



Influence of notch parameters on fracture behavior of notched component

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Abstract: In the present study, the influence of variation of notch parameters on the notch stress intensity factor K_{I0} is studied using CT- specimen made from steel construction. A semi-elliptical notch has been modeled and investigated and is applied to finite elements model. The specimen is subjected to a uniform uniaxial tensile loading at its two ends under perfect elastic-plastic behavior. The volumetric method and the Irwin models are compared using a finite elements method for determined the effective distance, effective stress and relative gradient stress which represent the elements fundamentals of volumetric method. Changing made to notch parameters affect the results of stress intensity factor and the outcomes obtained shows that the increase in size of minor axis reduces the amplitude of elastic-plastic stresses and effective stresses. In lengthy notches, the Irwin model remains constant with very little disturbance of outcomes.

Keywords: Notch; effective distance; notch stress intensity factor; effective stress; relative gradient; Irwin

I. Introduction

The role of stress concentration was first highlighted by Inglis (1913), [9] who gave a stress concentration factor for an elliptical defect, and later by Neuber (1958) [13]. The fracture phenomenon is created by these defects, if the fracture setting reaches the critical value and is observed in any geometric discontinuity. These kinds of failures take place in areas, which are called notches. The notch geometry and other notch characteristics have strong influence on fracture behavior. Notch effect results in the modification of stress distribution owed to the presence of a notch which changes the force flux. Near the notch tip the lines of force are relatively close together and this leads to a concentration of local stress which is at a maximum at the notch tip.

In fracture mechanics of cracked structures is dominated by the near-tip stress field, it is characterized by the stress intensity factors which describe the singular stress field ahead of a crack tip and govern the fracture of a specimen when a critical stress intensity factor is reached.

Nevertheless, the stress distribution at a notch tip is governed by the notch stress intensity factor (NSIF), which is the basis of Notch Fracture Mechanics [15] for which a crack is a simple case of a notch with a notch radius and notch angle equal to zero.

Notch Fracture Mechanics is associated with the volumetric method [21, 23] which postulates that fracture requires a physical volume, in this volume acts an average fracture parameter in term of stress, strain or strain energy density.

Several studies are mainly based on the volumetric method and focus on the notches effect, where Allouti et al, have addressed an analysis of these effects on the stress distribution [3]. Effective stresses are determined by two methods, the method of hot surface stresses (HS) and the volumetric method (VM). The model used for the test is done with a thin pipe of ductile material where the plastic relaxation induced a maximum distribution of stresses. The HS method is obtained by a linear extrapolation of the stress distribution for longitudinal or transversal surface defects in pipes under pressure. This interpolation uses discrete points where the stress concentration effect is dominant. The results led to a similarity of effective stress values. Pluvinaige et al., analysis the stress distribution at a notch root; they show a pseudo-singularity stress distribution governed by notch stress intensity factor (NSIF), K_{I0} . The result of this works and others studies indicate that this approach gives a good description in relation with the notch effect [24]. Under cyclic stress, a fatigue phenomenon is created and the damage of the area appears near the notches tips. The application of the volumetric approach has been extended to a problem of fatigue [2] and has been used to analyze fatigue parameters of notched specimens. Damage to the area due to fatigue depends not only on the peak stress at the notch root but also in areas where the damages caused to material accumulates [17]. The volumetric approach is classified as a macro-mechanical model [17, 20]. Its application depends on a number of considerations such as the elastoplastic stress distribution near the root of notch, notch geometry, loading, boundary conditions and the effect of plasticity and stress relaxation near the notch. According to this study a new concept contributes to a fatigue life assessment, based on volumetric approach and the YAO's concept (stress field intensity, SFI) [21, 22].

The objective of this work is to investigate the effect of short and lengthy notch in fracture behavior in plain specimen. For this purpose an elliptical notch applied to CT-specimen, plane stress, perfect elastic-plastic behavior in steel construction under mode-I loading conditions using two methods especially volumetric and Irwin models [10,11] applied to a notch specimen. The elliptical notch geometry is characterized by two

dimensional parameters, the minor axis '2b' and the major axis '2a' (Fig.1). The results have been extracted from elastic-plastic finite element code of the castem software to calculate the notch stress intensity factor as compared to the classical Irwin formula and analyzed the stress field that reigns at the notch root.

