



International Journal of Emerging Technologies in Computational and Applied Sciences (IJETCAS)

www.iasir.net

Estimation of Inputs for a Desired Output of a Cooperative and Supportive Neural Network

¹P. Raja Sekhara Rao, ²K. Venkata Ratnam, ³P.Lalitha

¹Department of Mathematics, Government Polytechnic, Addanki - 523 201, Prakasam, A.P., INDIA.

²Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad campus, Jawahar Nagar, Hyderabad-500078, INDIA.

³Department of Mathematics, St. Francis College for Women, Begumpet, Hyderabad, INDIA.

Abstract: In this paper a cooperative and supportive neural network involving time delays is considered. External inputs to the network are allowed to vary with respect to time. Asymptotic behavior of solutions of network system with variable inputs is studied with respect to its counterpart of constant inputs. With suitable restrictions on the inputs, it is noticed that solution of the network may be made to approach a pre-specified output.

Keywords: Co-operative and Supportive Neural Network, Variable Inputs, Desired Output, Convergence.

I. Introduction

This paper deals with the study of influence of time varying exogenous inputs on a cooperative and supportive neural network. A model of a cooperative and supportive network (CSNN, for short) is introduced by Sree Hari Rao and Raja Sekhara Rao [9]. It takes into account the collective capabilities of neurons involved with tasks divided and distributed to sub networks of neurons. Applications to such networks are many, for example, in industrial information management (hierarchical systems) which involve distribution and monitoring of various tasks. They are also useful in classification and clustering problems, data mining and financial engineering [6,7,8]. They are also utilized for parameter estimation of auto regressive signals and to decompose complex classification tasks into simpler subtasks and solve them.

In a recent paper [11], the authors considered time delays in transmission of information from sub-networks to main one as well as in processing of information in sub-network itself (before transmission of information to main network). Qualitative properties of solutions of the system are studied. Sufficient conditions for global asymptotic stability of equilibrium pattern of the system are established even in the presence of time delays. In the present paper, we wish to consider the CSNN model of [11] with time delays to study the influence of time varying inputs on the system. The motivation for this study stems from the observations of [10] that the applicability a neural network may be increased by the choice of inputs and inputs play a key role in attaining desired outputs. Proper choice of could be an alternative for modifying the neural network for each application and existing neural network may be utilized for different tasks, thus. Besides this, the presence of time varying inputs make the system non autonomous and the study enriches the literature. Mathematical studies of neural networks have been concentrated on stability of equilibrium patterns. Equilibria are stationary solutions of the system and correspond to memory states of the network. Stability of an equilibrium implies a recall of memory state. Thus, such stability analysis of neural networks is confined to recall of memories only and we may not reach the desired output for which the network is intended. In the present study, we deviate from this recall of memories but look for ways of reaching a desired solution.

An attempt is made in [9] to explain briefly the influence of variable inputs on asymptotic nature of solutions of CSNN model. The present study extends this work. We concentrate on the interplay between the inputs and outputs of the network. For this, several results are established for estimation or restriction of inputs for getting a desired or pre-specified output and understand the behavior of solutions in the presence of variable inputs. The work also extends the study of [10] carried out for BAM networks. As remarked in [10], convergence to a desired output for a given output explained here should not be confused with convergence of output function of the network. Results are available in literature which consider time varying inputs in various directions [1-3,5,12] but our emphasis here is on utilization of these inputs to make solutions of system approach an a priori value of output. We reiterate that this is not yet another usual study on qualitative behavior of solutions of the system under the influence of variable inputs.

The paper is organized as follows. In Section 2, the model under consideration is explained. Asymptotic behavior of solutions and their relation with the solutions of corresponding system with constant inputs are discussed. Section 3 deals with the input-output trade-off. Estimates on inputs are provided for approaching a desired, preset output for the network. A discussion follows in Section 4.

