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Bezier Surface Reconstruction using Artificial Neural Networks

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Abstract: Surface Reconstruction forms a key to many image processing and computer graphics applications. In the current work we have focused on the reconstruction of Bezier surfaces from a point cloud. The reconstruction is materialized using the learning and generalization capabilities of Radial Basis Function Neural Networks (RBFNN). The experimental results are shown to establish that RBF networks perform better than the traditional surface fitting methods. The reconstructed surface is a smooth surface approximating the given set of data points. The learning capabilities of RBF networks make surface reconstruction robust to noise and irregularities in sampling of the data.

Keywords: Surface Reconstruction, Point Cloud, Radial Basis Function Neural Networks, Surface Fitting.

I. Introduction

Modelling of curves and surfaces is a difficult and challenging problem to solve. Mathematics and numerical analysis forms the very base of curve and surface reconstruction problems. The advent of computer has given ease to the modelling and simulation of curves and surfaces. This problem is not as trivial as it seems. The reason is that the shapes can be easily identified by human eyes but for a computer it is very difficult to establish proper connectivity between the data points and also maintain the correct topology of the desired shape. In general we can define the modeling of curves and surfaces as the process of estimating the values of a curve or a surface at any location [1]. If the data is without noise then we can apply various interpolation techniques to generate the curve or surface. But practically the data is contaminated with noise due to various reasons, may be when the data is transmitted or when it is acquired through scanners.

Soft computing techniques have emerged to be more practical and flexible approaches towards the modeling of curves and surfaces. These methods have become popular due to their capability of handling imprecise and uncertain data. In the current work we have used the learning capabilities of a Radial Basis Function (RBF) neural network which was first introduced by Broomhead and Lowe [2]. A detailed survey of RBF neural networks for curve fitting and interpolation problems is given in [3, 4]. A B-Spline surface approximation using RBF networks is given by Liu [5]. The performance comparison of RBF neural networks to Back-propagation neural networks and generalized regression networks for function approximation is given in [6]. Comparative analysis of Radial basis function and Back-propagation neural networks for reconstruction of noisy Bezier curves is given in [7, 8]. In the presented work we have focused on the reconstruction of 3d-Bezier surfaces from dense point cloud. Bezier surfaces are frequently used in design work as they have the advantage of easily describing the complex and random shapes. Moreover these surfaces are invariant under affine transformations whether they are applied to control points or to the surface. Some other surfaces are also reconstructed and the results are presented. The results show that the reconstructed surface is a smooth surface that approximates well the given set of noisy data. The data in the input cloud is assumed to be non-uniformly distributed. So in general the surfaces are reconstructed from uniform as well as non-uniform data set.

II. Theoretical Concepts

A. Bezier Surface

Bezier surfaces are a class of surfaces which are suitable for ab-initio designs and also to the designing of free form surfaces. Given by Pierre Bezier a 3d-Bezier surface is defined by the control points and by using the basis function. The main properties of a Bezier surface are [9]:

- The basic functions are nonnegative.
- The sum of the basis functions is 1.
- The degree of the surface in a single parametric direction is one less than the number of defining polygon vertices in that direction.
- The continuity of the surface in a single parametric direction is two less than the number of defining polygons in that direction.

- The surface follows the general shape of the defining control polygon.
- Only the four corner points of the control polygon and the parameterized surface must be shared, coincident, or interpolated.
- The surface lies within the convex hull defined by the control points.
- The surface is invariant when affine transforms are applied to either the control points or parameterized surface.

