

Anisotropic Bianchi Type-II Cosmological Model with Varying Λ

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Abstract: We consider a locally rotationally symmetric (LRS) Bianchi type II model with a stiff fluid and a variable cosmological constant Λ . To solve the Einstein field equations we consider the cosmological term Λ to be proportional to H where H being Hubble parameter. The physical significance of the cosmological model is also discussed.

Keywords: LRS Bianchi type II models, Cosmological constant, stiff Fluid, Hubble's parameter.

I. Introduction

In recent year there have been considerable interests in cosmological constant problem in cosmology. Observations indicate that $\Lambda \sim 10^{-55} \text{ cm}^{-2}$ while the particle physics predication for Λ is greater than this value by a factor of order 10 to the power 120. This discrepancy is known as the cosmological constant problem. The simplest way out of this problem is to consider a varying cosmological term, which decays from a huge value at initial times to the small value observed in these days in an expanding universe [1-4]. Several phenomenological models have been suggested by considering Λ as a function of time [5-17], of special interest is the ansatz $\Lambda \propto \dot{a}^2$ where a is the scale factor of the Robertson- Walker metric [4], which has been considered/modified by several authors [5,8,18-20]. However, not all vacuum decaying cosmological models can predict acceleration. Researchers [21,22] have proposed a cosmological model with a cosmological constant of the term $\Lambda \propto H$ and $\Lambda \propto a^{-m}$, where a is the scale factor of the universe and m is constant.

Bianchi type-II space time has a fundamental role in constructing cosmological models suitable for describing the early stages of the evolution of the universe. Stiff fluid cosmological models create more interest in the study because for these models, the speed of light is equal to speed of sound and its governing equations have the same characteristics as those of gravitational field. Barrow has discussed the relevance of stiff equation of state $\rho = p$ to the matter content of the universe in the early state of evolution of universe. In this paper a new anisotropic LRS Bianchi type II stiff fluid cosmological model with variable Λ has been investigated by assuming a condition Λ proportional to the Hubble's parameter i.e. $\Lambda \propto H$. Cosmological consequences of the model are also discussed.

II. The Metric and Equations

We consider the LRS Bianchi type II metric in the form

$$ds^2 = -dt^2 + A^2(dy + xdz)^2 + B^2(dx^2 + dz^2) \quad (1)$$

Where A and B are functions of t only.

The energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} \quad (2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$

$$v_i v^i = 1 \quad (3)$$

We take the equation of state

$$p = \gamma \rho, \quad 0 \leq \gamma \leq 1 \quad (4)$$

Einstein's field equations with time dependent Λ and $8\pi G = C = 1$ given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} - \Lambda g_{ij} \quad (5)$$

For line-element (1) and energy momentum tensor (2) commoving system of coordinates, the field equations (5) yields.

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{A^2}{4B^4} = \rho - \Lambda \quad (6)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{3A^2}{4B^4} = -p - \Lambda \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{AB}{AB} + \frac{1A^2}{4B^4} = -p - \Lambda \quad (8)$$

The average scale factor $R(t)$ is defined as

$$R(t) = (AB^2)^{\frac{1}{3}} \quad (9)$$

The Hubble parameter and deceleration parameter are respectively defined as

$$H = \frac{\dot{R}}{R}, \quad q = -\frac{\ddot{R}}{\dot{R}^2} R \quad (10)$$

Hereafter a dot indicates an ordinary differentiation with respect to cosmic time t .

The expansion scalar θ and shear tensor σ^2 are defined as

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \quad (11)$$

$$\sigma^2 = \frac{1}{2} \left(\frac{A^2}{A^2} + 2\frac{B^2}{B^2} \right) - \frac{1}{6} \theta^2 \quad (12)$$

$$= \frac{1}{3} \left(\frac{A}{A} - \frac{B}{B} \right)^2$$

III. Solutions of the Field Equations

The field equations (6)-(8) are a system of three equations with five unknown parameter A , B , p , ρ , Λ . and two additional constraints relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion scalar θ in the model is proportional to the shear σ . This condition leads to

$$A = B^n \quad (13)$$

where n is constant.

