



International Journal of Emerging Technologies in Computational and Applied Sciences (IJETCAS)

www.iasir.net

Hybrid Sampling Technique for MRI Image Reconstruction

Tanuj Kumar Jhamb¹, Vinith Rejathalal², V.K. Govindan³

Department of Computer Science and Engineering,
National Institute of Technology Calicut, India

Abstract: The k -space data is obtained from signals generated by Magnetic Resonance Imaging (MRI) scanning machine, and these signals get captured at the radio frequency coils. Accuracy of reconstruction of MRI images involves many factors like data acquisition, data sampling and reconstruction algorithms. This paper investigates the effect of the type of sampling on the accuracy of the reconstruction. In this context, various sampling techniques have been reviewed, and the frequency encoding and the phase encoding for the k -space data have been explained. The related works in image reconstruction like, Sensitivity Encoding approach, SURE approach and signal acquisition protocol have been reviewed. The mechanism of k -space acquisition has been discussed, and a new approach for the k -space sampling has been proposed to improve the sampling of the captured k -space, and hence to provide a better reconstructed image. The performance of the proposed approach is evaluated using mean absolute error, mean squared error and peak signal-to-noise ratio. The results demonstrate that the proposed sampling approach, referred as “hybrid sampling of k -space”, can reconstruct better quality images when compared to the existing conventional sampling approaches.

Keywords: Frequency Encoding, Phase Encoding, FEG, PEG, CS, Radial Sampling, Hybrid Sampling

I. Introduction

MRI is a riveting imaging technique that pictures human body or part of it. It is non-invasive technique and does not cause any ionization in the human body. MRI evolved from various rudimentary inventions made during 1980's and it underwent enormous development since the last three decades. Indeed, the objects imaged by MRI have an impressive quality, even comparable to photographs taken by digital cameras. Now days, MRI is recognized as the prime modality for diagnosing many common diseases, including cancer and stroke. It is not only used for imaging high spatial resolution images, but also offering good contrast for soft tissues. It has advantages over other imaging techniques like SPECT (Single Photon Emission Computed Tomography), Doppler Ultrasound, PET (Positron Emission Tomography), and computed tomography (CT) which cause ionization in the human body. The examination procedure through MRI does not involve any risk to the patient's health. The basics of MRI imaging processes is briefly introduced in the following sub sections.

A. Frequency Encoding and Phase Encoding:

A main magnetic field B_0 is applied in the parallel direction of an object which aligns the nuclear spins of the protons. All the spins have the same Larmor frequency [1] and they are independent of the spatial location of the scanned object. A secondary magnetic field is applied in the form of gradient in Y and Z direction. The nuclei of an atom experiences stronger magnetic field in the positive X direction, and thus undergo with high Larmor frequency. Similarly, nuclei which are in the opposite (negative) X direction experiences low intensity magnetic field and thus undergo with lower Larmor frequency. This is referred to as frequency encoding. The field gradient is applied in Y direction; it induces spatial position dependent phase-difference. This is called phase encoding. Now, each spatial location of an object has a distinctive Larmor frequency and phase shift. An image can be reconstructed from these signals.

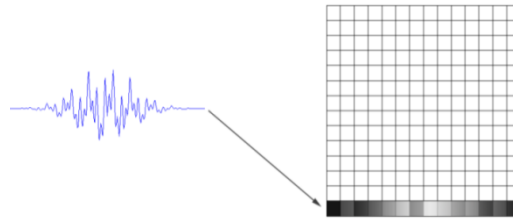
B. k -space Data Acquisition:

The k -space data acquisition in MRI takes place in following steps:

- (i) An object is placed in the MRI scanning device and a strong magnetic field B_0 is applied, which enables all protons of the object to undergo torque-induced precession at the Larmor frequency.
- (ii) The RF pulse applied, which tilts [2] the total magnetization present in the protons in a single direction and enables the in-phase precession for all protons in RF pulse.
- (iii) The PEG (Phase Encoding Gradient) is applied for making a phase difference with respect to the spatial position of an object.
- (iv) In this step, the FEG (Frequency Encoding Gradient) is applied which helps RF coils to capture the proton signal intensity from the excited protons when they return to their ground state.