A Bezier surface is given as [9]:

$$P(s,t) = \sum_{i=0}^n \sum_{j=0}^m B_{i,j} J_{n,i}(s) K_{m,j}(t) \quad 0 \leq s \leq 1$$

where $B_{i,j}$ represents the vertices of the defining polygon and $J_{n,i}(s)$, $K_{m,j}(t)$ are Bernstein basis functions in the parametric directions s and t , and are given by the following equations:

$$J_{n,i}(s) = \binom{n}{i} s^i (1-s)^{n-i}$$

$$K_{m,j}(t) = \binom{m}{j} t^j (1-t)^{m-j} \quad \text{with}$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}$$

$$\binom{m}{j} = \frac{m!}{j!(m-j)!}$$

B. Radial Basis Function Neural Networks (RBFNN)

RBF networks are a class of feed-forward networks which have only one hidden layer and the neurons in the hidden layer are called radial basis neurons. In general a RBF has three layers- input layer, hidden layer and output layer. The mapping between the input layer and hidden layer is non-linear whereas the mapping between the hidden layer and the output layer is linear. Normally a Gaussian function is used in the hidden layer as a Radial Basis Function. It is given as:

$$f_j = \exp\left(-\frac{\|X - c_j\|^2}{2\sigma^2}\right)$$

Where f_j depicts the output of the j^{th} radial basis neuron, x represents the input vector, c_j is the center of the j^{th} hidden neuron and σ is the parameter spread which determines the smoothness and it needs to be optimized for specific problem. The number of hidden layer neurons is not necessarily equal to the number of inputs. If the number of hidden layer neurons is less, than the accuracy of the results is affected and if it is more than that increases the complexity of the process and may affect the performance in terms of global generalizations of the network. Various clustering approaches are used to reduce the number of centers in the hidden layer of neurons and thus selecting the appropriate centers for modelling the data.

III. Simulation Results

In this section the simulation results are shown for various surfaces. Standard interpolation methods are used to approximate a point cloud and also the results of using RBF networks are presented. Various surfaces are reconstructed and the experimental results are shown in order to describe the approximation capabilities of RBF neural networks. Initially a Bezier surface is generated using sixteen control points. The control points which are considered are given in the form of matrices; first matrix gives the x-coordinates, second matrix gives the y-coordinates and third matrix gives the z-coordinates. The point cloud is generated experimentally by adding noise to the points of the surface. The noise level taken varies from 0.1 to 0.5. The noise is added to z-coordinate of the surface. The parameter in the RBF network is spread which is problem dependent. Spread basically determines the smoothness of the curve or the surface. RBF network is trained with the noisy data as the training set and the original surface is reconstructed using the original data set points which are without noise, noisy data points and the training output of the network. Fig.2 (a-e) shows the results for the Bezier surface. Control points, control polygon and the corresponding Bezier surface are shown in fig.2 (a), fig.2 (b) shows the surface. Noisy data of points is shown in fig.2(c), the noise of 0.1 being added to z-coordinate. Reconstructed surface using simple interpolation and RBFNN are shown in fig.2 (d) and fig.2 (e) respectively. This surface is generated using the following control points (Any set of control points can be taken to generate an initial Bezier Surface):

$$B(0,0) = (-5,1,-6), B(0,1) = (4,1,-3), B(0,2) = (-8.5,-1,1), B(0,3) = (-7,2,5);$$

$$B(1,0) = (-2,3,-6), B(1,1) = (-2.5,2,-3), B(1,2) = (-4.5,0,3), B(1,3) = (-3,4,6.5);$$

$$B(2,0) = (2,3,-6), B(2,1) = (2.5,2,-3), B(2,2) = (4.5,0,3), B(2,3) = (3,4,6.5);$$

$$B(3,0) = (5,1,-6), B(3,1) = (4,1,-3), B(3,2) = (8.5,-1,1), B(3,3) = (7,2,5).$$

Figure 3(a-d) shows the results for the surface defined by an explicit function: $z = 3/(x^2 + y^2)$ and in fig. 4(a-d) results of reconstruction of surface defined by $z = x \cdot \exp\{-(x^2 + y^2)\}$. The spread parameter is adjusted in each of the surface reconstruction in order to obtain a smooth surface. In first case i.e. for a Bezier Surface spread is taken as 1 whereas for the second and third surfaces spread is taken to be 0.3. The hidden neurons are taken

equal as the number of inputs. All the simulations are performed using MATLAB R2013a on Windows 7 platform.