II. The Model and Asymptotic Behavior

The following model is considered in [11],

$$\begin{aligned} x'_i &= -a_i x_i + \sum_{j=1}^n b_{ij} f_j(x_j) + \sum_{k=1}^{r_i} c_{iik} g_{i_k}(x_i, y_{i_k}(t - \tau_{i_k})) + I_i, i = 1, 2, \dots, n, \\ y'_{i_k} &= -c_{i_k} y_{i_k} + \sum_{l=1}^{r_i} d_{il} h_{i_l}(y_{i_l}(t - \tau_{i_l})) + J_{i_k}, k = 1, 2, \dots, r_i, 1 \leq r_i \leq n. \end{aligned} \quad (2.1)$$

In (2.1), x_i , $i=1,2,\dots,n$ denote a typical neuron in neural field X and y_{i_k} , $k = 1, 2, \dots, r_i$ denote a subgroup of neurons in another neuronal field Y and are attached to x_i . x_i 's may be considered to form the main group of neurons which are required to perform the task. y_{i_k} 's constitute a subgroup of neurons attached to each x_i to which x_i assigns some of its task. y_{i_k} 's support, coordinate and cooperate with x_i in completing the task. a_i and c_{i_k} are positive constants known as passive decay rates of neurons x_i and y_{i_k} respectively. b_{ij} and d_{il} are the synaptic connection weights and all these are assumed to be real or complex constants. c_{iik} denotes the rate of distribution of information between x_i and y_{i_k} . The weight connections connect the i th neuron in one neuronal field to the j th neuron in another neuronal field. The functions f_j , g_{i_k} and h_{i_l} are the neuronal output response functions and are more commonly known as the signal functions. The parameter $\tau_{i_k} > 0$ signifies the time delay in transmission of information from sub network neuron y_{i_k} to main network neuron x_i . The delay $\tau_{i_l} > 0$ in second equation represents the processing delays in the subsystems. I_i and J_{i_k} are exogenous inputs which are assumed to be constants in [11]. For more details of the terms and design of the CSNN, readers are referred to [9].

Introducing the time variables $I_i = I_i(t)$ and $J_{i_k} = J_{i_k}(t)$, $i = 1, 2, \dots, n$, in place of constant inputs I_i and J_{i_k} into the system (2.1), we get

$$\begin{aligned} x'_i &= -a_i x_i + \sum_{j=1}^n b_{ij} f_j(x_j) + \sum_{k=1}^{r_i} c_{iik} g_{i_k}(x_i, y_{i_k}(t - \tau_{i_k})) + I_i(t), \\ y'_{i_k} &= -c_{i_k} y_{i_k} + \sum_{l=1}^{r_i} d_{il} h_{i_l}(y_{i_l}(t - \tau_{i_l})) + J_{i_k}(t). \end{aligned} \quad (2.2)$$

The following initial functions are assumed for the system (2.2).

$$x_i(s) = \tilde{p}_i(s), \quad y_{i_k}(s) = \tilde{q}_i(s) \text{ for } s \in (-\tau, 0], \quad (2.3)$$

where $\tilde{p}_i, \tilde{q}_i$ are continuous, bounded functions on $(-\tau, 0]$ and $\tau = \max\{\tau_{i_k}, \tau_{i_l}\} \geq 0$. We assume that the response functions f_j, g_{i_k} and h_{i_l} satisfy conditions

$$|g_{i_k}(x_i, y_{i_k}) - g_{i_k}(\bar{x}_i, \bar{y}_{i_k})| \leq M_{1i_k} |y_{i_k} - \bar{y}_{i_k}| + M_{2i_k} |x_i - \bar{x}_i| \quad (2.4)$$

$$|f_j(x_j) - f_j(\bar{x}_j)| \leq p_j |x_j - \bar{x}_j| \quad (2.5)$$

$$|h_{i_l}(y_{i_l}) - h_{i_l}(\bar{y}_{i_l})| \leq q_{i_l} |y_{i_l} - \bar{y}_{i_l}| \quad (2.6)$$

where M_{1i_k}, M_{2i_k}, p_j and q_{i_l} 's are positive constants.

Under the conditions (2.4)-(2.6) on the response functions and with I_i and J_{i_k} bounded, continuous functions on $[0, \infty)$ it is not difficult to see that the system (2.2) possesses unique solutions that are continuable in their maximal intervals of existence ([11]).

Since (2.2) is non-autonomous it may not possess equilibrium patterns (constant solutions). A solution of (2.1) or (2.2) is denoted by $(x(t), y(t))$, where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $y(t) = (y_{1k}(t), y_{2k}(t), \dots, y_{n_k}(t))$ throughout.

Therefore, we study the asymptotic behavior of its solutions. We recall from [10] that two solutions (x, y) and $(\tilde{x}, \tilde{y})$ of the system (2.2) are asymptotically near if

$$\lim_{t \rightarrow \infty} (x(t), y(t)) - (\tilde{x}(t), \tilde{y}(t)) = 0.$$

In the following, we present results on asymptotic nearness of solutions of (2.2).