We also assume that the fluid obeys the stiff fluid equation of state, i.e.

$$p = \rho \quad (14)$$

Eliminating ρ from equations (6) and (8), we get

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = -2\Lambda \quad (15)$$

Without loss of generality taking $n = 1$,

From equation (13), we get

$$\frac{2\dot{B}^2}{B^2} + \frac{\ddot{B}}{B} = -\Lambda \quad (16)$$

From equation (9), we get

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} = -\Lambda \quad (17)$$

We assume that the cosmological constant Λ proportional to the Hubble parameter, i.e.

$$\Lambda \propto H$$

$$\Lambda = aH = a\frac{\dot{R}}{R} \quad (18)$$

where a is constant.

Form equation (17), we get

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{a\dot{R}}{R} = 0$$

$$\frac{\frac{d}{dt}\left(\frac{\dot{R}}{R}\right)}{\frac{\dot{R}}{R}} + 3\frac{\dot{R}}{R} + a = 0$$

or,

On integrating we get

$$\log\left(\frac{\dot{R}}{R}\right) + 3\log R + at = \log c_1$$

$$R = \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{\frac{1}{3}}$$

or,

(19)

From equation (10), we get

$$H = c_1 \exp(-at) \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-1} \quad (20)$$

From equation (18), we get

$$\Lambda = ac_1 \exp(-at) \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-1} \quad (21)$$

$$A = \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{\frac{1}{3}} \quad (22)$$

$$B = \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{\frac{1}{3}} \quad (23)$$

From equation (6), we get

$$\rho = 3c_1^2 \exp(-2at) \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-2} - \frac{1}{4} \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-\frac{2}{3}}$$

$$+ ac_1 \exp(-at) \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-1}$$

$$\theta = 3c_1 \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-1}$$

$$\sigma^2 = \frac{-\sqrt{3}c_1}{3} \left(-3\frac{c_1}{a}\exp(at) + 3c_2\right)^{-1}$$

IV. Discussion

We observe that at $t=0$, the parameter $(R, H, \Lambda, \rho, \sigma^2)$ are all constant and as t increases the scale factor R increases exponentially whereas the parameters $(H, \Lambda, \rho, \sigma^2)$ decreases. As t leads to infinite, the scale factor R become infinitely large whereas the parameters $(H, \Lambda, \rho, \sigma^2)$ converge to zero. This shows that the universe in the model has no initial singularity. The universe starts evolving with constant volume and expands at an exponential rate. Also $\frac{\rho}{\theta}$ constant at $t=0$. This shows that the matter is dynamically negligible near the origin. It can also be seen that the ratio $\frac{\sigma}{\theta} \rightarrow 0$ as $t \rightarrow \infty$. Thus universe in the model approaches isotropy at late times. It is also observe that the deceleration parameter q in the model is negative which indicates that the universe in the model is accelerating.

V. Conclusion

In this paper we have studied the LRS Bianchi type-II stiff fluid cosmological model with varying Λ term. The field equations have been solved exactly by using cosmological term Λ to be proportional to H with H being Hubble parameter. It is interesting that the proposed variation law provides an alternative approach to obtain

exact solution of Einstein's field equations. It presents a unified description of the evolution of the universe which starts with a decelerating expansion and expands with acceleration at late times. Recent observational data [24-27] strongly suggest that this acceleration as well as the cosmological constant $\Lambda(t) \propto 1/t^2$ follows the model in Refs. [12,23,28]. In the present model we obtain $\Lambda \sim H^2, \Lambda \sim R/\ddot{R}$ and $\Lambda \sim t^{-2}$, in accordance with the main dynamical laws for the decay of Λ . This form of Λ is physically reasonable because the observation suggests that Λ is very small in the present universe. Finally the present solution is new and useful for better understanding of the evolution of the universe in Bianchi type-II space time with variable Λ .

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