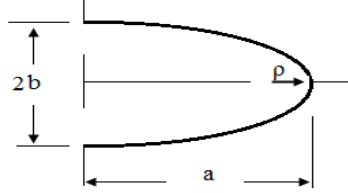


Fig. 1 Parameters of Semi-Elliptical notch

II. Finite element models of elliptical and circular notches

Fig.2 show the finite element model for elliptical and circular notch used in the elastic and elastic-plastic analysis, having the dimensions 20x20 [mm] with two given values of the major axis, the short length of the notch is 0.5[mm] and the long notch is 6[mm], subjected to a uniform uni-axial tension loading σ at its two ends with a value 125[MPa].

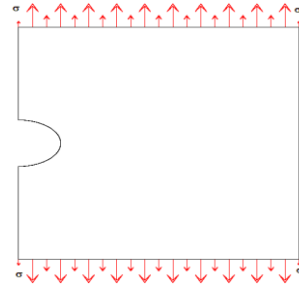


Figure 2 Finite element models

The material has the following mechanical properties: Young's modulus $E = 230E03\text{MPa}$, yield strength 670MPa, Poisson's coefficient $\nu = 0.293$.

An appropriate refinement of discretization applied by triangular elements with six nodes made around the tip of the notch. Fig.2a. Elliptical notch, Fig.2b Circular notch.

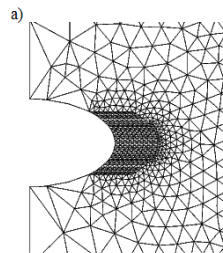


Fig.2a elliptical notch

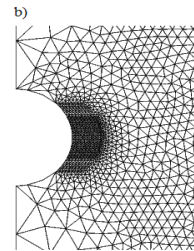


Fig.2b circular notch

III. Analysis of Elastic-Plastic Stress Field in Notched Bodies

The volumetric method has been classified as a critical distance method (TCD) [1, 4]. The main objective in volumetric method is to calculate "effective distance" and "effective stress" via extracted stress distribution at notch roots. The volumetric method takes damage accumulation in local damaged zone into consideration [2]. Investigations into the fatigue failure mechanisms have shown that the accumulation of fatigue damages depends not only on the peak stress at the notch roots but also an average stress in the damage zone and relative stress gradient [17].

In the volumetric method, the aforementioned average stress is named as "effective stress" and it is calculated using an effective distance. Traditionally, the effective distance has been obtained using volumetric bi-logarithmic diagram (Fig. 3). In fact, the stress distribution near notch roots versus ligament in bi-logarithmic diagram linearly behaves in certain zone like cracks [5] and the start point is considered as effective distance, this distance is considered the boundary of the stress relaxation. It can be found by means of the minimum point of relative stress gradient. The relative stress gradient for volumetric method can be written as:

$$\chi(x) = \frac{1}{\sigma_{yy}(x)} \frac{\partial \sigma_{yy}(x)}{\partial x} \quad (1)$$

Where $\chi(x)$ and $\sigma_{yy}(x)$, are the relative stress gradient, and maximum principal stress, or crack opening stress, respectively.

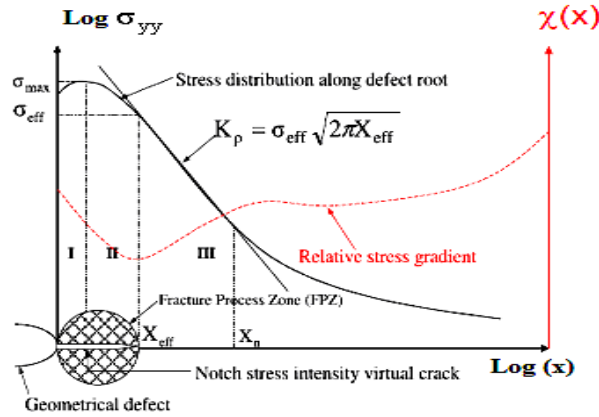


Fig. 3. A typical illustration of elastic-plastic stress along notch ligament and notch stress intensity virtual crack concept including relative stress gradient which signifies the effective distance position

The effective stress for fracture is then considered as the average volume of the stress distribution over the effective distance. The bi-logarithmic elastic-plastic stress distribution (Fig. 3) along the ligament exhibits three distinct zones which can be easily distinguished [18]. The elastic-plastic stress primarily increases and it attains a peak value (zone I) then it gradually drops to the elastic plastic regime (zone II). Zone III represents linear behaviour in the bi-logarithmic diagram. It has been proof by examination of fracture initiation sites that the effective distance correspond to the beginning of zone III which is in fact an inflexion point on this bi logarithmic stress distribution [7]. A graphical method based on the relative stress gradient $\chi(x)$ associated the effective distance to the minimum of $\chi(x)$.

