu = Pu = z$.

Since $Q(X) \subset ST(X)$, there exists a point $v \in X$ such that $z = STv$. Then, again using (4)

$$\begin{aligned} M(Pu, Qv, kt) &\geq M(ABu, Pu, t) \star M(STv, Qv, t) \star M(STv, Pu, \beta t) \star M(ABu, Qv, (2 - \beta)t) \star M(ABu, STv, t) \text{ and} \\ N(Pu, Qv, kt) &\leq N(ABu, Pu, t) \diamond N(STv, Qv, t) \diamond N(STv, Pu, \beta t) \diamond N(ABu, Qv, (2 - \beta)t) \diamond N(ABu, STv, t) \end{aligned}$$

Proceeding limit as $n \rightarrow \infty$, we have for $\beta = 1$, $Qv = z$. Therefore, $ABu = Pu = STv = Qv = z$.

Since pair (P, AB) is occasionally weakly compatible, therefore,

$Pu = ABu$ implies that $PABu = ABPu$

i.e., $Pz = ABz$.

Now we show that z is a fixed point of P . For $\beta = 1$, we have

$$\begin{aligned} M(Pz, Qv, kt) &\geq M(ABz, Pz, t) \star M(STv, Qv, t) \star M(STv, Pz, \beta t) \star M(ABz, Qv, (2 - \beta)t) \star M(ABz, STv, t) \\ &= I \star I \star M(z, Pz, t) \star M(Pz, z, t) \star M(Pz, z, t). \end{aligned}$$

And

$$\begin{aligned} N(Pz, Qv, kt) &\leq N(ABz, Pz, t) \diamond N(STv, Qv, t) \diamond N(STv, Pz, \beta t) \diamond N(ABz, Qv, (2 - \beta)t) \diamond N(ABz, STv, t) \\ &= 1 \diamond 1 \diamond N(z, Pz, t) \diamond N(Pz, z, t) \diamond N(Pz, z, t). \end{aligned}$$

Therefore, we have by Lemma 2.2,

$$Pz = z.$$

$$\text{Hence } Pz = z = ABz.$$

Similarly, pair of map $\{Q, ST\}$ is occasionally weakly compatible, we have $Qz = STz = z$.

Now we show that $Bz = z$, by putting $x = Bz$ and $y = x_{2n+1}$ with $\beta = 1$ in for (4) we have

$$\begin{aligned} M(PBz, Qx_{2n+1}, kt) &\geq M(AB(Bz), P(Bz), t) \star M(STx_{2n+1}, Qx_{2n+1}, t) \star M(STx_{2n+1}, PBz, t) \star M(AB(Bz), Qx_{2n+1}, t) \star \\ &\quad M(AB(Bz), STx_{2n+1}, t). \end{aligned}$$

and

$$\begin{aligned} N(PBz, Qx_{2n+1}, kt) &\geq N(AB(Bz), P(Bz), t) \diamond N(STx_{2n+1}, Qx_{2n+1}, t) \diamond N(STx_{2n+1}, PBz, t) \diamond N(AB(Bz), Qx_{2n+1}, t) \diamond \\ &\quad N(AB(Bz), STx_{2n+1}, t). \end{aligned}$$

Proceeding limits as $n \rightarrow \infty$ and using Lemma 2.2, we have $Bz = z$. Since $ABz = z$, therefore,

$$Pz = ABz = Bz = z = Qz = STz.$$

Finally, we show that $Tz = z$, by putting $x = z$ and $y = Tz$ with $\beta = 1$ in (4).

$$M(Pz, Q(Tz), kt) \geq M(ABz, Pz, t) \star M(ST(Tz), Q(Tz), t) \star M(ST(Tz), Pz, t) \star M(ABz, Q(Tz), t) \star M(ABz, ST(Tz), t) \text{ and}$$

$$N(Pz, Q(Tz), kt) \geq N(ABz, Pz, t) \diamond N(ST(Tz), Q(Tz), t) \diamond N(ST(Tz), Pz, t) \diamond N(ABz, Q(Tz), t) \diamond N(ABz, ST(Tz), t)$$

Therefore, $Tz = z$.

$$\text{Hence, } ABz = Bz = STz = Tz = Pz = Qz = z.$$

Uniqueness follows easily. If we put $B = T = I$, the identity map on X , in Theorem 3.1, we have the following:

Corollary 1: Let $(X, M, N, \star, \diamond)$ be a complete intuitionistic fuzzy metric space with $t \star t \geq t$ for all $t \in (0, 1)$

and let A, S, P and Q be the mapping from X into itself such that

$$(5) \quad P(X) \subset A(X), Q(X) \subset S(X).$$

(6) The pairs (A, S) and (Q, S) are occasionally weakly compatible mappings.