The FEG shifts the Larmor frequency of the protons which is determined by the position of the object and then

Figure 1 Generation of Raw Data Matrix (k-space) [1]



the signal represents spectrum of frequencies. The phase also differs from the spatial position of the protons. Generally, each excitation and the "fall back" of the proton energy to ground state is read-out from RF coils and collected in the raw data matrix in row major order as shown in Figure 1. Basically, the entries in the raw data matrix are the frequency components of the proton density image.

C. Image Reconstruction Algorithm

Image Reconstruction is the process of converting k-space data to an image that is visible and pictorially meaningful. An image reconstruction algorithm is required to generate the spatial domain images from the sampled k-space data obtained from MRI scan. There exist many algorithms [3] to reconstruct images from k-space data like Zero Padding Method, Phase Correction using Conjugate Symmetry method, Homodyne Reconstruction Algorithm, and Projections onto Convex set (POCS) method. Formally, the k-space obtained from the scanning machine can be represented in the form of continuous domain [5] that relates the density of protons $m(r)$ to the signal received $p(k)$. The $m(r)$ is related to $p(k)$ by the following equation:

$$p(k) = \int_{FOV} m(r) e^{-i2\pi k \cdot r} dr \quad 1$$

where 'k' represents the frequency domain point and the 'r' represents the spatial domain point

II. Related Work

An enormous amount of work has been done in the field of reconstruction of images from the signals acquired by scanning devices. There has been staggering improvement in the quality of reconstruction since 1970 [4]. A few of the notable works are reviewed in the following subsections:

A. Sensitivity Encoding

One of the classical works in image reconstruction is that by Cheng and Dewey [6]. The authors described the concept of reconstruction of image from projections or raw data as an inverse problem, and implemented in cases where a large number of images to be reconstructed. Expectation maximization is employed for determining the class values that are not available. MRF (Markov Random Field) model of dynamic objects is used to achieve local regularity and increased smoothness during reconstruction. Pruessmann et al. [7] proposed a new approach for collecting the k-space data referred to as SENSE (Sensitivity Encoding). In classical technique, Fourier Imaging collects the data on RF coil at a very slow rate, but with the help of SENSE, the capability of Fourier Imaging can be enhanced at the double rate. This also reduces the time for scanning. In the normal Fourier encoding, the k-space data is sampled and transferred to the RF coil. If the Fourier Imaging is used with SENSE, it reduces the sampling density and collects the k-space more efficiently. Authors observed that SENSE enhanced gradient encoding and reduced scan time as compared to Fourier imaging.

B. Stein's Unbiased Risk Estimator (SURE)

Alfred et al. [8] reviewed the theoretical results from sparse solutions of systems that are linear. Harmonic analysis of wavelets and applications of sparsity to signal in image processing are discussed. The sparsity based solution of linear equations in the application of image compression and image reconstruction is also discussed. The other concepts discussed in the paper are LU Decomposition, and Least Square method. Raich and Alfred [9] proposed sparse image reconstruction in radio astronomy. They proposed three sparse image reconstruction methods; the first uses Stein's unbiased risk estimator (SURE) for selecting the hyper-parameter for L1 estimator, and the other two are based on sparse prior; they used empirical Bayes denoising. These three methods are simulated and found to produce better results than sparse Bayesian learning (SBL) [10]. Sparse prior is the magnetization intensity falling on the body. Using sparse prior with SURE method enhanced the quality of signals leading to better reconstructed images.

C. Signal Acquisition Protocol

Candes et al. [11] presented how to reconstruct an image from numerical optimization from the full-length signal/image using the raw data (k-space) that is also called projections. They also described a very simple and

efficient signal acquisition protocol that samples the signal which is independent of the rate of change in the projections. The final set obtained after the sampling allows improving the reconstruction power of sparse data, and permits better reconstruction of images.

