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Design of FIR Digital Differentiator using Grey Wolf Optimization Technique

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Abstract: The Finite Impulse Response (FIR) digital differentiator gives the rate of change or slope. Due to inherent stability and phase linearity, the FIR digital differentiators have been widely used in various signal processing applications, biomedical equipments and high frequency radars etc. In this paper, the design of FIR digital differentiator is designed using a novel optimization technique named as Grey Wolf Optimization (GWO) technique. The coefficients of FIR digital differentiators are optimized by the proposed method. The optimized coefficients obtained by using the proposed method are compared with those obtained by Cuckoo Search Algorithm (CSA). A performance comparison of the proposed method in terms of accuracy is carried out using MATLAB.

Keywords: Cuckoo Search Algorithm, Digital Differentiators, Error Function, FIR, Grey Wolf Optimization, MATLAB

I. Introduction

Differentiation is an important signal processing tool. It gives the time derivative of the signal. There are many signal processing applications which require the determination of the time derivatives of a signal. In this paper, design of Finite Impulse Response (FIR) digital differentiators using Grey Wolf Optimization (GWO) technique has been presented. The proposed technique improves the global search ability of the algorithm. The balance between exploration and exploitation is maintained by using two random parameters $\vec{A}$ which decides the convergence and divergence from the best solution and the parameter $\vec{C}$, which provides random weights to the prey (current solution) so as to emphasize or de-emphasize the search process.

The FIR digital differentiators can be designed by using various methods like minimax method [1], eigen filter method ([3],[4]), least squares method. The least squares method [9] results in lower computational complexity than minimax and Eigen filter methods.

Later on, evolutionary algorithms were developed which constituted- the genetic algorithm (GA), the differential evolution (DE), the particle swarm optimization (PSO) were used to design FIR digital differentiators [5]. It has been observed that while PSO technique results in faster convergence among all methods, the DE technique has better performance as compared to the performance of PSO and GA. It is simple, robust, converges fast, and finds the optimum value of the solution. Moreover, having few parameters to set, and use of same settings for many different problems are few things which are definitely in favour of DE. Based on the advantages of using lesser number of parameters and easy to understand approach, a novel optimization technique has been proposed in this paper named as Grey Wolf Optimization technique [12].

This paper, presents the novel optimization technique i.e Grey Wolf Optimization technique to design FIR digital differentiators. The problem formulation has been discussed in Section II. The method proposed in this paper i.e. Grey Wolf Optimization is described in section III. The design of FIR based digital differentiators of various orders have been discussed in section IV. Finally, the conclusion has been discussed in section V.

II. Problem Formulation

(A) FIR Digital Differentiator

The impulse response of a digital differentiator ([7],[10]) is defined by:

$$H(z) = \sum_{n=-N_1}^{N_1} h(n)z^{-n} \quad (1)$$

where, n is the order of differentiator

N_1 is the length of differentiator

The difference equation for a digital FIR filter is:

$$y[m] = h[0]x[m] + h[1]x[m-1] + h[2]x[m-2] + \dots + h[N-1]x[m-(N-1)] \quad (2)$$

To achieve the design of FIR filters with a linear phase[8], the following four different types of structures are generally considered which are listed in table 1.

Table 1. Types of Linear Phase FIR filter

Type	$p(\omega)$	M	n_0	N	Response	$b(n)$
I	$\cos(n\omega)$	$(N-1)/2$	0	odd	Symmetric $h(n)=h(N-1-n)$	$2h[(N-1)/2-n]$ for $1 \leq n \leq (N-1)/2$ and $h[(N-1)/2]$ for $n=0$
II	$\cos(n-1/2)\omega$	$N/2$	1	even	Symmetric $h(n)=h(N-1-n)$	$2h(N/2-n)$ for $1 \leq n \leq N/2$
III	$\sin(n\omega)$	$(N-1)/2$	1	odd	Anti-Symmetric $h(n)=-h(N-1-n)$	$2h[(N-1)/2-n]$ for $1 \leq n \leq (N-1)/2$
IV	$\sin(n-1/2)\omega$	$N/2$	1	even	Anti-Symmetric $h(n)=-h(N-1-n)$	$2h(N/2-n)$ for $1 \leq n \leq N/2$

where, N represents the length of the filter,

$b(n)$ is the filter coefficients

$p(\omega)$ is the trigonometric function[2]