A. Weight function

Weight function deals with stress contribution in elaborated damage accumulation zone; the weight function explicitly depends on stress, stress gradient and distance from notch root in the elaborated zone and implicitly depends on notch geometry, loading type, boundary conditions and material properties [1]. Weight function is essential to distinguish between Stress Field Intensity (SFI) weight function and other weight function concepts which are utilized in Fracture Mechanics based on Green's function for boundary problem [8, 12]. The weight function should satisfy the following conditions:

- a) $0 \leq \varphi(r, \chi) \leq 1$
- b) $\varphi(0, \chi(0)) = 1$
- c) $\varphi(r_{\max}, \chi(r_{\max})) = 1$

Three weighting functions have been proposed, available in volumetric method [1]:

- Unit weight function: $\varphi(x, \chi) = 1$
- Delta weight function: $\varphi(x, \chi) = \delta(x - X_{\text{eff}})$
- Gradient weight function: $\varphi(x, \chi) = 1 - x \cdot \chi$

An analytical expression allowing the discrete points modelling of the stress distribution is expressed by the polynomial interpolation [1] :

$$\sigma_{yy}(x) = \sum_{i=0}^n a_i x^i \quad (2)$$

The relative stress gradient can be derived by Eq. (2) as:

$$\chi(x) = \frac{1}{\sigma_{yy}(x)} \frac{\partial \sigma_{yy}(x)}{\partial x} = \frac{\sum_{i=0}^n i a_i x^{i-1}}{\sum_{i=0}^n a_i x^i} \quad (3)$$

According to the polynomial formulation, effective distance which corresponds to minimum point of relative stress gradient can be obtained as below:

$$\frac{\partial \chi(x)}{\partial x} = -\frac{1}{\sigma_{yy}^2(x)} \left(\frac{\partial \sigma_{yy}(x)}{\partial x} \right)^2 + \frac{1}{\sigma_{yy}(x)} \frac{\partial^2 \sigma_{yy}(x)}{\partial x^2} = 0 \quad (4)$$

B. Effective stress

The average stress value within the fracture process zone is then obtained by a line method [8] which consists to average the opening stress distribution over the effective distance. One obtains the second fracture criterion

parameter called the effective stress σ_{ef} . However, it is necessary to take into account the stress gradient due to loading mode and specimen geometry. This is done by multiply the stress distribution by a weight function $\varphi(x, \chi)$; where x is the distance from notch tip and χ . The effective stress is finally defined as the average of the weighted stress inside the fracture process zone [25,26]:

$$\sigma_{eff} = \frac{1}{X_{eff}} \int_0^{X_{eff}} \sigma_{yy}(x) \varphi(x, \chi) dx \quad (5)$$

The volumetric method effective distance can be numerically solved using the presented volumetric method effective distance characteristics equation. With substitution of Eq. (2) into (5) and calculated effective stress, it can be rewritten including unknown weight function as follows:

$$\sigma_{eff} = \frac{1}{X_{eff}} \int_0^{X_{eff}} \left(\sum_{i=0}^n a_i x^i \right) \times \varphi(x, \chi) . dx \quad (6)$$

The mentioned polynomial stress distribution can be utilized to calculate the effective stress for all proposed weight functions. Eq. (6) can be rewritten using unit weight function as follows [1]:

$$\sigma_{eff} = \frac{1}{X_{eff}} \sum_{i=0}^n \frac{a_i X_{eff}^{i+1}}{i+1} \quad (7)$$

By changing the weight function and replacing the unit weight function by delta function $\varphi = \delta(x - X_{eff})$, the new relationship of effective stress becomes:

$$\sigma_{eff} = \sigma_{yy}(X_{eff}) \quad (8)$$

Including another weight function, which uses the relative stress gradient, is taken as: $\varphi(x, \chi) = 1 - x \cdot \chi$

