



Plasma magnetization by a nonuniform electromagnetic beam

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Abstract: Relativistic and non-relativistic plasma outflows are quite ubiquitous in astrophysical scenarios, as well as in laboratory plasmas. The propagation of relativistic and non-relativistic charged particle beams in background plasmas provides return currents in the opposite direction and interactions between the currents then drive several plasma instabilities involving the longitudinal (electrostatic instabilities) and transverse (electromagnetic instability) modes. Such instabilities have been accepted as possible mechanisms for generating spontaneous magnetic fields in extreme astrophysical environments, such as the gamma-ray bursts (GRBs), pulsar magnetosphere, active galactic nuclei (AGN), as well as in laboratory plasmas such as those in inertial confinement fusion schemes. In the present paper we have described the physics of the ponderomotive force of nonuniform intense electromagnetic beams as a possible mechanisms for generating seed magnetic fields in plasmas. This mechanism has the potential of generating both non-stationary and stationary magnetic fields, the strength of which critically depends on the plasma conditions (e.g. the electron temperature and density gradients and the intensity of the electromagnetic beams).

Keywords: GRBs, pulsar magnetosphere, AGN.

I. Introduction

Understanding the origin of magnetic fields in cosmic and laser produced plasmas has been of great theoretical as well as experimental interest [1–3]. The generation of seed magnetic fields in the early universe is a long-standing mystery in astrophysics. A very weak magnetic field (typically of the order of one to thirty microgauss) in the early universe has been of much interest to the scientific community. Our galaxy and many other spiral galaxies are endowed with coherent magnetic fields ordered on scales = 10 kpc with typical strength $B_G = 3 \times 10^{-6} G$ [4,5]. Furthermore, the cosmic magnetic field is strongly correlated with the large-scale structures (e.g. currents filaments and sheets) of the Universe [6, 7]. The origin of cosmological, galactic and large-scale extragalactic magnetic fields is one of the main unresolved problems of astrophysics and cosmology. The existence of these magnetic field are relevant to the processes which took place in the early universe. Because of the Universe's high conductivity, two important quantities are almost conserved during the evolution of the Universe: the magnetic flux and the magnetic helicity. The magnetic field in ionized media is generated on account of electrical currents or of rotational electric fields. The self-excitation of large scale magnetic fields B in astrophysical bodies, such as the Sun, stars, galaxies, etc [8, 9], is a challenging issue. The main problem of most particle physics mechanisms of the origin of seed magnetic fields is how to produce them coherently on cosmological (large) scales [10].

During the last few decades, the generation of magnetic fields has been studied in different areas of physics, such as in cosmic environments (e.g. supernova remnants, gamma-ray bursts), and in laser produced plasmas. Thus, magnetic fields are found to be important in every scale hierarchy of various plasma systems. The usual question posed is: How can an initially magnetic-field free plasma give rise to a non-zero “seed magnetic field”? The answer to this question rests on identifying suitable mechanisms that are responsible for creating the electron current j_e and the electric field E that are required for generating the seed magnetic field B . Neglecting the displacement current in Maxwell's equations, we have

$$\nabla \times B = \mu_0 J \equiv -\mu_0 e(n_e v_e - n_i v_i) \quad (1)$$

and

$$\frac{\partial B}{\partial t} = -\nabla \times E \quad (2)$$

where the electron and ion fluid velocities, v_e and v_i , in turn, are controlled by the electromagnetic fields E and B , as well as by the forces involving the gradients of the plasma and electromagnetic wave intensity. Thus, the key issue is how to tear apart the electrons and ions, so that there will be electric fields and currents due to the motion of the electrons against the ions.

It has been shown that the seed magnetic fields in plasmas can be generated by several mechanisms, e. g. due to non-parallel electron density and temperature gradients ($\nabla n_e \times \nabla T_e$) (known as the Biermann battery [11]), by electron temperature anisotropy (known as the Weibel instability [12]), by counterstreaming charged particle

beams (known as the current filamentation instability [13]), due to the inverse Faraday effect [14, 15], and by the ponderomotive forces of intense laser beams [16–18]. These mechanisms describe how the motion of electrons generates the space charge electric field and currents that are sources for the seed magnetic fields in plasmas. We will discuss in more detail each of these mechanisms for the magnetic field generation in the following subsections.