(B) An ideal Fir based Digital Differentiator

An ideal k^{th} order differentiator has the following frequency response:

$$D(\omega) = \begin{cases} (\frac{j\omega}{2\pi})^p, & 0 \leq \omega \leq \omega_p \\ (\frac{j(2\pi-\omega)}{2\pi})^p, & 2\pi - \omega_p \leq \omega \leq 2\pi \end{cases} \quad (3)$$

where p represents the order of the differentiator and the ω_p is the highest frequency for which differentiation [2] is required.

$$H(\omega) = j \sum_{k=1}^{N/2} b[k] \sin(k - \frac{1}{2}) \omega \quad (4)$$

where, $b[k]=2h[N/2-k]$, $k=1,2,\dots,N/2$ (5)

Thus, the error function [6] to be minimized is given as:

$$E = |D(\omega) - H(\omega)|^2 \quad (6)$$

where $D(\omega)$ is the desired frequency response and,

$H(\omega)$ is the actual frequency response of the differentiator.

III. Grey Wolf Optimization (GWO)

The proposed method is a new meta-heuristic called as Grey Wolf Optimization which is inspired by Grey Wolves (*Canis lupus*). Grey wolves are apex predators. The grey wolves live in packs . The pack size is 5-12 on average.

There are four categories of grey wolves- alphas, betas, deltas and omegas. The alphas are the leaders of the pack. They are two in number. A male and a female. They decide the place of hunting, time to wake, sleeping place etc. The entire pack follows orders of alphas. The alpha are not necessarily the strongest members of the pack but they surely are best in terms of managing the pack. This means that organization of the pack is foremost important.

The betas assist the alphas in hunting and other chores[12]. The beta are the best candidates for becoming an alpha in case one or both alphas passes away. They brace alpha's commands to the rest of the pack and gives feedback to alpha.

The third category[12] among the grey wolves is the deltas. They have to surrender to alphas and betas but they dominate the omegas. These further consists of five types of wolves i.e. the Scouts, sentinels , hunters and caretakers. The duties of these wolves are watching the boundaries of the territory, protecting guaranteeing the safety of the pack, helping alphas or betas in hunting, caring for the weak or ill wolves in the pack respectively. The last type of wolves are the elders who were earlier alphas or betas.

The last category of grey wolves is the omegas. The omegas act as sitting duck . They have to submit to all the other wolves in the pack. They are considered unimportant in the pack, hence they are allowed to eat after other wolves have finished their meal.

The hierarchy of grey wolves has been described in Fig.1.

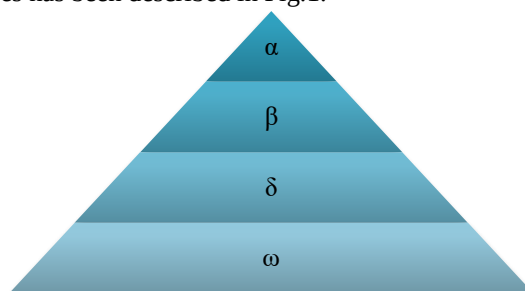


Figure 1. Hierarchy of Grey Wolves (Dominance decreases from top down) [12]

(C) Phases Of Grey Wolf Hunting

The main phases of grey wolf hunting are given as under:

- Tracking, chasing and approaching the prey
- Pursuing, encircling and harassing the prey till the time it stops moving
- Attacking the prey



Figure 2. Hunting behaviour of grey wolves: A) chasing, approaching, and tracking prey (B-D) pursuing[12]. harassing, and encircling (E) stationary situation and attack [2]

First of all, the grey wolves chase the prey, after approaching the prey, the prey is tracked. After that, the grey wolves wait for the prey to move. If the prey starts to move the prey is chased by the grey wolves. If the prey remains stationary, the wolves encircle the prey and when the prey becomes stationary, the prey is attacked by grey wolves.