Inspite of the large amount of existing development works in MRI image reconstruction, there still has issues like complexity in k-space acquisition process and requirements of large scan time for generating good quality images. Thus, there are further scopes for improvements in the quality of reconstructed images by introducing improvements in k-space acquisition process and reducing the scan time by enhancing gradient encoding. This paper proposes a hybrid sampling technique to improve the k-space acquisition process.

III. Proposed k-space Sampling Approach

Generally, the sampling of k-space can be done in two ways: (i) Cartesian Sampling, and (ii) Radial Sampling. The order in which the RF signals are detected plays an important role in MRI image reconstruction. During the scanning of an object from the MR camera, the k-space data is collected at different times, which have different spatial intensities (state) of an object. As discuss in subsections given below, the central region of k-space is having more structural information about the scanned part and the farthest region of k-space (peripheral region) contains less information about the structural part of the object. In view of the above fact, to take care of this non-uniformity of the k-space, we proposed a combined approach for sampling in which the central region is sampled by radial sampling and the peripheral region is sampled by cartesian sampling. The basics of conventional sampling approaches and the proposed combined approach are presented in the following subsections:

A. Cartesian Sampling

In Cartesian sampling, the sampling is done by scanning each spatial location of k-space data in a rectilinear fashion row by row as shown in Figure 2. All the sampled points are equidistant from each other. Hence, the central regions are not much focussed for the reconstruction, which results in a poor quality reconstructed image. The samples obtain by Cartesian approach are all equidistant from each other which treat k-space uniformly. Hence, there should be more samples from the central region as compared to peripheral regions, but here it is not so. So, this sampling approach is not that acceptable for sampling k-space.

Figure 2 Cartesian sampling of k- space data [12]

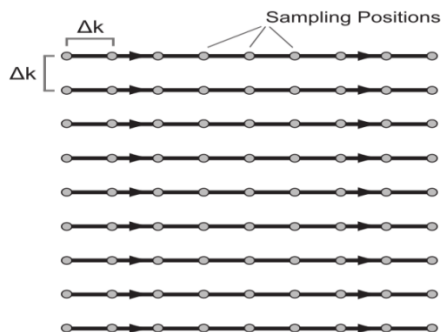


Figure 3 Radial sampling of k-space data [12]

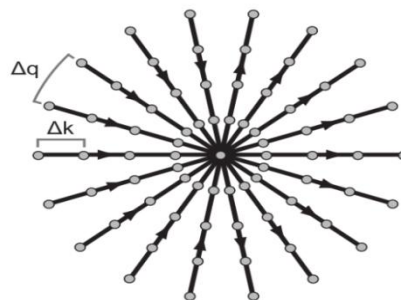


Figure 4 Co-ordinate of k-space in 2 Dimension [13]

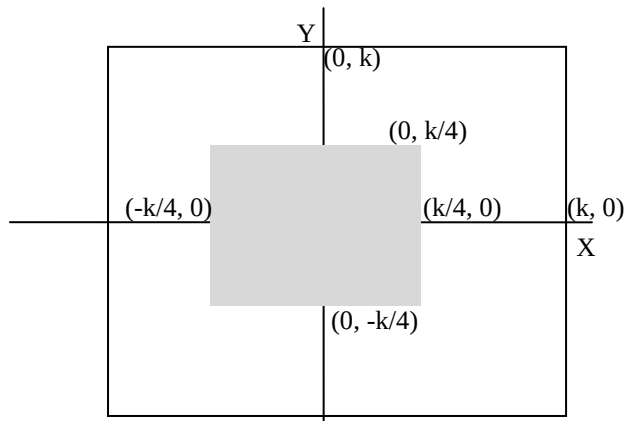
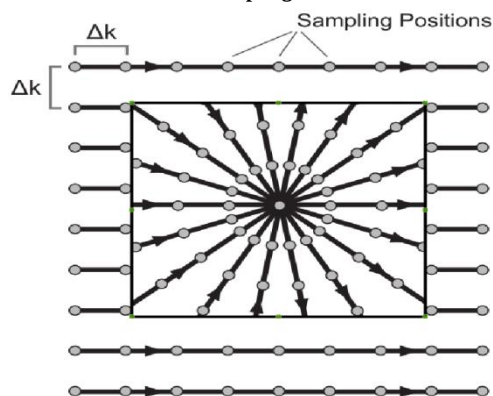


