



Determining of the resonant transverse oscillations of a beam fixed at one end

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Abstract: Methods to determine own resonant transverse oscillations of a beam fixed at one end have been investigated. The beam is viewed as a system of parameters. The resonant frequencies have been defined both theoretically and experimentally and a comparison between them has been made. The obtained results can be used to determine (the modulus) of elasticity of the tested beam. The experimental measurements were made with an electromagnetic vibration table.

Keywords: beam; transverse vibrations; modulus of elasticity

I. Introduction

Oscillation processes play an important role in engineering. According to their physical nature oscillations can be divided into mechanical, thermal and electrical. As it is well known, the following types of oscillations can be further distinguished - free continuous oscillations, freely dying oscillations, induced oscillations, parametric oscillations, self-excited oscillations, etc. Regardless of their type they are defined by the same physical laws and properties. Quite frequently oscillation processes in engineering are a sum of free oscillations, which makes the study of the latter extremely important in practice. Such oscillations are the lateral oscillations of a beam, a bar or a strip fixed at one end. These oscillations are used not only in engineering but also in some musical instruments like harmonica, xylophone and tuning fork

II. Presentation

When we consider a real beam we should take into account both its inertial and elastic properties. That's why we should take into consideration the mass and the elasticity of the beam. This makes the beam a complex system with a large number of variables, where the position of each part of the beam during its movement in space is defined by individual coordinates.

The aim of this article is to explore methods of determining the natural frequencies of free, lateral oscillations of a beam fixed at one end; to create graphs to demonstrate the relationship between the natural oscillations and the length and thickness of the beam; to compare the theoretical and experimental curves.

The lateral oscillations of a beam are normally caused by the elastic bending strains. They are defined by a partial-differential equation. For the solution of the equation to be one-valued the respective initial and boundary conditions are used. The differential equation of free lateral oscillations of straight prismatic beams with distributed mass is worked out by using dynamic force analysis. It is known from theory that the differential equation of the elastic line is as follows:

$$EI \frac{\partial^2 y}{\partial x^2} = M_b \quad (1)$$

where I is the geometric inertia moment of the beam; E – modulus of elasticity; $y=y(x,t)$ - deflection, M_b – bending moment.

The dependency $q = \frac{\partial^2 M_b}{\partial x^2}$ (2)

is used when spread load with intensity $q(x,t)$ is present.

With free lateral oscillations the intensity of the lateral oscillations is determined by the apparent forces

$$q = S\rho \frac{\partial^2 y}{\partial t^2} \quad (3)$$

where S is the cross sectional area, ρ – the density of the material.

The condition of a prismatic beam is $S = \text{const}$ and $I = \text{const}$. Double differentiating of (1) in relation to x and comparing (1), (2) and (3) results in

$$\frac{EI}{S\rho} \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \quad (4)$$

Equation (4) defines free lateral oscillations of a straight prismatic beam. It is a linear, homogenous, partial-differential equation with constant coefficients. Its solution is of the type:

$$y(x, t) = X(x) \cdot T(t) \quad (5)$$

where $X(x)$ is function, which depends of x ; $T(t)$ is function, which depends of t .

The displacement and the angle of inclination at the built-in end are equal to zero. The following boundary conditions are derived:

$$X(0) = 0 \text{ and } \frac{\partial X}{\partial x}(0) = 0 \quad (6)$$

for the built-in end

$$X(\ell) = 0 \text{ and } \frac{\partial X}{\partial x}(\ell) = 0 \quad (7)$$

for the free end.

For the natural frequencies of the lateral oscillations of a beam fixed at one end the following expression is derived:

$$f_i = 0,046(2i-1)^2 \frac{\pi^2 h}{4\ell^2} \sqrt{\frac{E}{\rho}} \quad i = 1, 2, 3, 4, \quad (8)$$

where h is the thickness of the beam and ℓ is the length of the beam.

For the first three natural frequencies of the lateral oscillations of a beam fixed at one end the following expression is derived

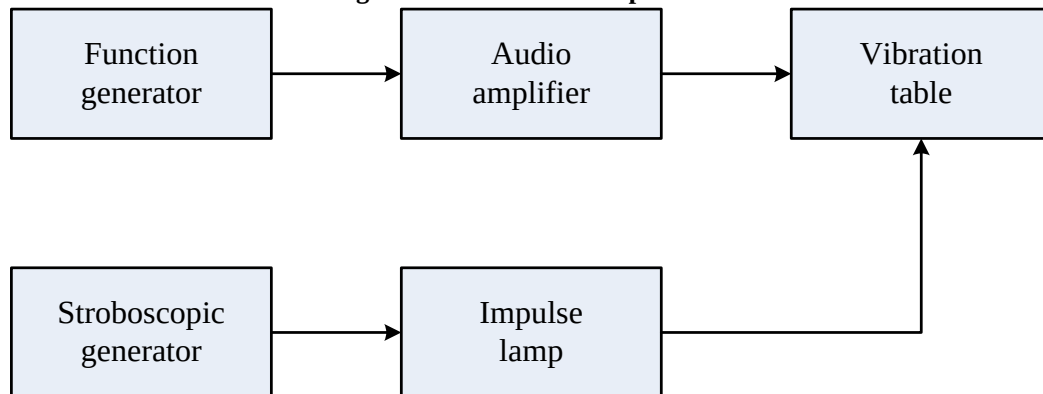
$$f_1 = 0,046 \frac{\pi^2 h}{4\ell^2} \sqrt{\frac{E}{\rho}} \quad (9)$$