The encircling behaviour of wolves is represented by equation [12]:

$$\vec{D} = |\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)| \quad (7)$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (8)$$

Where t indicates the current iteration, $\vec{A}$ and $\vec{C}$ are coefficient vectors, $\vec{X}(t+1)$ is position of current search agent, $\vec{X}_p$ is position vector of prey and $\vec{X}$ indicates position vector of a grey wolf.

The vectors $\vec{A}$ and $\vec{C}$ are calculated as[12]:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (9)$$

$$\vec{C} = 2 \cdot \vec{r}_2 \quad (10)$$

Where components of $\vec{a}$ linearly decrease from 2 to 0 over the course of iterations and $\vec{r}_1, \vec{r}_2$ are random vectors in $[0,1]$.

After tracking and encircling the prey, hunting is started. The hunting is guided by alphas. The betas assist the alphas in hunting. In order to simulate the hunting behavior of grey wolves, it is assumed that alphas are the best candidate solutions, betas are the second best solutions and deltas are the third best solutions. Rest of the solutions are termed as omegas. In this way, the three best solutions are saved and other solutions are ignored.

These three best solutions update their positions according to the position of best search agent. The position updating formulas are given as:

$$\vec{D}_\alpha = |\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}| \quad (11)$$

$$\vec{D}_\beta = |\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}| \quad (12)$$

$$\vec{D}_\delta = |\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}| \quad (13)$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha) \quad (14)$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta) \quad (15)$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta) \quad (16)$$

$$\vec{X} = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (17)$$

Where, $\vec{D}_\alpha, \vec{D}_\beta$ and $\vec{D}_\delta$ are the distances of α, β , and δ respectively, when measured with respect to ω ,

$\vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{C}_1, \vec{C}_2, \vec{C}_3$ are the coefficient vectors,

$\vec{X}$ is the position vector of a grey wolf,

$\vec{X}_1, \vec{X}_2, \vec{X}_3$ is position vector of prey,

$\vec{X}_\alpha$ is the first best search agent,

$\vec{X}_\beta$ is second best search agent,

$\vec{X}_\delta$ is third best search agent

$\vec{X}(t+1)$ is position of current search agent,

In third phase, the attacking of prey (exploration) takes place. This means, the global minima is calculated in this phase. In this phase, two parameters i.e. $\vec{A}$ and $\vec{C}$ come into play. The range of $\vec{A}$ is any random value within $[-a, a]$, where a is linearly decreased from 2 to 0. The values of $\vec{A}$ are taken as $[-1, 1]$. If $|\vec{A}| < 1$ then the wolves attack the prey, if $|\vec{A}| > 1$ or $|\vec{A}| < -1$, then wolves diverge from the current prey (current solution) and finds the fitter prey (better solution). Another component of GWO, which is used for exploration is $\vec{C}$ is any random value between $[0, 2]$. This component provides random weights to the prey (current solution) in order to stochastically emphasize ($\vec{C} > 1$) or de-emphasize ($\vec{C} < 1$). This process is known as exploitation. Generally, while grey wolves hunt, a lot of obstacles come in their way which prevents them to carry out hunting or exploration of any prey or optimal solution respectively. The $\vec{C}$ does exactly the same. It serves the purpose of an obstacle in the hunting path of grey wolves.

In nutshell, the search process is started by creating a random population of grey wolves or candidate solutions in GWO algorithm. As iterations progress, alpha, beta and delta find the potential position of the prey. Each candidate solution update its distance from the prey. The parameter a is continuously decreased from 2 to 0 so as to accentuate exploration and exploitation respectively. The candidate solutions diverge from the prey when if $|\vec{A}| > 1$ and converge towards the prey when $|\vec{A}| < 1$. Finally, the GWO algorithm is terminated by using an end criteria.

The pseudo code of Grey Wolf Optimization algorithm is given as below:

Initialize the grey wolf population (Number of wolves=20)

Initialize $a=[0,2]$, $A=[-a, a]$ and $C=[0,2]$

Calculate the fitness of each search agent

X_α =Best search agent

X_β =Second best search agent

X_δ =Third best search agent

while(iterations count < Maximum number of iterations(Here, 100))

for each search agent

Update the position of the current search agent by equation (17)

end for

Update a , A and C

Calculate the fitness of all search agents

Update X_α , X_β , X_δ

iteration=iteration+1

end while

return X_α

Multiply X_α with objective function created in another MATLAB file(.m file)

Optimized coefficients are displayed in the command window and frequency response is obtained

IV. Simulation Results

The simulation is performed on MATLAB for design of FIR digital differentiator. Design examples are presented to validate the design of FIR digital differentiators of different orders.