Figure 5 Diagrammatic representation of hybrid k-space sampling



B. Radial Sampling

In radial sampling, the k-space data is processed from the low frequency region (origin) and continue in angular fashion as shown in Figure 3. The data near to the origin in k-space have the most information about the structure of an image like contrast, and the SNR, whereas the data at farther region helps in reconstructing smaller structures in an image [12]. With the help of radial k-space sampling, the low frequency regions (central regions) of an image get processed at every scan. Hence, using the radial sampling of k-space can generate good reconstructed images, but the peripheral regions are not getting processed much which sometimes result in lower quality reconstruction of small structures of an image.

C. Hybrid Sampling

It has been discussed that the reconstructed images from the radial sampling of k-space are lacking information of small structural regions of the scanned object. Also, the drawback of cartesian sampling is that it assumes, the information in the k-space, as uniformly distributed. We can eliminate the drawbacks of the above two approaches by combining both the radial and Cartesian sampling techniques. In this approach, the central region of k-space data (Figure 4) is sampled using radial approach and the peripheral region is sampled using cartesian approach. As it can be seen from the Figure 5, the central region of k-space will get processed frequently and the distance between the samples is also very less. Here, the peripheral regions are also getting processed better when compared with a radial sampling approach.

IV. Algorithm for Hybrid approach

It has been proved [13] that the most of the information in k-space lies up in *region* $-k/4 \leq x \leq k/4$ and $-k/4 \leq y \leq k/4$ as indicated in Figure 4. There is an assumption that the centroid of k-space data is the origin of 2D coordinates. Algorithm for hybrid sampling is presented as follows:

Algorithm 1: Hybrid Sampling Algorithm:

Given: S_1 : Cartesian sampled raw data, S_2 : Radial Sampled raw data, S : k-space data which is used to represent pictorial view of k-space, RI : Reconstructed image after applying hybrid sampling reconstruction algorithm

Input: S_1, S_2

Output: RI, S

1. Fix the extreme points for radial sampling as $-k/4$ and $+k/4$ at X and Y axes as shown in Figure 4. The rest of $k \times k$ space will be sampled in rectangular Cartesian form.
2. Start sampling from $(-k, k)$ to $(k, -k)$ in row by row order:
 - For $I = 0$ to $k-1$
 - For $J = 0$ to $k-1$
 - if $(-k/4 \leq I \leq k/4)$ and $(-k/4 \leq J \leq k/4)$: Reconstruct using radial sampling (S_2, I, J, S)
 - else: Reconstruct using cartesian sampling (S_1, I, J, S)
3. Image and k-space data can be displayed using RI and S .

(i) Reconstruction using radial sampling: [14]

Input: S_2, I, J, S

Output: Updated RI and Updated S Compute the image reconstructed

- 1) Each sample point is having area ' dk ', which can be represented in continuous domain as:

$$\hat{m}(r) = \int_{k-space} p(k) e^{i2\pi k \cdot r} dk \quad 2$$

$$\text{where } p(k) = \int_{FOV} m(r) e^{-i2\pi k \cdot r} dr \quad 3$$

Here, ' k ' represents the frequency domain point and the $r = (I, J)$ represents the spatial domain point and, $\hat{m}(r)$ is the proton density and $p(k)$ is the signal received. In the discrete domain, the above can be written, with a weighing function to account for the non-uniform sampling, as:

$$\hat{m}(r) = \sum W(k_{I,J}) p(k_{I,J}) e^{i2\pi k \cdot r} \quad 4$$

where $W(k_{I,J})$ is the weighing function

Weighing function (refer [3]) is used for determining the non-uniform sampling and it is used to compensate the data points from radial lines into grids. Without the weighing, the radial sampled data cannot be processed further. The uniform distribution of spatial frequency in k-space data can be obtained by multiplying weighing function to the partial k-space in the equation above.