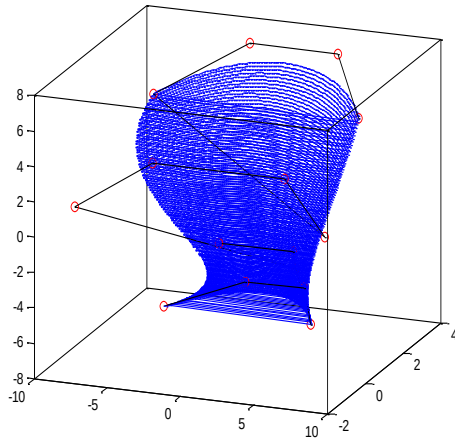


Fig.2 (a). Bezier Surface with Control points and Control Polygon (Box view) .

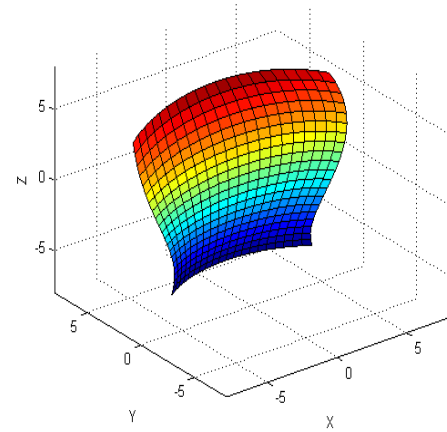


Fig. 2 (b). Bezier Surface

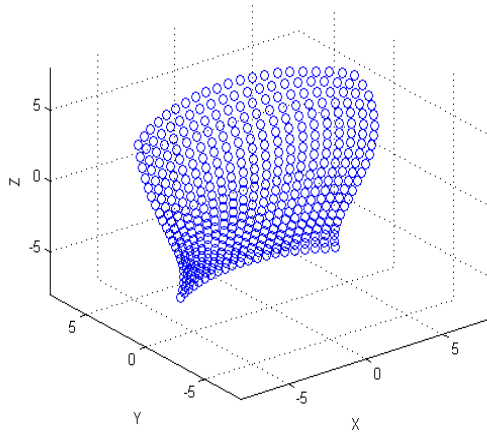


Fig. 2(c). Noisy data points of the Bezier surface

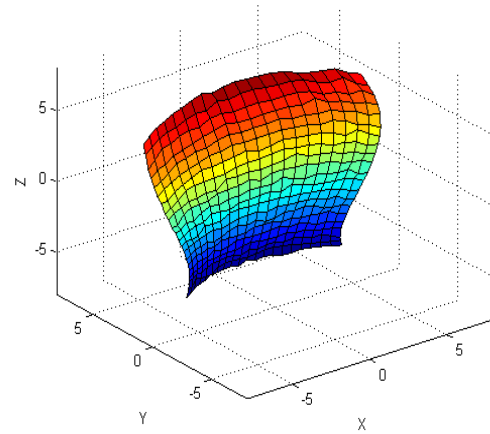


Fig. 2 (d). Reconstructed Bezier Surface from the noisy data using traditional interpolation

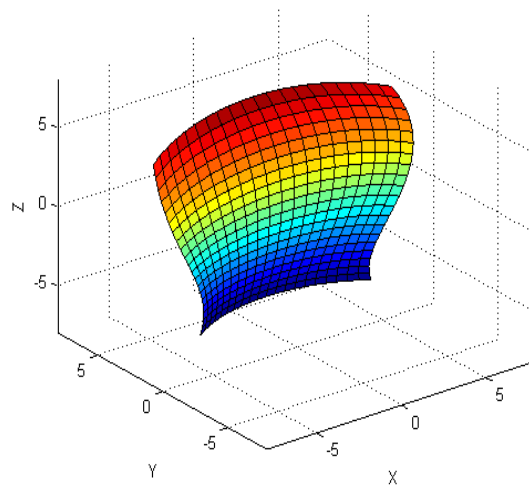


Fig. 2(e). Reconstructed Bezier Surface from the noisy data using RBF Neural Networks
Results of the surface given by $z = 3/(x^2 + y^2)$ are shown as below in Fig. 3(a-d)