Effective stress will be [1]:

$$\sigma_{eff} = \frac{1}{X_{eff}} \sum_{i=0}^n \frac{a_i X_{eff}^{i+1}}{i+1} - \frac{1}{X_{eff}} \cdot \chi \times \sum_{i=0}^n \frac{a_i X_{eff}^{i+2}}{i+2} \quad (9)$$

In bi-logarithmic diagram, at limit of zone II and $x = X_{eff}$, the notch stress intensity factor is expressed as function of X_{eff} and σ_{eff} [6,25]:

$$K_{I\rho} = \sigma_{eff} \sqrt{2\pi \cdot X_{eff}} \quad (10)$$

Where $K_{I\rho}$, σ_{eff} and X_{eff} are the notch stress intensity factor, the effective stress, and the effective distance respectively.

C. Irwin's models

The elastic stress distribution in an elliptical defect was determined by Inglis [9]. Calculating the stress intensity factor (SIF) of a crack can be done without analyzing the singular stresses field near the tip of notch if the crack is replaced by an elliptical notch of the same size. The relationship between K_I , σ_{max} and ρ can be written as [11,14]:

$$K_I = \lim_{\rho \rightarrow 0} \frac{\sigma_{max}}{2} \sqrt{\pi \rho} \quad (11)$$

Where K_I , σ_{max} and $\rho = b^2/a$, are stress intensity factor in opening mode, maximum elastic stress, and curvature radius of elliptical notch respectively.

In corrected approach, Irwin argued that the presence of a crack tip plastic zone makes the crack behave as if it were longer than its physical size, and the distribution of stress is equivalent to that of an elastic crack with a length $(a + r_E)$ that is [19]:

$$K_{I_{eff}} = \sigma \sqrt{\pi(a + r_E)} \quad (12)$$

Where σ , a and r_E : applied tension, major axis of ellipse and plastic zone size respectively.

IV. Analysis of Elastic Stress Field in Notched Bodies

Fig. 4 shows an elastic stress distribution analysis of the short and lengthy notch, having a semi-elliptical shape with dimension b equal to 0.1, 1 and 5 [mm] and for the major axis can take the value 0.5mm for short notch, and for lengthy notch having a value equal to 6 [mm]. It shows a comparison of analytically obtained results with those obtained by the finite element method. Various formalisms representing the stress distribution can be found in literature and are presented as follows [16]:

- Usami 1985:

$$\sigma_{yy} = \frac{\sigma_{max}}{3} \left[1 + \frac{1}{2} \left(1 + \frac{x}{\rho} \right)^{-2} + \frac{3}{2} \left(1 + \frac{x}{\rho} \right)^{-4} \right]$$

- Chen and Pen, 1978:

$$\sigma_{yy} = \sigma_{\max} \sqrt{\frac{\rho}{\rho + 8x}}$$

- Neuber and Weiss1962:

$$\sigma_{yy} = \sigma_{\max} \sqrt{\frac{\rho}{\rho + 4x}}$$

- Kujawski,1991

$$\sigma_{yy} = f \sigma_{\max} \left[\left(1 + \frac{2x}{\rho} \right)^{-1/2} + \left(1 + \frac{2x}{\rho} \right)^{-3/2} \right]$$

$$\text{if } \frac{x}{\rho} < 0.2 \Rightarrow f = 1 ;$$

$$\text{if } \frac{x}{\rho} \geq 0.2 \Rightarrow f = 1 + \frac{\tan(\pi / 2k_t)}{2.8} \left(\frac{x}{\rho} - 0.2 \right)$$

The results are similar near the end of notch where the stress concentration is high (Fig. 4a, Fig. 4b and Fig. 4c). However, at remote distances from notch bottom, i.e. where the stress gradient is lower, the results obtained by finite element method decrease rapidly towards lower values. It is also known that the flattened notches behave the same way as crack, generate a higher stress concentration, and consequently significant elastic stresses reach maximum values. In the lengthy notches ($a=6[\text{mm}], b=1\text{mm}$ and 5mm) away from the notch root, stresses are reduced to lower values then the results obtained by Chen Pen , Neuber and Kujawski, decrease regularly and get closer mutually (Fig. 4b and Fig. 4c). If the semi-elliptical notch tends towards a semi-circular notch, the maximum elastic-plastic stresses tend to have lower values. For short notch configurations, the Chen Pen and Neuber results show a convergence those obtained by finite elements, while for lengthy notch, $b=1\text{mm}$ Neuber and Kujawski obtained better results and the extent of high stress region becomes smaller ($\leq 1 [\text{mm}]$) compared to lengthy notches for $b=5\text{mm}$ which is less than $2 [\text{mm}]$.