Our first result is

Theorem 2.1: For any pair of solutions (x, y) and $(\tilde{x}, \tilde{y})$ of (2.2), we have

$$\lim_{t \rightarrow \infty} |(x, y) - (\tilde{x}, \tilde{y})| = 0$$

provided $\tilde{A} = \min\{\bar{A}, \bar{B}\} > 0$ holds, where

$$\begin{aligned} \bar{A} &= \{\min [a_i - \sum_{j=1}^n |b_{ji}| p_j - \sum_{k=1}^{r_i} |c_{iik}| M_{2i_k}] > 0, i = 1, 2, \dots, n\} \\ \bar{B} &= \{\min [c_{i_k} - \sum_{l=1}^{r_i} |d_{il}| q_{i_l} - |c_{iik}| M_{1i_k}], 1 \leq i \leq n.\} \end{aligned} \quad (2.7)$$

Proof: Consider the functional,

$$V(t) = \sum_{i=1}^n \left[|x_i - \bar{x}_i| + \sum_{k=1}^{r_i} |y_{ik} - \bar{y}_{ik}| + \sum_{k=1}^{r_i} \left[|c_{ik}| M_{1ik} \int_{t-\tau_{ik}}^t |y_{ik}(s) - \bar{y}_{ik}(s)| ds \right. \right. \\ \left. \left. + \sum_{l=1}^{r_i} d_{il} \int_{t-\tau_{il}}^t |h_{il}(y_{il}(s)) - h_{il}(\bar{y}_{il}(s))| ds \right] \right]$$

Along the solutions of (2.2), the upper right derivative of V is given by

$$D^+V(t) \leq \sum_{i=1}^n \left[-a_i |x_i - \bar{x}_i| + \sum_{j=1}^n |b_{ij}| |f_j(x_j) - f_j(\bar{x}_j)| + \sum_{k=1}^{r_i} |c_{ik}| |g_{ik}(x_i, y_{ik}(t - \tau_{ik})) - g_{ik}(\bar{x}_i, \bar{y}_{ik}(t - \tau_{ik}))| \right. \\ \left. + \sum_{k=1}^{r_i} \left[-c_{ik} |y_{ik} - \bar{y}_{ik}| + \sum_{l=1}^{r_i} |d_{il}| |h_{il}(y_{il}(t - \tau_{il})) - h_{il}(\bar{y}_{il}(t - \tau_{il}))| \right] \right. \\ \left. + \sum_{k=1}^{r_i} |c_{ik}| M_{1ik} |y_{ik} - \bar{y}_{ik}| - \sum_{k=1}^{r_i} |c_{ik}| M_{1ik} |y_{ik}(t - \tau_{ik}) - \bar{y}_{ik}(t - \tau_{ik})| \right. \\ \left. + \sum_{k=1}^{r_i} \left[\sum_{l=1}^{r_i} |d_{il}| |h_{il}(y_{il}) - h_{il}(\bar{y}_{il})| - \sum_{l=1}^{r_i} |d_{il}| |h_{il}(y_{il}(t - \tau_{il})) - h_{il}(\bar{y}_{il}(t - \tau_{il}))| \right] \right] \\ \leq \sum_{i=1}^n \left[-a_i |x_i - \bar{x}_i| + \sum_{j=1}^n |b_{ij}| |p_j| |x_j - \bar{x}_j| + \sum_{k=1}^{r_i} |c_{ik}| M_{2ik} |x_i - \bar{x}_i| + \sum_{k=1}^{r_i} |c_{ik}| M_{1ik} |y_{ik}(t - \tau_{ik}) - \bar{y}_{ik}(t - \tau_{ik})| \right. \\ \left. + \sum_{k=1}^{r_i} \left[-c_{ik} |y_{ik} - \bar{y}_{ik}| + \sum_{l=1}^{r_i} |d_{il}| |h_{il}(y_{il}(t - \tau_{il})) - h_{il}(\bar{y}_{il}(t - \tau_{il}))| \right] \right. \\ \left. + \sum_{k=1}^{r_i} |c_{ik}| M_{1ik} |y_{ik} - \bar{y}_{ik}| - \sum_{k=1}^{r_i} |c_{ik}| M_{1ik} |y_{ik}(t - \tau_{ik}) - \bar{y}_{ik}(t - \tau_{ik})| \right. \\ \left. + \sum_{k=1}^{r_i} \left[\sum_{l=1}^{r_i} |d_{il}| |q_{il}| |y_{ik} - \bar{y}_{ik}| - \sum_{l=1}^{r_i} |d_{il}| |h_{il}(y_{il}(t - \tau_{il})) - h_{il}(\bar{y}_{il}(t - \tau_{il}))| \right] \right]$$

using (2.4)-(2.6).