Table 2: FIR Digital Differentiator design specifications

Differentiator specifications	Values
Pass band frequency	0.1π
Stop band frequency	π
Cut off frequency	0.9π
Order	4,8,10,31

Table 3: Control parameters of the Grey Wolf Optimization technique

Parameter	Grey Wolf Optimization
Number of search agents	20
Number of iterations	100
$\vec{A}$	$[-a, a]$
$\vec{C}$	$[0, 2]$

Example 1: The design of FIR digital differentiator of order 4, length $N=5$, cut-off frequency $= \pi$. Figure 4 shows the frequency response of the designed differentiator. The filter coefficients of the designed differentiator are listed in table 4.

Example 2: The design of FIR digital differentiator of order 8, length $N=9$, cut-off frequency $= \pi$. Figure 5 shows the frequency response of the designed differentiator. The filter coefficients of the designed differentiator are listed in table 5.

Example 3: The design of FIR digital differentiator of order 10, length $N=11$, cut-off frequency $= \pi$. Figure 6 shows the frequency response of the designed differentiator. The filter coefficients of the designed differentiator are listed in table 6.

Example 4: The design of FIR digital differentiator of order 31, length $N=32$, cut-off frequency $= \pi$. Figure 7 shows the frequency response of the designed differentiator. The filter coefficients of the designed differentiator are listed in table 7.

Table 4: Filter coefficients of digital differentiator of order 4

$h(n)$	CSA	GWO
$h(1)$	-0.1427	-0.0069
$h(2)$	0.9999	0.0640
$h(3)$	-0.7365	0
$h(4)$	0.0522	-0.0640
$h(5)$	-0.0758	0.0069

Table 5: Filter coefficients of digital differentiator of order 8

$h(n)$	CSA	GWO
$h(1)$	0.0066	-0.0086
$h(2)$	-0.0119	0.0312
$h(3)$	-0.0073	-0.0799
$h(4)$	-0.0322	0.2124
$h(5)$	0.0699	0
$h(6)$	-0.7892	-0.2124
$h(7)$	0.9549	0.0799
$h(8)$	0.1804	0.0312
$h(9)$	0.0532	-0.0086

Table 6: Filter coefficients of digital differentiator of order 10

$h(n)$	CSA	GWO
$h(1)$	-0.0637	0.0044
$h(2)$	0.2948	-0.0129
$h(3)$	-0.8388	0.0285
$h(4)$	0.8125	-0.0588
$h(5)$	0.3571	0.1404
$h(6)$	0.0529	0
$h(7)$	-0.0803	-0.1404
$h(8)$	-0.0424	0.0588
$h(9)$	-0.0453	-0.0285
$h(10)$	-0.0183	0.0129
$h(11)$	-0.0124	-0.0044

Table 7: Filter coefficients of digital differentiator of order 31

$h(n)$	$b(n)$
$h(1)=-h(32)$	0.0000
$h(2)=-h(31)$	-0.0000
$h(3)=-h(30)$	0.0000
$h(4)=-h(29)$	-0.0000
$h(5)=-h(28)$	0.0000
$h(6)=-h(27)$	-0.0000
$h(7)=-h(26)$	0.0000
$h(8)=-h(25)$	-0.0000
$h(9)=-h(24)$	0.0000
$h(10)=-h(23)$	-0.0000
$h(11)=-h(22)$	0.0000

$h(12)=-h(21)$	-0.0000
$h(13)=-h(20)$	0.0002
$h(14)=-h(19)$	-0.0003
$h(15)=-h(18)$	0.0009
$h(16)=-h(17)$	-0.0081

The simulation results for the design of FIR digital differentiators of order 4,8,10 and 31 are shown in Fig. 4-7. It is observed that the magnitude response at low orders deviates from the ideal response which is a straight line passing through origin, varying linearly with respect to frequency. As the order of the FIR digital differentiator is increased, the deviation between actual and ideal responses minimises to a great extent. The results shows that the proposed method to design FIR digital differentiator is effective in terms of optimization of coefficients and convergence rate. The optimized coefficients are given in Table 3-6. The value of mean square error by using proposed method is compared with the value obtained by using cuckoo search algorithm (CSA)[11].

