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FRW-Cosmological Model with Variable Deceleration Parameter

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Abstract: We have studied homogenous and isotropic FRW space time with variable deceleration parameter. The deceleration parameter is assumed to be trigonometric of function of time. The cosmological model describes a unified expansion history of universe indicating initial decelerating expansion and late time accelerating phase. Cosmological consequences of the model are also discussed.

Keywords: FRW model, deceleration parameter

I. Introduction

The cosmological constant problem is a difficult and fascinating problem for cosmologist and quantum field theory researchers [1-6] observations of the CMBR indicate that we live in a spatially flat universe [7-13] with total energy density which equals to the critical density. Chen and Wu [14] considered as Λ is a function of cosmic time. Carvalho et al. [15] have studied by taking Λ is a function of Hubble parameter.

On the other hand, gravitational measurements of matter density in the galaxies lead to an average as the cosmological scale approximately equals to one third of the critical density. It is widely believed that a consistent unification of all fundamental forces in nature representing the energy density of vacuum is variable dynamic degree of freedom which being are initially very large went down to its small present value in an expanding of universe. The last one can be measured through the discrepancies between the small infinite value that the cosmological constant has for the present universe and the values expected by the standard models.

Solution of the field equation may also be generated by law of variation of scale factor which was proposed by Pavon, D [16] Einstein field equations with Robertson-Walker line element has been the subject of numerous studies.

There are numerous research workers that shown example of Phenomenological Λ decay laws. The studies comprises of Λ depending on temperature, time, Hubble parameter and scale factor [17-29]. A dynamically decaying cosmological constant with cosmic expansion have been earlier reported by Borges and Carneiro [30], They considered as cosmological term is proportioned to the Hubble Parameter in FRW model with variable Λ . Recently Tiwari R.K. [31,32,33] considered as cosmological term is proportional to Hubble Parameter in Bianchi type-I model with varying G and Λ .

An idea of a variable gravitational constant G in the framework of general relativity was proposed by Sisterio [21] based on a modification linking the variation of G with that Λ . This modification allows us to use Einstein field equations, which are unchanged, since variation in Λ is accompanied by G. Several authors investigated Friedmann Robertson Walker (FRW) and Bianchi Models using this approach (Abdel Rahman [19]; Berman & Som [20]; Sisteiro [21]; Kalligas [22]; Abdussattar & Vishwakarma [23]; Vishwakarma [24,25]; Pradhan & Ostarod [26]; Singh et al. [27]; Singh & Tiwari [28]; Borges & Carneiro [30]. They considered that the cosmological term is proportional to the Hubble Parameter in FRW model and also Bianchi type-I model with variables G and Λ .

II. Metric and Field Equations.

We consider spatially homogeneous and isotropic Robertson-Walker line element given by

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

Matter components of the cosmic fluid consists of perfect fluid represented by energy momentum tensor

$$T_{ij} = (\rho + p)v_i v_j - p g_{ij} \quad (2)$$

$$p = (\omega - 1)\rho \quad 1 \leq \omega \leq 2 \quad (3)$$

Where p and ρ are the density & pressure respectively v_i is the unit flow vector satisfying $v_i v^i = 1$, Einstein's fields equations with time dependent cosmological constant $\Lambda(t)$ & gravitational constant $G(t)$

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G T_{ij} + \Lambda g_{ij} \quad (4)$$

Where the units such that $c=1$. Einstein's equation (4) for the metric (1) leads to the following equations

$$-\frac{2\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} = 8\pi G p - \Lambda \quad (5)$$

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G \rho + \Lambda \quad (6)$$

Where the subscript stands for ordinary definition with respects to time t .

$$8\pi G \left\{ \rho + 3(\rho + p) \frac{\dot{R}}{R} \right\} + 8\pi \rho \dot{G} + \dot{\Lambda} = 0 \quad (7)$$

The usual energy conservation equation (7) shows that decaying vacuum term leads to matter production

$$T_{i;j}^j = 0 \text{ yields}$$

$$\dot{\rho} + 3(\rho + p) \frac{\dot{R}}{R} = 0 \quad (8)$$

Therefore

$$(7) \text{ reduces to } 8\pi \rho \dot{G} + \dot{\Lambda} = 0 \quad (9)$$

$$\text{From (3) \& (8) we get } \rho = \frac{R_1}{R^{3\omega}} \quad (10)$$

$$\text{where } q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2} \text{ and expansion}$$

scalar $\theta = 3H$, the critical density (ρ_c) vacuum density (ρ_v) and density parameter (Ω) as

$$\rho_c = \frac{3H^2}{8\pi G} \quad (11)$$

$$\rho_v = \frac{\Lambda}{8\pi G} \quad (12)$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2} \quad (13)$$

And which are respectively the critical density vacuum density parameter.