$$D^+V(t) \leq \sum_{i=1}^n \left[\left[a_i - \sum_{j=1}^n |b_{ji}| p_j - \sum_{k=1}^{r_i} |c_{ik}| M_{2ik} \right] |x_i - \bar{x}_i| + \sum_{k=1}^{r_i} \left[c_{ik} - \sum_{l=1}^{r_i} |d_{il}| q_{il} - |c_{ik}| M_{1ik} \right] |y_{il} - \bar{y}_{il}| \right] \\ \leq -\tilde{A} \sum_{i=1}^n [|x_i(t) - \bar{x}_i(t)| + \sum_{k=1}^{r_i} |y_{ik}(t) - \bar{y}_{ik}(t)|] < 0.$$

Integrating both sides with respect to t,

$$V(t) + \tilde{A} \int_0^t \sum_{i=1}^n \left[|x_i(s) - \bar{x}_i(s)| + \sum_{k=1}^{r_i} |y_{ik}(s) - \bar{y}_{ik}(s)| \right] ds \leq V(0).$$

Thus, V (t) is bounded on $[0, \infty)$ and $\int_0^\infty [|x_i(s) - \bar{x}_i(s)| + \sum_{k=1}^{r_i} |y_{ik}(s) - \bar{y}_{ik}(s)|] ds < \infty$, for $i = 1, 2, \dots, n$. But $|x_i - \bar{x}_i|$ and $|y_{ik} - \bar{y}_{ik}|$ are also bounded on $[0, \infty)$. Hence, it follows that their derivatives are also bounded on $[0, \infty)$.

Therefore, $|x_i - \bar{x}_i|$ and $|y_{i_k} - \bar{y}_{i_k}|$, $i = 1, 2, \dots, n$ are uniformly continuous on $[0, \infty)$. Thus, we may conclude that $|x_i - \bar{x}_i|$ and $|y_{i_k} - \bar{y}_{i_k}| \rightarrow 0$ as $t \rightarrow \infty$ (e.g., [4]). This concludes the proof.

The following result provides conditions under which all solutions of (2.2) are asymptotic to the solutions of (2.1), which shows that for a proper choice of input functions the stability of system (2.1) is not altered by the presence of time dependent inputs.

Theorem 2.2: Assume that the parametric conditions (2.7) hold. Further, let inputs satisfy $\int_0^\infty \sum_{i=1}^n |\tilde{I}_i(t) + \sum_{k=1}^{r_i} \tilde{J}_{i_k}(t)| dt < \infty$ where $\tilde{I}_i(t) = I_i(t) - I_i$, $\tilde{J}_{i_k}(t) = J_{i_k}(t) - J_{i_k}$. Then for any solutions (x, y) of (2.2) and $(\bar{x}, \bar{y})$ of (2.1) we have $\lim_{t \rightarrow \infty} (x, y) = (\bar{x}, \bar{y})$

Proof: To establish this, we employ the same functional as in Theorem 2.1, that is,

$$V(t) = \sum_{i=1}^n \left[|x_i - \bar{x}_i| + \sum_{k=1}^{r_i} |y_{i_k} - \bar{y}_{i_k}| + \sum_{k=1}^{r_i} \left[|c_{i i_k}| M_{1 i_k} \int_{t-\tau_{i_k}}^t |y_{i_k}(s) - \bar{y}_{i_k}(s)| ds \right. \right. \\ \left. \left. + \sum_{l=1}^{r_i} |d_{i_l}| \int_{t-\tau_{i_l}}^t |h_{i_l}(y_{i_l}(s)) - h_{i_l}(\bar{y}_{i_l}(s))| ds \right] \right]$$

Proceeding as in Theorem 2.1 we get after a simplification and rearrangement

$$D^+V(t) \leq -\sum_{i=1}^n \left[|a_i - \sum_{j=1}^n |b_{ji}| p_i - \sum_{k=1}^{r_i} |c_{i i_k}| M_{2 i_k}| x_i - \bar{x}_i| + \sum_{k=1}^{r_i} [c_{i k} - \sum_{l=1}^{r_i} d_{i_l} q_{i_l} - |c_{i i_k}| M_{1 i_k}] |y_{i_k} - \bar{y}_{i_k}| \right] \\ + \sum_{i=1}^n |\tilde{I}_i(t) + \sum_{k=1}^{r_i} \tilde{J}_{i_k}(t)| \\ \leq -\tilde{A} \sum_{i=1}^n [|x_i(t) - \bar{x}_i(t)| + \sum_{k=1}^{r_i} |y_{i_k}(t) - \bar{y}_{i_k}(t)|] + \sum_{i=1}^n |\tilde{I}_i(t) + \sum_{k=1}^{r_i} \tilde{J}_{i_k}(t)|$$

Rest of the argument is same as that of Theorems 2.1, and hence, omitted. Thus, the conclusion follows.