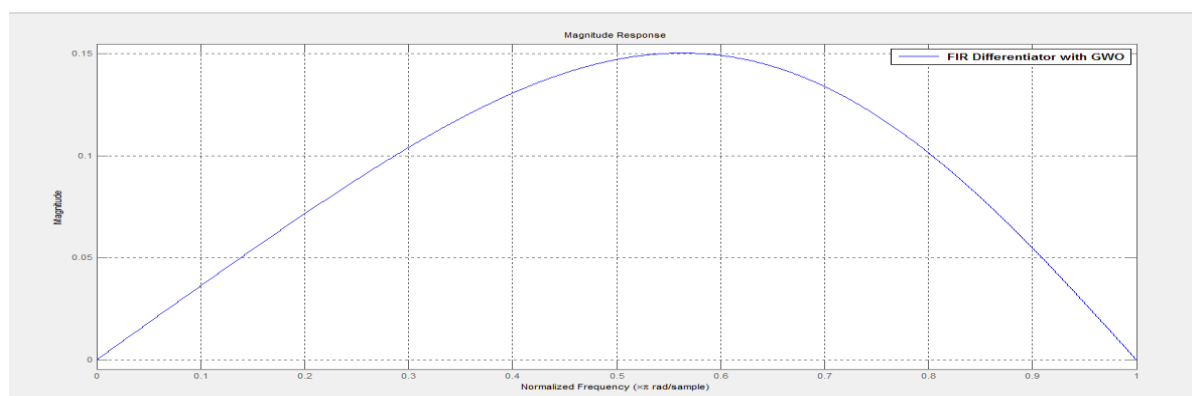


Figure 4. Magnitude Response of FIR digital differentiator of order 4

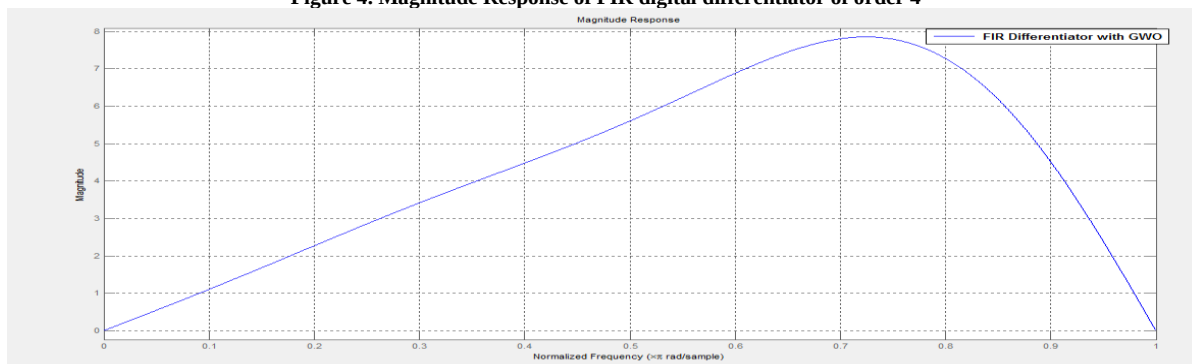


Figure 5. Magnitude Response of FIR digital differentiator of order 8

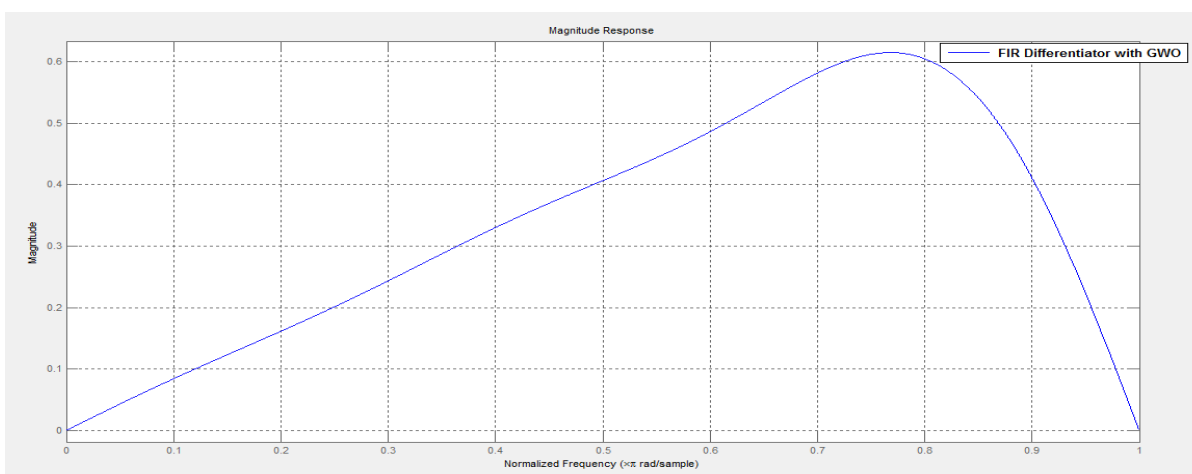


Figure 6. Magnitude Response of FIR digital differentiator of order 10

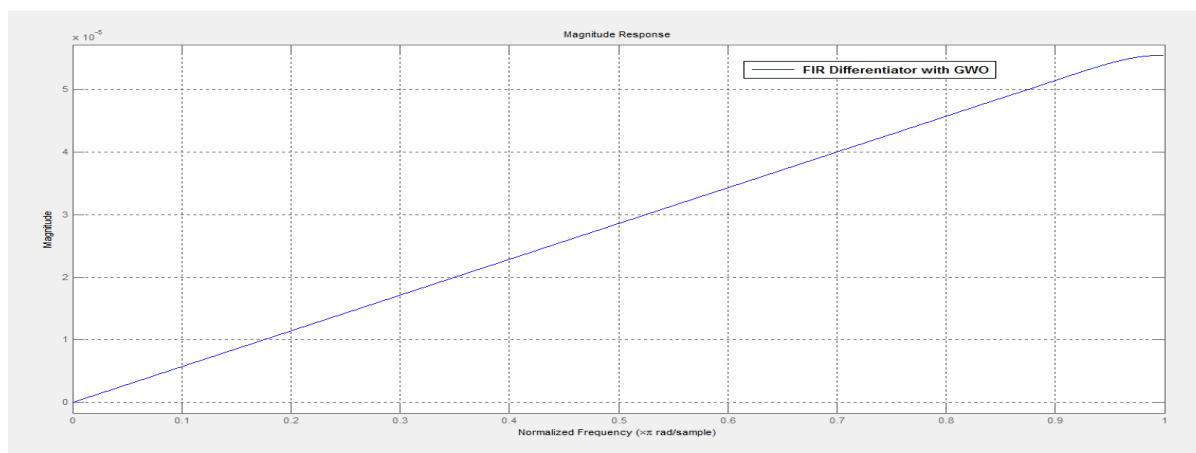


Figure 7. Magnitude Response of FIR digital differentiator of order 31

Table 8. Comparison summary of magnitude error for different order FIR Digital Differentiator using CSA and GWO

Order	CSA	GWO
4	2.2225×10^{-5}	9.2923×10^{-7}
8	1.9257×10^{-5}	2.32348×10^{-7}
10	1.0894×10^{-5}	1.4868×10^{-7}

Table 9. Other results using GWO

Order	Magnitude Error
31	-6.3640×10^{-7}

V. Conclusion

The design of FIR digital differentiator using the Grey Wolf Optimization technique has been discussed in this paper. The results show that the designed differentiator's coefficients are optimal in comparison to those obtained by using cuckoo search algorithm. Smaller and thus better value of the magnitude error is obtained. An increase in the accuracy of the proposed technique has been observed by using the proposed method when compared with cuckoo search algorithm.

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