3 Solution of the field equations:

We have the three independent equation (3), (5) and (6) connecting five unknown variable (R, ρ, p, Λ and G). Thus two more relation connecting these variables are needed to solve these equations.

$$q = \alpha \sec^2 \beta t - 1 \quad (14)$$

where α, β are constants and $\alpha < 1$

$$R = (\sin \beta t)^{1/\alpha} \quad (15)$$

A. Matter dominated solution (Cosmology for $\omega=1$)

In this case the matter density ρ and pressure p . This equation of state has been widely used in general relativity to obtain stellar and cosmological models for utterly dense matter

$$\rho = \frac{R_1}{(\sin \beta t)^{3/\alpha}} \quad (16)$$

$$p = 0 \quad (17)$$

$$G = \frac{\beta^2}{4\pi k_1 \alpha} (\sin \beta t)^{\frac{3-2\alpha}{\alpha}} + \frac{k}{8\pi k_1} (\sin \beta t)^{\frac{1}{\alpha}} \quad (18)$$

$$\Lambda = -\frac{2\beta^2}{\alpha} \operatorname{cosec}^2 \beta t + \frac{3\beta^2}{\alpha^2} \cot^2 \beta t + \frac{k}{(\sin^{2/\alpha} \beta t)} \quad (19)$$

$$\theta = \frac{3\beta}{\alpha} \cot \beta t \quad (20)$$

The density parameter is given by

$$\Omega = \frac{2\beta^2}{\alpha \sin^2 \beta t} + \frac{k}{(\sin \beta t)^{2/\alpha}} \quad (21)$$

The vacuum energy density ρ_c and critical density ρ_v are given by

$$\rho_v = - \frac{\frac{2\beta^2}{\alpha} \operatorname{cosec}^2 \beta t + \frac{3\beta^2}{\alpha^2} \cot^2 \beta t + \frac{k}{(\sin \beta t)^{2/\alpha}}}{\frac{2\beta^2}{k_1 \alpha} (\sin \beta t)^{\frac{3-2\alpha}{\alpha}} + \frac{k}{k_1} (\sin \beta t)^{1/\alpha}} \quad (22)$$

$$\rho_c = \frac{3k_1 \beta^2 \cot^2 \beta t}{\alpha^2 \left[\frac{2\beta^2}{\alpha} (\sin \beta t)^{\frac{3-2\alpha}{\alpha}} + k (\sin \beta t)^{1/\alpha} \right]} \quad (23)$$

B. Cosmology for $\omega=2$

The spatial volume V , matter density ρ , pressure p , gravitational constant G and cosmological constant Λ are given by

$$V = (\sin \beta t)^{3/\alpha} \quad (24)$$

$$\rho = p = \frac{k_1}{(\sin \beta t)^{6/\alpha}} \quad (25)$$

$$G = \frac{\beta^2}{8\pi k_1 \alpha} (\sin \beta t)^{\frac{6-2\alpha}{\alpha}} \quad (26)$$

$$\Lambda = \frac{-\beta^2}{\alpha} \operatorname{cosec} \beta t + \frac{3\beta^2}{\alpha^2} \cot^2 \beta t + \frac{2k}{(\sin \beta t)^{2/\alpha}} \quad (27)$$

The expansion scalar θ is given by

$$\theta = \frac{3\beta}{\alpha} \cot \beta t \quad (28)$$

The density parameter is given by

$$\Omega = \frac{\alpha}{3} \sec^2 \beta t \quad (29)$$

The vacuum density ρ_v and matter density ρ_c are given by

$$\rho_v = \frac{k_1}{(\sin \beta t)^{6/\alpha}} \left[-1 + 3k_1 \cos^2 \beta t + \frac{2k}{(\sin \beta t)^{\frac{2(1-\alpha)}{\alpha}}} \right] \quad (30)$$

$$\rho_c = \frac{3k_1}{\alpha} \frac{\cos^2 \beta t}{(\sin \beta t)^{6/\alpha}} \quad (31)$$

We note that the initial time of the Universe in the model is free from initial singularity. Also Hubble's parameter H , expansion scalar θ , isotropic pressure p and energy density ρ all are infinite at the initial time ($t = t'$) whereas the shear scalar σ is constant. Now as t increases scale factor a also increases while the physical parameters $H, \theta, p, \rho, \sigma$ all decrease and in the limit of large t , scale factor a becomes infinitely large but the parameters $H, \theta, p, \rho, \sigma$ converge to zero. This shows that the Universe in model starts from a non-singular state and expands exponentially with cosmic time t present accelerating phase and shows the

largest value of deceleration parameter q hence the fastest rate at which the Universe is undergoing expansion for large t . This future scenario of the Universe is also shown by recent cosmological observations.

III. CONCLUSION

In the present paper, spatially homogeneous and isotropic FRW space-time is considered. We examined a cosmological scenario proposing a variation law in which the deceleration parameter q is assumed to be a simple trigonometric function. We have obtained the cosmological models in which the Universe starts from a non-singular state and expands exponentially with cosmic time t till late times. The deceleration parameter q in the model is found to be time dependent. It is seen that q shows a transition from initial decelerating phase to the present accelerating phase of expansion and supplies the largest value and the fastest rate at which the universe is expanding. Same is also observed by the researchers. The cosmological term Λ approaches to zero as t tends to infinite also shown by recent observations

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