(7) There exists a number $k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq M(Ax, Px, t) \star M(Sy, Qy, t) \star M(Sy, Px, \beta t) \star M(Ax, Qy, (2 - \beta)t) \star M(Ax, Sy, t); \text{ and}$$

$$N(Px, Qy, kt) \leq N(Ax, Px, t) \diamond N(Sy, Qy, t) \diamond N(Sy, Px, \beta t) \diamond N(Ax, Qy, (2 - \beta)t) \diamond N(Ax, Sy, t);$$

for all $x, y \in X, \beta \in (0, 2)$ with $t > 0$.

If the range of the one subspaces is complete then A, S, P and Q have a unique common fixed point in X . If we put $A = B = S = T = I$ in Theorem 3.1, we have the following:

Corollary 2: Let $(X, M, N, \star, \diamond)$ be a complete intuitionistic fuzzy metric space with $t \star t \geq t$ for all $t \in (0, 1)$

and let P and Q be occasionally weakly compatible mapping from X into itself. If there exists a constant

$k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq M(x, Px, t) \star M(y, Qy, t) \star M(y, Px, \beta t) \star M(x, Qy, (2 - \beta)t) \star M(x, y, t) \text{ and}$$

$$N(Px, Qy, kt) \leq N(x, Px, t) \diamond N(y, Qy, t) \diamond N(y, Px, \beta t) \diamond N(x, Qy, (2 - \beta)t) \diamond N(x, y, t)$$

for all $x, y \in X, \beta \in (0, 2)$ with $t > 0$.

If the range of the one subspaces is complete then P and Q have a unique common fixed point in X .

If we put $P = Q$, $A = S$ and $B = T = I$ in Theorem 3.1, we have the following:

Corollary 3: Let $(X, M, N, \star, \diamond)$ be a complete intuitionistic fuzzy metric space with ,

$$t \star t \geq t \text{ for all } t \in [0, 1]$$

and let P, S be occasionally weakly compatible maps on X such that $P(X) \subset S(X)$ and satisfy the following condition:

$$M(Px, Py, t) \geq M(Sx, Px, t) \star M(Sy, Py, t) \star M(Sy, Px, \beta t) \star M(Sx, Py, (2 - \beta)t) \star M(Sx, Sy, t) \text{ and}$$

$$N(Px, Py, t) \leq N(Sx, Px, t) \diamond N(Sy, Py, t) \diamond N(Sy, Px, \beta t) \diamond N(Sx, Py, (2 - \beta)t) \diamond N(Sx, Sy, t),$$

for all $x, y \in X$, $\beta \in (0, 2)$ and $t > 0$. If the range of the one subspaces is complete then P and S have a unique common fixed point in X .

Example 3.1: Let $X = [0, 1]$ with usual metric d and for each $t \in [0, 1]$. Define

$$M(x, y, t) = \frac{t}{t + |x - y|}, \quad N(x, y, t) = \frac{|x - y|}{t + |x - y|}$$

$$M(x, y, 0) = 0 \text{ and } N(x, y, 0) = 1 \text{ for all } x, y \in X$$

Clearly $(X, M, N, \star, \diamond)$ is a complete fuzzy metric space where $\star$ is defined by $a \star b = ab, a \diamond b = \min \{1, a + b\}$

Let A, B, S, T, P and Q be defined by $Ax = x, Bx = x/2, Sx = x/5, Tx = x/3, Px = x/6$ and $Qx = 0$, for all $x, y \in X$.

Then $P(X) = [0, 1/6] \subset [0, 1/2] = AB(X)$ and $Q(X) = 0 \subset [0, 1/5] = STX$.

If we take $k = 1/2, t = 1$ and $\beta = 1$, we see that all conditions of Theorem 3.1 are satisfied. Moreover, the pair $\{P, AB\}$ and $\{Q, ST\}$ are occasionally weakly compatible.

IV. Conclusion

Theorem 3.1 is a generalization of the result of Sharma [16] in Intuitionistic fuzzy metric space, in the sense that condition of compatibility of type(A) of the pairs of self mappings has been restricted to occasionally weakly compatible self mappings and continuity of the mappings have been completely removed.

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