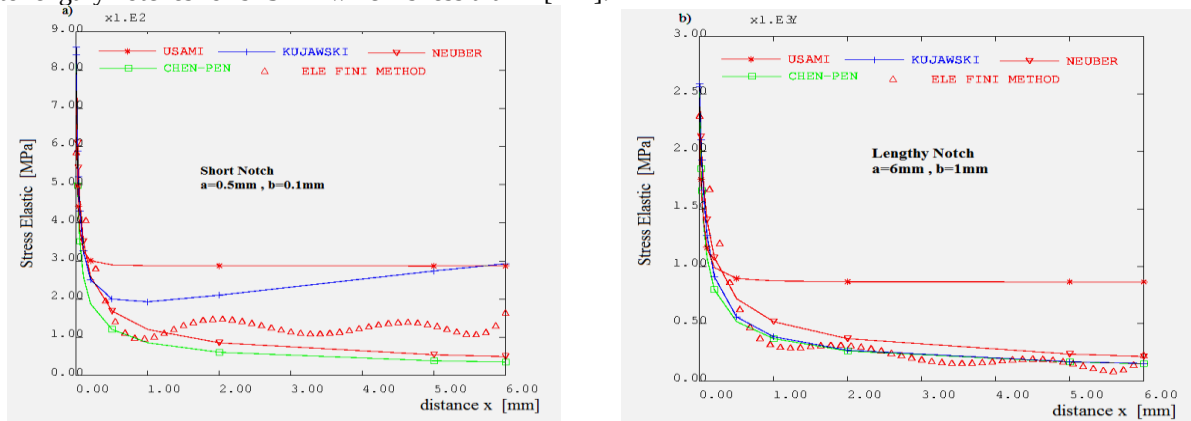


Fig. 4 Elastic stresses distribution, a) Short notch $a=0.5\text{mm}$ and $b=0.1\text{mm}$, b) Lengthy notch $a=6\text{mm}$ and $b=1\text{mm}$

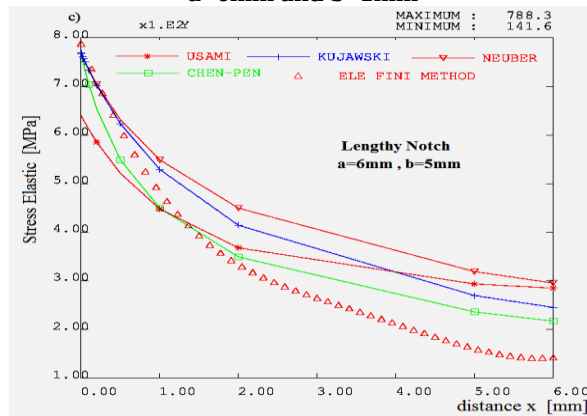


Fig. 4c Elastic stresses distribution, lengthy notch $a=6\text{mm}$ and $b=5\text{mm}$

On the other hand, in the case of deep notches having an elastic behaviour, increasing the size of minor axis (b) reduces the effect of stress concentration. The elastic distribution of stress gives the maximum value at notch bottom; unlike the maximum of elastic-plastic stress is far from the notch tip.

V. Analyzing notch stress intensity factor

The phenomenon of rupture must indeed be considered in its physical dimension. It requires a certain elaboration volume of process failure [15]. In this volume, a zone is governed by the notch stress intensity factor, which depends at the same time on the effective distance, the effective stress, the relative stress gradient and the weight function. The results obtained are compared with those calculated in accordance to the two Irwin models [10,14].

Figures 5a and 5b show the notch stress intensity factor evolution versus the ratio b/a calculated by using the two Irwin models and the three weight functions: unit, delta and gradient functions.