We now recall from [11] that the system (2.1) has a unique equilibrium pattern (x^*, y^*) for any set of input vectors (I, J) provided the parameters satisfy,

$$a_i > \sum_{j=1}^n |b_{ji}| p_i + \sum_{k=1}^{r_i} |c_{i i_k}| M_{2 i_k}, \quad c_{i_k} > \sum_{l=1}^{r_i} |d_{i_l}| q_{i_l} + \sum_{k=1}^{r_i} |c_{i i_k}| M_{1 i_k}$$

Then we have,

Corollary 2.3: Assume that all the hypotheses of Theorem 2.2 are satisfied. Further if (2.1) possesses equilibrium pattern (x^*, y^*) then all solutions (x, y) of (2.2) approach (x^*, y^*) .

Proof: The result obviously follows from the observation that the equilibrium pattern (x^*, y^*) is also a solution of (2.1) and the choice $(\bar{x}, \bar{y}) = (x^*, y^*)$ in Theorem 2.2.

The following example illustrates the above results.

Example 2.4: Consider the following system having two neurons in X each supported by two neurons in Y involving time delays as given by

$$\begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} = - \begin{pmatrix} 1.9x_1 \\ 2.2x_1 \end{pmatrix} + \begin{pmatrix} 0.2 & 0.32 \\ 0.6 & 0.25 \end{pmatrix} \begin{pmatrix} f_1(x_1) \\ f_2(x_2) \end{pmatrix} + \begin{pmatrix} 0.23 & 0.44 \\ 0.31 & 0.26 \end{pmatrix} \begin{pmatrix} g_{1_1}(x_1, y_{1_1}(t - \tau_{1_1})) & g_{2_1}(x_1, y_{2_1}(t - \tau_{2_1})) \\ g_{1_2}(x_1, y_{1_2}(t - \tau_{1_2})) & g_{2_2}(x_1, y_{2_2}(t - \tau_{2_2})) \end{pmatrix} \\ + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \\ \begin{pmatrix} y'_{1_1} \\ y'_{1_2} \end{pmatrix} = - \begin{pmatrix} 1.5y_{1_1} \\ 2.0y_{1_2} \end{pmatrix} + \begin{pmatrix} 0.22 & 0.12 \\ 0.31 & 0.25 \end{pmatrix} \begin{pmatrix} h_{1_1}(y_{1_1}(t - \tau_{1_1})) \\ h_{1_2}(y_{1_2}(t - \tau_{1_2})) \end{pmatrix} + \begin{pmatrix} J_{1_1} \\ J_{1_2} \end{pmatrix} \\ \begin{pmatrix} y'_{2_1} \\ y'_{2_2} \end{pmatrix} = - \begin{pmatrix} 1.8y_{2_1} \\ 1.6y_{2_2} \end{pmatrix} + \begin{pmatrix} 0.5 & 0.2 \\ 0.3 & 0.1 \end{pmatrix} \begin{pmatrix} h_{2_1}(y_{2_1}(t - \tau_{2_1})) \\ h_{2_2}(y_{2_2}(t - \tau_{2_2})) \end{pmatrix} + \begin{pmatrix} J_{2_1} \\ J_{2_2} \end{pmatrix}.$$

Choose $f_i(x_i) = \tanh(x_i)$, $h_{i_k}(y_{i_k}) = \tanh(y_{i_k})$ and $g_{i_k}(x_i, y_{i_k}) = x_i + y_{i_k}$. Then $p_j = q_{i_k} = M_{1 i_k} = M_{2 i_k} = 1$, $i = 1, 2$, $k = 1, 2$.

Clearly conditions for both the existence of unique equilibrium and its stability are satisfied for any pair of constant inputs I_i, J_{i_k} .