$$f_2 = 0,046 \frac{9\pi^2 h}{4\ell^2} \sqrt{\frac{E}{\rho}} \quad (10)$$

$$f_3 = 0,046 \frac{25\pi^2 h}{4\ell^2} \sqrt{\frac{E}{\rho}} \quad (11)$$

III. Scheme of the experiment and tasks

Figure 1 Scheme of the experiment



The sample is fixed on the vibrating table and the frequency of oscillations is changed by function generator while the beam begins to vibrate with maximum amplitude – until a resonance is achieved. The frequency of the blink of the pulse lamp of the stroboscopic generator changes until the image of the beam becomes stationary. Then the frequency is given from the stroboscopic generator.

The following tasks are to be accomplished:

IV. To determine the relationship between the natural oscillations of a beam fixed at one end and its length – $f = f(\ell)$ and to check it with the theoretical relationship $f_{theor} = f(\ell)$.

Determine the relationship $f_1 = f_1(\ell)$ for the first natural frequency of a steel beam with the following characteristics:

modulus of elasticity $E = 3,12 \cdot 10^{11} \text{ N/m}^2$;
 density $\rho = 7,74 \cdot 10^3 \text{ kg/m}^3$;
 thickness $h = 2,80 \text{ mm}$.
 Experimental results:

Table 1

ℓ , mm	265	250	235	220	205	190	175	160
f_{1e} , Hz	29,1	32,7	37,0	42,2	48,6	56,5	66,6	79,5
f_{1b} , Hz	28,7	32,3	36,5	41,7	48,0	55,9	65,9	78,8

Determine the relationship $f_1 = f_1(\ell)$ for the first natural frequency of a synthetic resin- bonded paper beam with the following characteristics:

modulus of elasticity $E = 2,19 \cdot 10^{10} \text{ N/m}^2$;
 density $\rho = 1,294 \cdot 10^3 \text{ kg/m}^3$;
 thickness $h = 3,00 \text{ mm}$.
 Experimental results:

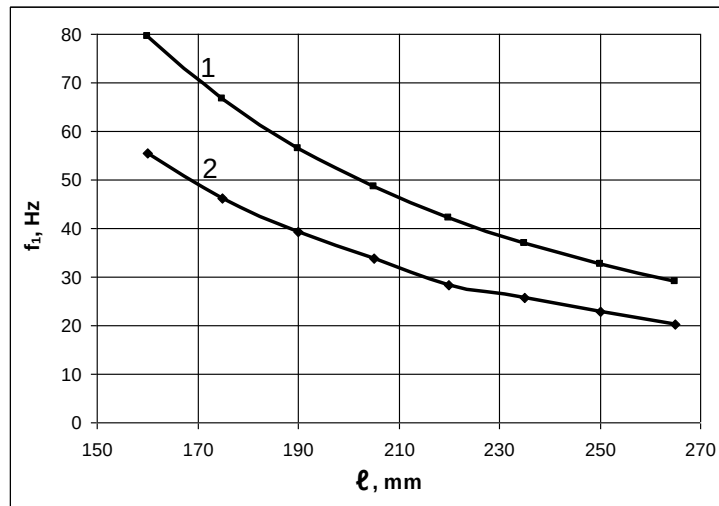
Table 2

ℓ , mm	265	250	235	220	205	190	175	160
f_{1e} , Hz	20,3	22,8	25,8	28,4	33,8	39,4	46,3	55,4
f_{1b} , Hz	19,9	22,4	25,4	28,9	33,3	38,8	45,7	54,7

From Table 1 and Table 2 shows that the experimental dependence $f_1 = f_1(\ell)$ for a beam of steel and synthetic resin- bonded paper hardly differs from the theoretical.

Fig. 2 presents graphs of the dependence $f_1 = f_1(\ell)$ for steel and synthetic resin- bonded paper.

Figure 2 Graphs of the experimental dependence $f_1 = f_1(\ell)$: 1 – for steel; 2 – for synthetic resin- bonded paper.



V. To determine the relationship between the natural oscillations of a beam fixed at one end and its thickness – $f = f(h)$ and to check it with the theoretical relationship $f_{theor} = f(h)$.