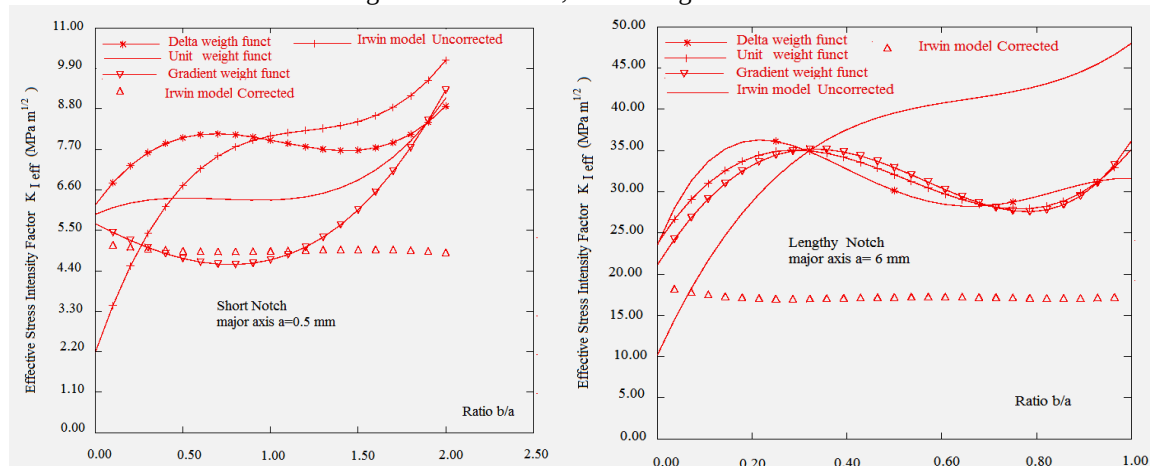


Fig. 5 Evolution the notch stress intensity factor for, a) short notch, $a=0.5\text{mm}$, b) lengthy notch, $a=6\text{mm}$

The changes made to notch parameters (a , b and, ρ) affects the evolution of stress intensity factor versus the ratio b/a . The figures show its variation for elastic-plastic behaviour; (note that the volumetric method was applied initially for brittle materials, the extension of its use to ductile materials has gave the good approximations).

Figure 5 depicts at low values of (b/a) i.e. for an open notch. The results calculated by the volumetric method are convergent mutually by approaching each other with the results of Irwin corrected model. In particular the weight gradient function gives approximately the same results with the corrected Irwin model.

For a short notch ($a = 0.5\text{mm}$) and beyond a higher ratio (b/a), a divergence in the results of the volumetric method is observed compared to Irwin corrected, but a convergence to the uncorrected Irwin model has appeared.

The made of significant plasticizing (Fig.9) that is starting at the bottom of the notch in lengthy notches, the results of volumetric method (VM) describe a similarity of approximation of the results with re-convergence to the uncorrected -Irwin model and excessive divergence from Irwin – corrected. This latter remains constant with very little disturbance of outcomes, we can conclude that the volumetric method is very sensitive to notch parameters variation.

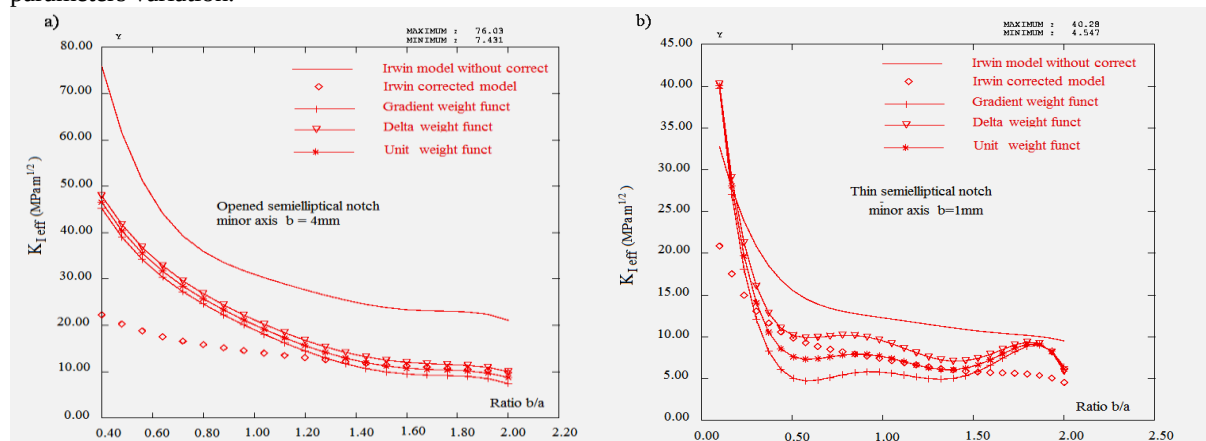


Fig. 6 Evolution of the notch stress intensity factor for: a) Opened semielliptical notch; b) Thinned semielliptical notch

If a plastific area widens near the end of the notch, the Irwin corrected model can not detect the change that has occurred to the notch stress intensity factor (NSIF) parameters.