Now choose $I_1(t) = I_1 + \frac{1}{1+t^2}$, $J_{i_1}(t) = J_{i_1} + e^{-t} \sin t$; $I_2(t) = I_2 + e^{-t}$, $J_{i_2}(t) = J_{i_2} + e^{-t}$

It is easy to see that since the condition $\int_0^\infty |\tilde{f}_i(t) + \tilde{f}_{i_k}(t)| dt < \infty$ holds, and we have

- (i). Conditions of Theorem 2.1 are satisfied and all solutions of the system are asymptotic to each other.
- (ii). Conditions of Corollary 2.3 are satisfied and all solutions of the system approach the equilibrium pattern of corresponding system with constant inputs.

III. Estimations on Inputs for a Pre-specified Output

In this section, we try to estimate our inputs, depending on the output given, that help the solutions to approach the given output. For an easy understanding of the concept, we avoid the complicated notation. We, therefore, rearrange our system (2.2) suitably. We use the following notation

For $\tau_{i_k} = 0$, (2.2) may be represented as

$$\begin{aligned} x'(t) &= -ax(t) + bf(x(t)) + c_1g(x, y) + I(t) \\ y'(t) &= -cy(t) + dh(y(t)) + J(t) \end{aligned} \quad (3.1)$$

in which

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t)), \quad y(t) = (y_{1_k}(t), y_{2_k}(t), \dots, y_{n_k}(t)), \quad a = (a_1, a_2, \dots, a_n), \quad c = (c_{1_k}(t), c_{2_k}(t), \dots, c_{n_k}(t)),$$

$$bf(x) = (\sum_{j=1}^m b_{1j}f_j(x_j), \sum_{j=1}^m b_{2j}f_j(x_j), \dots, \sum_{j=1}^m b_{nj}f_j(x_j))$$

$$c_1g(x, y) = \left(\sum_{k=1}^{r_i} c_{11_k}g_{1_k}(x_1, y_{1_k}), \sum_{k=1}^{r_i} c_{22_k}g_{2_k}(x_2, y_{2_k}), \dots, \sum_{k=1}^{r_i} c_{nn_k}g_{n_k}(x_n, y_{n_k}) \right)$$

$$dh(y) = \left(\sum_{l=1}^{r_i} d_{1l}h_{1l}(y_{1l}), \sum_{l=1}^{r_i} d_{2l}h_{2l}(y_{2l}), \dots, \sum_{l=1}^{r_i} d_{nl}h_{nl}(y_{nl}) \right)$$

$$I(t) = (I_1(t), I_2(t), \dots, I_n(t)) \text{ and } J(t) = (J_1(t), J_2(t), \dots, J_n(t)).$$

We assume that (η, δ) is the desired output of the network. Further that both η and δ are fixed with respect to t and are arbitrarily chosen. We now arrange (3.1) as

$$x'(t) = (x - \eta)' = -a(x - \eta) + b(f(x) - f(\eta)) + c_1(g(x, y) - g(\eta, \delta)) - a\eta + bf(\eta) + c_1g(\eta, \delta) + I(t)$$

$$y'(t) = (y - \delta)' = -c(y - \delta) + d(h(y) - h(\delta)) + J(t) - c\delta + dh(\delta) \quad (3.2)$$

Conditions on response functions (2.2) to (2.6) may be modified as $|f(x) - f(y)| \leq k|x - y|$, $|g(x, y) - g(\bar{x}, \bar{y})| \leq l|x - \bar{x}| + m|y - \bar{y}|$ and $|h(y) - h(\bar{y})| \leq n|y - \bar{y}|$ for some $l, m, n > 0$, denoting appropriate norm. We have

Theorem 3.1. Assume that the parameters of the system and the response functions satisfy the condition

$$0 < \alpha \equiv \min \{a - bk - c_1l, \quad c - c_1m - dn\} < \infty.$$

For an arbitrarily chosen output (η, δ) , the solutions of system (3.1) converge to (η, δ) provided the inputs satisfy either of the conditions

$$(i). \int_0^\infty [|I(s) - a\eta + bf(\eta) + c_1g(\eta, \delta)| + |J(s) - c\delta + dh(\delta)|] ds < \infty \text{ or}$$

$$(ii). \lim_{t \rightarrow \infty} (I(t), J(t)) = (a\eta - bf(\eta) - c_1g(\eta, \delta), \quad c\delta - dh(\delta)).$$