Determine the relationship $f_1 = f_1(h)$ for the first natural frequency of a steel beam with the following characteristics:

modulus of elasticity $E = 3,12 \cdot 10^{11} \text{ N/m}^2$;
 density $\rho = 7,74 \cdot 10^3 \text{ kg/m}^3$;
 length $\ell = 250 \text{ mm}$.
 Experimental results:

Table 3

h , mm	2,20	2,40	2,60	2,80	3,00	3,20	3,40	3,60
f_{1e} , Hz	25,0	27,2	29,4	31,7	34,1	36,4	38,6	40,9
f_{1b} , Hz	25,4	27,7	30,0	32,3	34,6	36,9	39,2	41,5

Determine the relationship $f_1 = f_1(h)$ for the first natural frequency of a synthetic resin- bonded paper beam with the following characteristics:

modulus of elasticity $E = 2,19 \cdot 10^{10} \text{ N/m}^2$;
 density $\rho = 1,294 \cdot 10^3 \text{ kg/m}^3$;

length $\ell = 250$ mm.
Experimental results:

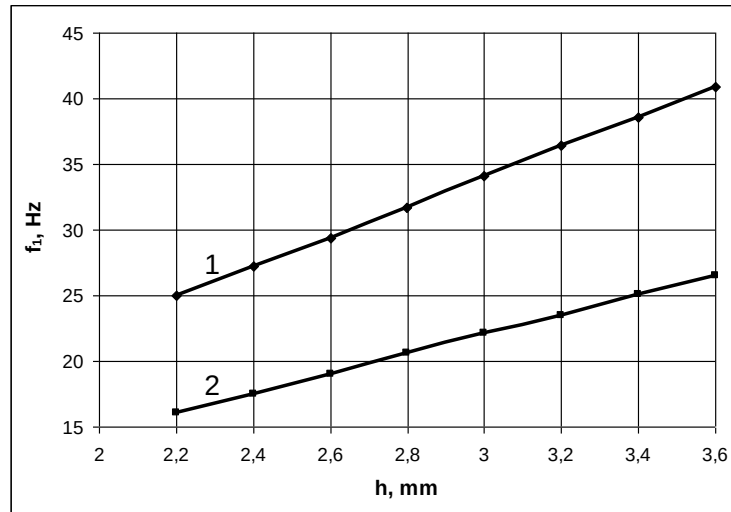
Table 4

h , mm	2,20	2,40	2,60	2,80	3,00	3,20	3,40	3,60
f_{1e} , Hz	16,1	17,5	19,0	20,6	22,1	23,5	25,1	26,5
f_{1b} , Hz	16,4	17,9	19,4	20,9	22,4	23,9	25,4	26,9

From Table 3 and Table 4 shows that there is good correlation between experimental and theoretical results of dependence $f_1 = f_1(h)$ for steel and synthetic resin- bonded paper beams.

Fig. 3 presents graphs of the dependence $f_1 = f_1(h)$ for steel synthetic resin- bonded paper.

Figure 3 Graphs of the experimental dependence $f_1 = f_1(h)$: 1 – for steel; 2 – for synthetic resin - bonded paper.



The beams used are made of steel and synthetic resin - bonded paper. The modulus of elasticity can be determined by means of obtained experimental results and the following equation:

$$E = \frac{f_1^2 l^4 \rho}{k_l^2 0.046^2 h^2} \quad (12)$$

where $k = (2i - 1)^2$

VI. Conclusion

The proposed analytical method for determining the resonance frequencies of transverse oscillations of a beam fixed at one end gives results that are very close to the experimentally obtained. The system in figure 1 can be used for measuring resonant frequencies of more complex parts which would be difficult to determine by means of analytical methods.

VII. References

- [1] P. V. Fegade, J. A. Mahajan, G. P. Bhole "Study on Vibration Analysis of an Un-crack and Cracked Cantilever Beam", International Journal of Emerging Technologies in Computational and Applied Sciences, 10(4), SeptemberNovember, 2014, pp. 340-343
- [2] Karadzhov, Ts., N. Angelov, Determining the Lateral Oscillations Natural Frequency of a Beam Fixed at One End. University of Rousse. Union of Scientists – Rousse, 2011, Vol. 8, ISSN 1311-9974.
- [3] Petkov, S.I., D.A.Dichev, Chr.C.Koev. Estimation of bias error by modal testing of mechanical vibrating structures. Maritime Industry, Ocean Engineering and Coastal Resources. Taylor&Francis Group, London, UK, 2007. ISBN 13-978-0-415-45523-7.
- [4] Dichev, D., S. Petkov, H. Koev, A. Andreev. Vibration force excitation on dynamical behaviour of sensors and instruments for ships' roll measurement. 13th Congress of Intl. Maritime Assoc. of Mediterranean IMAM 2009, İstanbul, Turkey, 12-15 Oct. 2009. ISBN 978-975-561-355-0.
- [5] Dichev, D., Hr. Koev. Models for analysis of dynamic error for instruments, measuring parameters of moving objects. International virtual journal for science, technics and innovations for the industry MTM' 2011, issue 9/2011. ISSN 1313-0226
- [6] Pisarev A. Mechanical vibrations, Technika, Sofia, 1985
- [7] Lilkova-Markova S., D. Lolov Numerical-analytical method to determine their own forms of rights beam with concentrated masses, Jubilee Conference of VSU, Sushodol, 2003
- [8] Cheschanov B. Theory of oscillation, Technical University of Sofia, 1992, ISBN 954-438-001-9