The increase in size of the major axis (a) and the reduction of the minor axis (b) correspond to a deeper and flattened semi-elliptical notch; creating a region where the stress concentration is high and generates an increased stress gradient as a result of the increase of the stress intensity factor values. Thinned and deepened semi-elliptical notches are much more dangerous than short semi-elliptical and semi-circular notches.

If the stress gradient is low (b/a high), Figures 6a and 6b show that the results obtained by VM and Irwin corrected model of the SIF- K_I are closer and the curves show an identical appearance in Figure 6a. The Irwin corrected model gives converging results with results of unit weight function (Fig. 6a and Fig. 6b).

For a short notch, at great stress gradient (b/a low), i.e. decreasing the major axis (a); the results obtained by the three weight functions move away from those obtained by the Irwin corrected model and are approaching to those of Irwin uncorrected model (Fig. 6b).

For values of the minor axis b equal to 4 [mm], the elliptical notch is classified as an open notch (Fig. 6a), this gives a rearrangement of stress concentration distribution. The opening of the elliptical notch reduces the stress concentration in the area near the bottom of the notch (increased b / a).

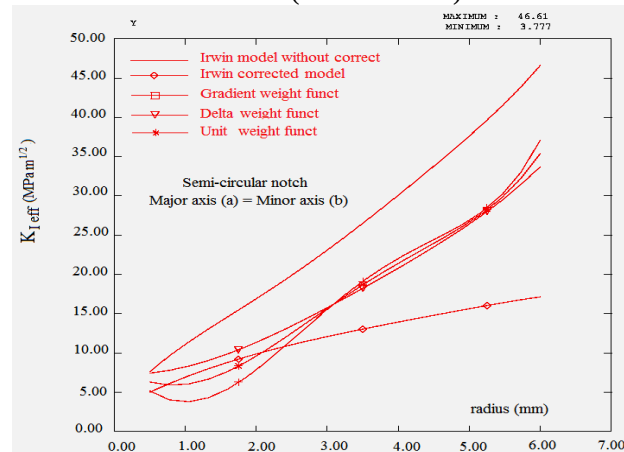


Fig. 7 Evolution of the notch stress intensity factor for a semi-circular notch

From a high stress gradient, the values obtained using the two Irwin models, diverge excessively relative to the volumetric approach (Fig. 6a). In the semi-circular notch (Fig. 7), the results of SIF, (calculated according to the Irwin corrected model) are intermediate between those obtained by the weight, delta and unit functions for a radius lower than 2 [mm].

VI. Effective Stress and Maximum Elastic-Plastic Stress

The effective stress is the average value of the stress distribution over the effective distance, weighted by the relative stress gradient inside the elaboration volume of fracture. Figures 8a and 8b show the evolution of effective stress calculated using the three weight functions (unit, delta and gradient) with the maximum elastic-plastic stress, according to the ratio (b/a), relating to the short and lengthy semi-elliptic notches in order to determine the function which can approximate the maximum elastic-plastic stress.

At low values of (b / a), where the notch tends to flatten the effective stress calculated by the VM and maximum elastic-plastic stress values increases to the peak, and then follow a regular decrease to low values of stress, where the stress gradient is lower.