Proof: Let $V(x(t)) = |x(t) - \eta| + |y(t) - \delta|$. Then the upper right derivative of V along the solutions of (3.1), using (3.2), we have

$$\begin{aligned} D^+V &\leq -a|x - \eta| + b|f(x) - f(\eta)| + c_1|g(x, y) - g(\eta, \delta)| + |I(t) - a\eta + bf(\eta) + c_1g(\eta, \delta)| \\ &\quad -c|y - \delta| + d|h(y) - h(\delta)| + |J(t) - c\delta + dh(\delta)| \\ &\leq -a|x - \eta| + bk|x - \eta| + c_1l|x - \eta| + c_1m|y - \delta| + |I(t) - a\eta + bf(\eta) + c_1g(\eta, \delta)| \\ &\quad -c|y - \delta| + dn|y - \delta| + |J(t) - c\delta + dh(\delta)| \\ &\leq -\alpha V + |I(t) - a\eta + bf(\eta) + c_1g(\eta, \delta)| + |J(t) - c\delta + dh(\delta)|. \end{aligned}$$

This gives rise to

$$V' + \alpha V \leq |I(t) - a\eta + bf(\eta) + c_1g(\eta, \delta)| + |J(t) - c\delta + dh(\delta)|.$$

Rest of the argument is similar to that of Theorem 2.1 and invoking the condition (i) on inputs.

Again, it is easy to see from the last inequality above that

$$V(t) \leq [|I(t) - a\eta + bf(\eta) + c_1g(\eta, \delta)| + |J(t) - c\delta + dh(\delta)|][1 - e^{-\tilde{\beta}t}] + V(0)e^{-\tilde{\beta}t} \rightarrow 0$$

for large t using conditions (ii) on $I(t)$ and $J(t)$. Hence, in either of the cases, $V \rightarrow 0$ follows.

The proof is complete.

The following example illustrates the effectiveness of this result.

Example 3.2: Consider

$$\begin{aligned}x'(t) &= -6x(t) + 3 \tanh(x(t)) + 2(x(t) + y(t)) + I(t) \\y'(t) &= -4y(t) + 2 \tanh(y(t)) + J(t),\end{aligned}$$

Now choose $I(t) = (6\eta - 3 \tanh(\eta) - 2(\eta + \delta)) \left(1 + \frac{1}{1+t^2}\right)$ and $J(t) = (5\delta - 2 \tanh(\delta))(1 + te^{-t})$.

Clearly, for given (η, δ) we have all the conditions of Theorem 3.1 are satisfied, and hence, $(x, y) \rightarrow (\eta, \delta)$ for sufficiently large t .

We shall now consider the time delay system corresponding to (2.2),

$$\begin{aligned}x'(t) &= -ax(t) + bf(x(t)) + c_1g(x, y(t-\tau)) + I(t) \\y'(t) &= -cy(t) + dh(y(t-\tau)) + J(t)\end{aligned}\tag{3.3}$$

As has been done earlier for a given (η, δ) , we write (3.3) as

$$\begin{aligned}x'(t) = (x - \eta)' &= -a(x - \eta) + b(f(x) - f(\eta)) + c_1(g(x, y(t-\tau)) - g(\eta, \delta)) - a\eta + bf(\eta) \\&\quad + c_1g(\eta, \delta) + I(t) \\y'(t) = (y - \delta)' &= -c(y - \delta) + d(h(y(t-\tau)) - h(\delta)) + J(t) - c\delta + dh(\delta)\end{aligned}\tag{3.4}$$

We have

Theorem 3.3: Assume that the parameters of the system and the response functions satisfy the condition

$$0 < \min \{a - bk - c_1l, \quad c - c_1m - dn\} < \infty.$$

For an arbitrarily chosen output (η, δ) , solutions of system (3.3) converge to (η, δ) provided the inputs satisfy the condition $\int_0^\infty [|I(s) - a\eta + bf(\eta) + c_1g(\eta, \delta)| + |J(s) - c\delta + dh(\delta)|] ds < \infty$;

Proof: Employing the functional

$$\begin{aligned}V(x(t)) &= |x(t) - \eta| + |y(t) - \delta| \\&\quad + b \int_{t-\tau}^t |g(x, y(s)) - g(\eta, \delta)| ds + d \int_{t-\tau}^t |h(y(s)) - h(\delta)| ds\end{aligned}$$

using (3.4) and proceeding as in Theorem 2.1 and Theorem 3.1, the conclusion follows.