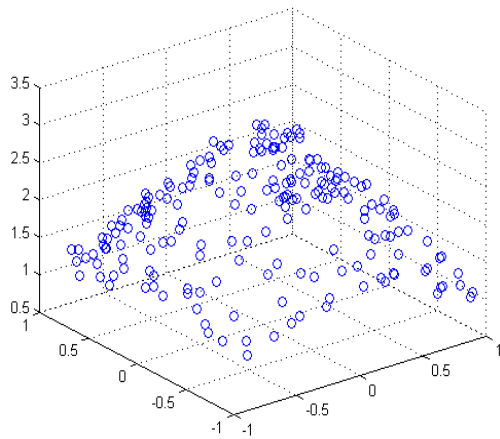


Fig. 3(a). Noisy data points of a surface with noise 0.1.

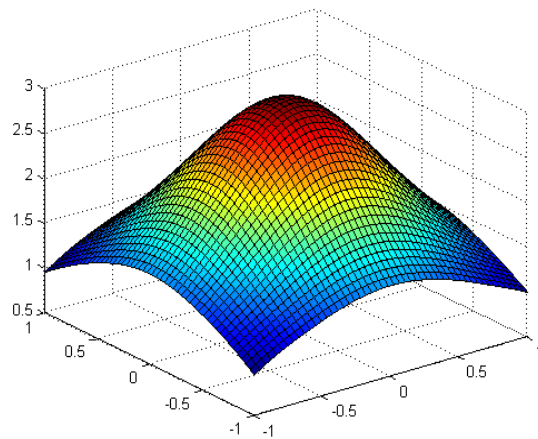


Fig. 3(b). Original Surface.

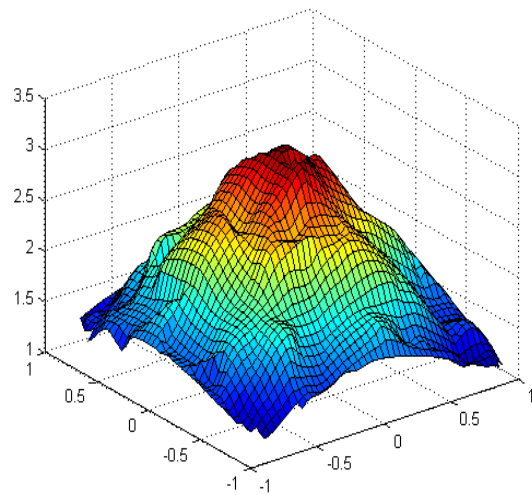


Fig.3(c). Reconstructed Surface using Cubic Interpolation.

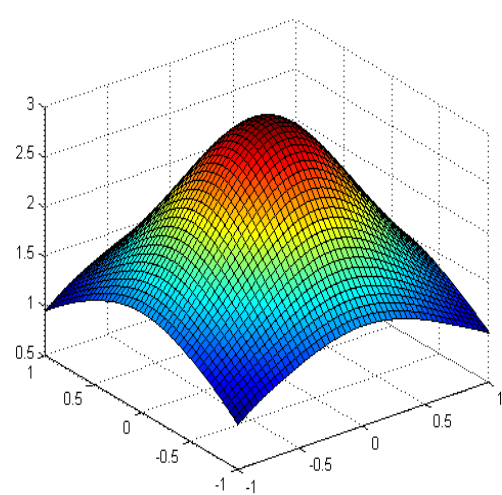


Fig.3(d). Reconstructed Surface using RBF Neural Networks.