- 2) Update $RI(I, J)$ with $\hat{m}(r)$, which is the contribution of k-space $p(k_{I,J})$ to the image, $RI(I, J)$.

3) Update S with the grinding procedure [5] applied to $p(k_{i,j})$

(ii) *Reconstruction using cartesian sampling: [15]*

Input: S_2, I, J, S

Output: Updated RI and Updated S

1) All the sample points S_1 are equidistant from each other. They all are in the form of Cartesian ordinate space; the proton density (image) can be given as follows:

$$\hat{m}(r) = \sum p(k_{i,j}) e^{i2\pi k \cdot r} \quad 5$$

where parameters are same as equation -4

2) Update RI(I, J) with $\hat{m}(r)$ which is the contribution of k-space $p(k_{i,j})$ to the image, RI(i,j).

3) Update S with $p(k_{i,j})$

V. Experiment

The experiment is performed using *Matlab*, 7.10.0.499 (R2010a), 32-bit (*glnxn86*) running under Linux Operating System (ubuntu 32-bit, kernel 3.13.0.46-generic). The cartesian sampled raw data can be downloaded from <https://github.com/zacination/KickSat-AsteroidDemo/blob/master/Matlab%20Code/RawData.mat> and the radial sampled raw data from MRI machine can be downloaded from <https://github.com/zacination/KickSat-AsteroidDemo/blob/master/Matlab%20Code/rawdata.mat>. The simulation of Cartesian sampling, radial sampling, and hybrid sampling (the proposed approach) are performed using the downloaded raw data (both cartesian and radial sampled data). The experimental results of reconstruction with various sampling approaches considered are given in Figure 6, Figure 7, and Figure 8, respectively, for cartesian sampling, radial sampling and hybrid sampling. The corresponding images for all the k-space data are also given. It can be observed from the pictorial representations of k-space that the central region varies in all the sampling approaches.

Figure 6 Cartesian sampling image reconstruction

(a) Gives the pictorial view of k-space obtained after cartesian sampling, (b) Red color portion represents the high intensity, black dots indicate the uniform distribution of samples and yellow area represents the intensity of sample points. It can be seen that all the region near to center and away from center has uniform distributed samples, (c) Represents the final reconstructed image from this k-space.

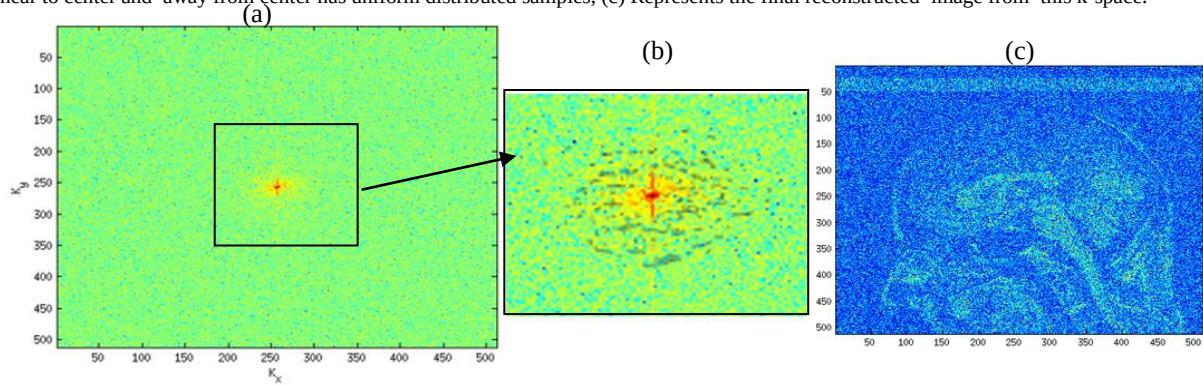


Figure 7 Radial sampling image reconstruction

(a) Gives the pictorial view of k-space obtained after radial sampling, (b) Red color portion represents the high intensity, black dots indicate the uniform distribution of samples and yellow area represents the intensity of samples points. It can be seen that the center region is denser than peripheral regions, (c) Represents the final reconstructed image from this k-space.