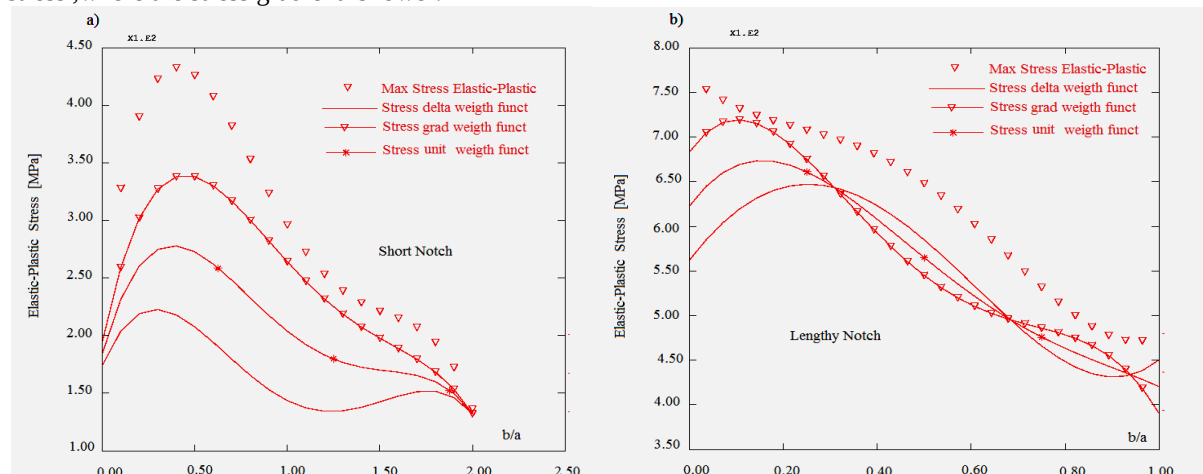


Fig. 8 Evolution of effective stress & maximum elastic-plastic stress for short notch and lengthy notch

In the case of deep notch which develops a high stress, induces a high stress gradient and displays an extensive of lower values ($b/a < 0.70$) (Fig. 8b), than that of the short notch ($b/a < 1.2$) (Fig. 8a) and the effective stresses calculated by the weight gradient function get closer to stress maximum elastic-plastic. The outcomes of maximums elastic-plastic stress is greater than the effective stresses value calculated by the volumetric approach. The opening of the elliptical notch (b high) reduces the amplitude of the effective stresses and maximum elastic-plastic stresses and brings up a similarity of approximation of the results of effective stresses calculated by the different weight functions (Fig. 8b).

If the minor axis 'b' of semi-elliptical notch increases, the approximation obtained by the three weight functions converge between them for both notch configurations (short and lengthy).

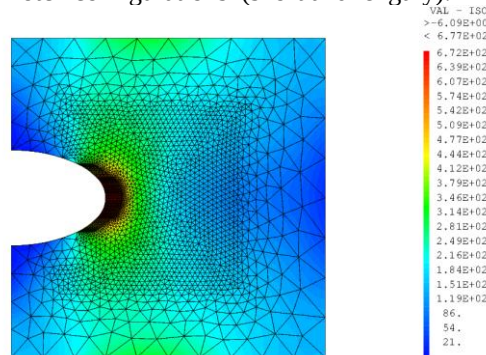


Fig. 9 Plastic zone at the tip of the lengthy notch according to VON MISES criterion; notch $a=6\text{mm}$

VII. Conclusion

In the present study, the volumetric method is investigated and compared with Irwin models using the finite element method to explain the influence of notch parameters variation in weight functions, effective distance, effective stress, and in the stress gradient (consequently on the fracture behaviour). Extension of the application of volumetric method in the case of an elastic-plastic stress distribution has taken into account the effect of these changes made to notch parameters.

The main outcomes can be summarized as:

- Changing the notch parameters (a , b and ρ) creates stress field disturbances near the notch root. Therefore, it affects the evaluation of the stress intensity factor.
- Lengthy notches have a significant plasticizing near the notch root, and the corrected- Irwin model remains constant with very little disturbance of outcomes. The volumetric method is very sensitive to notch parameters variation. The outcomes of volumetric method (VM) describe a similarity of approximation of the results with re-convergence to the uncorrected -Irwin model and excessive divergence from the Irwin corrected model.
- If the notch tends to open, the results achieved through volumetric method and the uncorrected Irwin model, mutually converge.
- The deeper and flattened notch creates a region where the stress concentration is high and generates an increased stress gradient. The thinned and lengthy semi-elliptical notches are much more dangerous than the short semi-elliptical and semi-circular notches.
- The results obtained show that the increase in size of minor axis of elliptical notch reduces the amplitude of elastic-plastic stresses and effective stresses.
- The elastic stress distribution is characterized by maximum stress at the notch root and in the case of elastic-plastic distribution it is characterized by a stress relaxation.

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