Results of the surface given by $z = x \cdot \exp\{-(x^2 + y^2)\}$ are shown as below in Fig. 4(a-d)

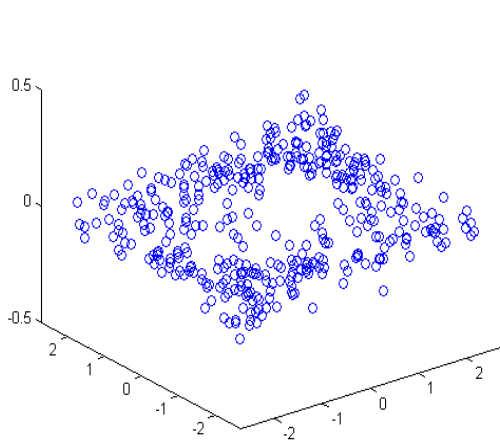


Fig.4(a). Noisy data points of a surface with noise 0.1.

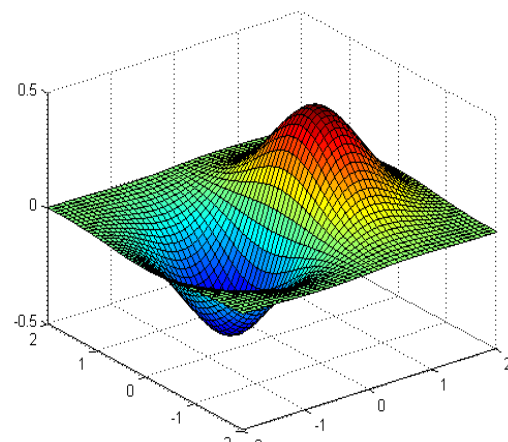


Fig. 4(b). Original Surface.

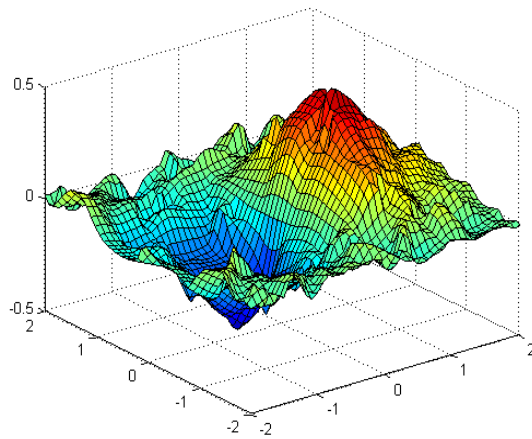


Fig.4(c). Reconstructed Surface using Cubic Interpolation.

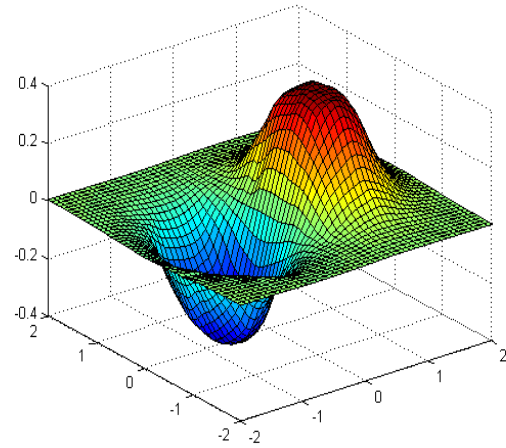


Fig. 4(d). Reconstructed Surface using RBF Neural Networks

IV Conclusions

In this paper we have reconstructed various surfaces including a Bezier Surface from the point cloud. The learning capabilities of Radial Basis Function are exploited to learn the patterns in the data and finally reconstruct the original surface. A comparison of traditional surface fitting to the RBF based reconstruction is shown for the noisy data set. The results show that the surface reconstructed using RBF networks is a good approximation to the original surface and also the learning/training speed of the network is fast. The simulation results are shown for reconstructed surface by using cubic interpolation and RBF networks. However in the current approach we have used the entire data set for training the network, the future work would focus on reducing the training data to a set of few feature points.

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