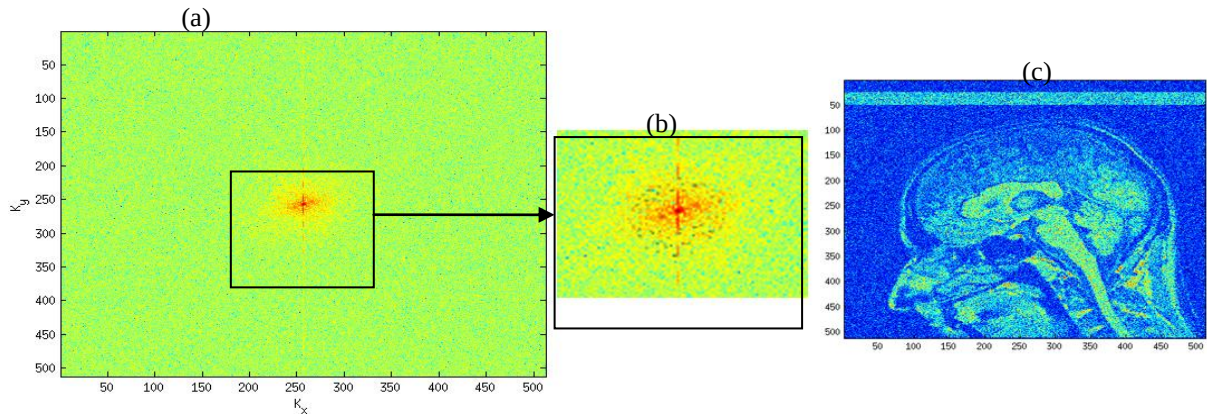
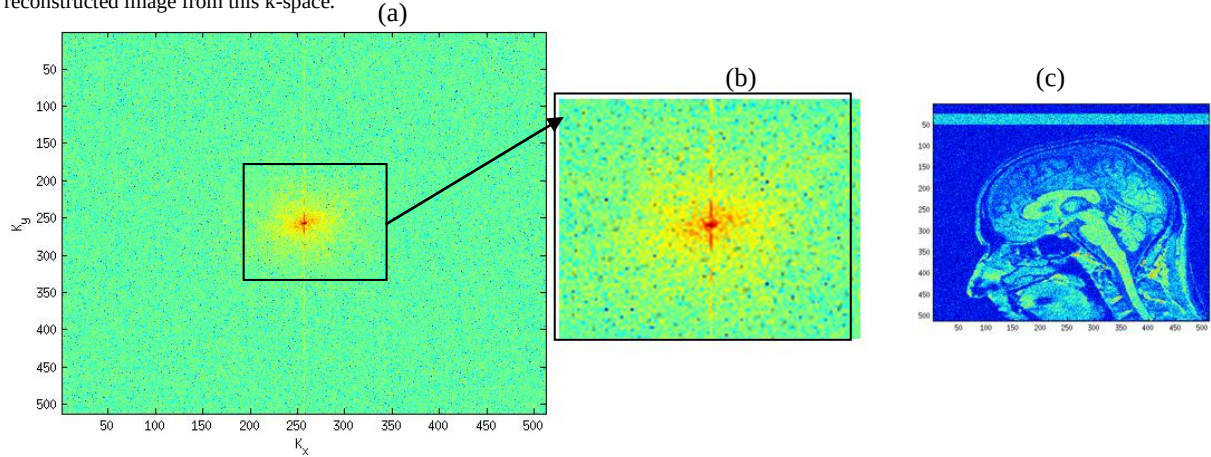


Figure 8 Hybrid sampling image reconstruction

(a) Gives the pictorial view of k-space obtained after hybrid sampling, (b) Red color portion represents the high intensity, black dots indicate the uniform distribution of samples and yellow area represents the intensity of sample points. It can be seen that the samples are denser at central region due to radial sampling and there is also better sampling at peripheral region due to cartesian sampling, (c) Represents the final reconstructed image from this k-space.