Since the conditions on parameters and input functions in Theorem 3.1 and 3.3 are the same, it may imply that delays have no effect on convergence here

Example 3.4: Consider

$$\begin{aligned}x'(t) &= -9x(t) + 4 \tanh(x(t)) + 3(x(t) + y(t-\tau)) + I(t) \\y'(t) &= -8y(t) + 3 \tanh(y(t-\tau)) + J(t),\end{aligned}$$

Now choose $I(t) = (9\eta - 4 \tanh(\eta) - 3(\eta + \delta))(1 + e^{-2t})$ and $J(t) = (8\delta - 3 \tanh(\delta))(1 + e^{-t})$.

Clearly, for given (η, δ) we have all the conditions of Theorem 3.1 are satisfied, and hence, $(x, y) \rightarrow (\eta, \delta)$ for sufficiently large t .

Remark 3.5: Now consider the system

$$\begin{aligned}x'(t) &= -ax(t) + bf(x(t)) + c_1g(x, y(t-\tau)) + I \\y'(t) &= -cy(t) + dh(y(t-\tau)) + J\end{aligned}\tag{3.5}$$

$I = a\eta - bf(\eta) - c_1g(\eta, \delta)$, $J = c\delta - dh(\delta)$ are constant inputs, given η, δ . It is easy to observe that (η, δ) is an equilibrium pattern of (3.5). Then from Corollary 2.4, we have, solutions of (3.3) approach (η, δ) whenever the variable inputs of (3.3) are well near those of (3.5) as specified in Corollary 2.4.

Thus, by varying the external inputs of the system in the parameter space defined by (η, δ) as specified by Theorems 3.1 and 3.3, the solutions of the network approaches pre-specified output (η, δ) .

IV. Discussion

In this article, we have extended the concept of approaching a desired output of a given network by suitable selection of inputs based on the given output for a cooperative and supportive neural network that was studied earlier for a BAM network ([10]). With the help of suitable Lyapunov functionals, results are established for asymptotic nearness and boundedness of solutions of the system also. It is noticed that inputs define a new space of equilibria for the network while they run through a space defined by output parameters. This way memory states of brain that are usually ignored by constant inputs may be recalled by varying the inputs to brain appropriately. Since the input-output relation is not direct but includes system parameters and functional responses, dynamics of entire system are involved in this process. It is hoped that this concept helps in utilizing the same network for different applications without altering its architecture. This shows how designed structures may be made emergent structures which are adaptive and flexible. Since the results hold good for all time delays (delay independent criteria) the results are applicable to delay-free case as well, i.e., models of [9].

V. References

- [1] H. Bereketoglu and I. Gyori, Global asymptotic stability in a nonautonomous Lotka-Volterra type system with infinite delay, *Journal of Mathematical Analysis and Applications*, 210(1997), 279-291.
- [2] Q.X. Dong, K. Matsui and X.K. Huang, Existence and stability of periodic solutions for Hopfield neural network equations with periodic input, *Nonlinear Analysis*, 49(2002), 471-479.
- [3] M. Forti, P. Nistri and D. Papini, Global exponential stability and global convergence in finite time of delayed neural networks with infinite gain, *IEEE TNN* 16(6): 1449-1463, 2005.
- [4] K. Gopalsamy and Xue-Zhong He, Delay-independent stability in bidirectional associative memory networks, *IEEE TNN*, 5(1994), 998-1002.
- [5] S. Hu and D. Liu, On global output convergence of a class of recurrent neural networks with time varying inputs, 18(2005), 171-178.
- [6] B. Kosko, "Neural Networks and Fuzzy Systems - A Dynamical Systems Approach to Machine Intelligence", Prentice-Hall of India, New Delhi, 1994.
- [7] F.-L. Luo, R. Unbehauen, *Applied Neural Networks for Signal Processing*, Cambridge Univ. Press, Cambridge, UK, 1997.
- [8] B.B. Misra and S. Dehuri, Functional link artificial neural network for classification task in data mining, *J. Computer Science*, 3(12), 2007, 948-955.
- [9] V. Sree Hari Rao and P. Raja Sekhara Rao, Cooperative and Supportive Neural Networks, *Physics Letters A* 371 (2007) 101–110.
- [10] V. Sree Hari Rao and P. Raja Sekhara Rao, Time Varying Stimulations to Simple Neural Networks and Convergence to Desired Outputs, Communicated.
- [11] P. Raja Sekhara Rao, K.Venkata Ratnam and P. Lalitha, Delay Independent Stability of Co-operative and Supportive Neural Networks, Communicated.
- [12] Zhang Yi, J.C. Lv and L. Zhang, Output convergence analysis for a class of delayed recurrent neural networks with time varying inputs, *IEEE TSMC*, 36(1), 87-95, 2006.