From the pictorial view of the results, it can be observed that the image reconstructed from k-space obtained using hybrid sampling provides better quality images. The objective evaluation and comparative analysis of hybrid sampling with radial sampling, and cartesian sampling are presented in the next section.

V. Performance Evaluation and Comparative analysis

There are various measures for evaluating the quality of reconstructed images. This paper uses the Root Mean Square (RMS), Mean Absolute Error (MAE), and Peak Sound to Noise ratio (PSNR) for evaluating the quality of the reconstructed image. The various performance measures obtained for Cartesian sampling, radial sampling, and hybrid sampling are presented in Table 1. The performance of different approaches is graphically represented as in Figures 9. It is observed that the proposed radial approach is superior in terms of MSE, MAE and PSNR.

Mean Squared Error (MSE): MSE is an average of the square of the absolute error.

$$MSE = \frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{n}$$

where y_i is the actual value, $\hat{y}_i$ is the experimental value or predicted value at the i^{th} observation.

Mean Absolute Error (MAE): MAE is an average of the absolute error.

$$MAE = \frac{\sum_{i=1}^N |p_i - q_i|}{N}$$

where p_i is the actual value, q_i is the experimental value or predicted value at the i^{th} observation.

Peak Signal to Noise ratio (PSNR): Peak signal-to-noise ratio (in dB) is the ratio of maximum intensity (power) of pixel (signal) at a sample point to the intensity (power) of corrupting noised image.

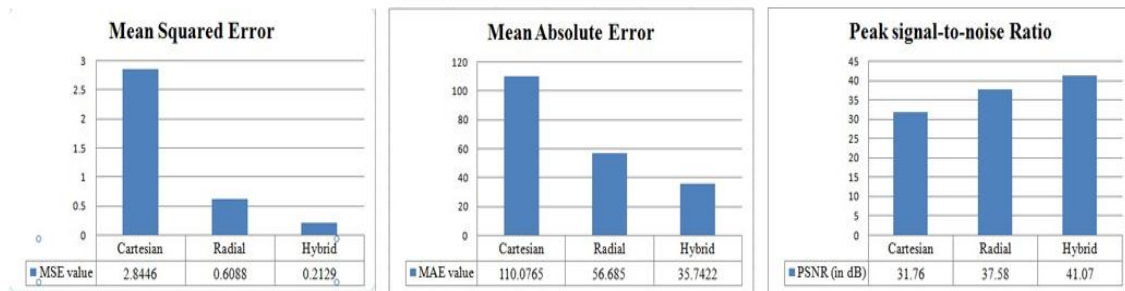
$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX^2}{MSE} \right)$$

where MSE is the mean squared error, MAX is the maximum pixel intensity.

Table -1: Performance Evaluation

Sampling	MSE	MAE	PSNR (in dB)
Cartesian	2.8446	110.0765	31.76
Radial	0.6088	56.6850	37.58
Hybrid	0.2129	35.7422	41.07

Figure 9 Comparative Analysis of hybrid sampling with radial and cartesian sampling in terms of MSE, MAE and PSNR



From the bar diagram of MSE, MAE and PSNR, it can be seen that the MSE and MAE of our proposed sampling approach provides low error values when compared to Cartesian and Radial sampling approaches. Also, the PSNR for the proposed sampling approach is higher than that for Cartesian and radial sampling techniques.

VII. Conclusion

This paper proposes a hybrid sampling technique for improving the quality of MRI reconstructed images. The raw data acquisition of k-space has been briefly discussed. The various features of k-space acquisition like phase encoding, phase shift, frequency encoding have been analyzed. The drawback of cartesian sampling is that it assumes uniformly distribution of k-space. The radial sampling of k-space gives better reconstructed images, but has few disadvantages like the small structures of objects are not properly reconstructed. The disadvantages of cartesian and radial sampling are eliminated by introducing the proposed hybrid sampling technique. The algorithm for hybrid sampling technique is implemented and the performances are compared with cartesian and radial sampling. The results demonstrate that the hybrid technique provides better performance figures in terms of PSNR, mean squared error and mean absolute error when compared to Cartesian and radial approaches.

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