II. The Weibel instability

The Weibel instability [12] is considered to be the most important example of a purely growing electromagnetic mode in an unmagnetized plasma with electron temperature anisotropy. In his classic paper, Weibel found that an anisotropic plasma having a directional dependence of the temperature is unstable against a magnetic field perturbation. Transverse modes are amplified in a collisionless plasma even in the absence of an external magnetic field. The free energy stored in electron temperature anisotropy produces a purely growing magnetic field. The dispersion relation in an unmagnetized plasma has been derived by using a closed set of linearized Vlasov-Maxwell equations. We have

$$\omega^2 - k^2 c^2 - \int_0^\infty \int_{-\infty}^\infty \left(\frac{\partial f_0}{\partial v_0} - \frac{v_0 k}{(\omega + k v_z)} \frac{\partial f_0}{\partial v_z} \right) v_0^2 dv_0 dv_z = 0 \quad (3)$$

where ω is the angular frequency of the electromagnetic mode, and the wave vector k is parallel to the z -direction, and $f_0(v) = F(v_0, v_z)$ is an anisotropic distribution functions for the electrons with $v_0^2 = v_x^2 + v_y^2$. Specifically, we have chosen a bi-Maxwellian electron distribution function (an electron temperature anisotropy $(u_0/u_z)^2 - 1 > 0$) of the form

$$f_0 = \frac{n_0}{v_0^2 u_z (2\pi)^{3/2}} \exp \left(-\frac{v_0^2}{2u_0^2} - \frac{v_z^2}{2u_z^2} \right) \quad (4)$$

where u_0 and u_z are the unperturbed thermal velocities corresponding to v_0 and v_z , respectively. In the limit $\omega/k u_z \gg 1$, Eq.(3) reads

$$\omega^4 - (\omega^2 + k^2 c^2) \omega^2 - k^2 u_0^2 \omega_{pe}^4 = 0 \quad (5)$$

The solution of Eq. (5) is

$$\omega = \pm \left(\frac{1}{2} [(\omega_{pe}^2 + k^2 c^2) \pm ((\omega_{pe}^2 + k^2 c^2)^2 + 4k^2 u_0^2 \omega_{pe}^2)^{1/2}] \right)^{1/2} \quad (6)$$

The negative imaginary solution of Eq. (6) shows the growth rate of self-excited magnetic fields. Using $k u_z / \omega \ll 1$ and $u_0 \gg u_z$ we can simplify the expression for the growth rate Γ , obtaining

$$\Gamma = \text{Im}(\omega) = \frac{k u_0 \omega_{pe}}{(\omega_{pe}^2 + k^2 c^2)^{1/2}} \quad (7)$$

which is valid only for $k u_z / \omega \ll 1$. The latter implies that $u_0 \gg u_z$. Hence, a large electron temperature anisotropy is required for instability.

The physical picture of the Weibel instability in the presence of counterstreaming electron beams in plasmas was interpreted by Fried [19]. To understand the physical mechanism of the Weibel instability, Fried studied the anisotropy for electrons and assumed immobile ions in an electron-ion plasma. Due to their anisotropic temperatures, the electrons are moving with the thermal velocity $v = \pm a \hat{x}$ along the x -direction (see Fig.1). The anisotropic electron distribution function is given by

$$f_0(v) = \delta(v_x^2 - a^2) \delta(v_y) \delta(v_z) \quad (8)$$

Now, let us assume some initial perturbation in the magnetic field $B = \hat{z} B_z \cos(ky)$, which is polarized along the z -axis and propagating along the y -axis, arising from noise. The electrons will be deflected by the magnetic field and will acquire in time an additional z -component of velocity & $\delta v_z = \delta t \omega_L a$, where $\omega_L = e B_1 / m_e$ is the electron gyrofrequency associated with the oscillating magnetic field. The latter would grow at a rate $\Gamma = \omega_{pe} a / c$. Since the magnetic field changes in time, it generates an electric field perpendicular to the magnetic field $E_x = (\omega/k) B_z$ (assuming $\exp(-i\omega t)$ time dependence), due to Faraday's law. By incorporating these effects, Fried obtained a similar quadratic dispersion relation for ω^2 , as Weibel found,

$$\omega^4 - (k^2 + \omega^2) - k^2 a^2 \omega_{pe}^4 = 0 \quad (9)$$

Now, we can allow the thermal velocities to be also in the y and z directions by considering $f_0(v) = \delta(v_x^2 - a_1^2) \delta(v_y^2 - a_2^2) \delta(v_z^2 - a_2^2)$. The net effect is simply to replace a^2 with $a_2^2 - a_1^2$ in Eq. (9). The instability could occur only when $a_2 > a_1$. But, if $a_2 = a_1$, unstable modes would no longer occur and one would have a stable electromagnetic wave propagating in an unmagnetized plasma.

The Weibel instability has a wide range of applicability in astrophysical plasmas, such as gamma ray burst sources, supernovae and galactic cosmic environments. Medvedev and Loeb [20] suggested that the Weibel

instability is capable of generating a strong small-scale magnetic field in an initially unmagnetized plasma, arising from electron temperature anisotropy. The Weibel instability has been extensively studied in both non-relativistic and relativistic regimes. The case of the Weibel instability with ultra-relativistic electron streams (viz., $v_e/c \sim 1$) has been investigated by Yoon and Davidson [21, 22], who obtained the maximum growth rate

$$\Gamma = \frac{\omega_{pe}}{\gamma^{1/2}} \quad (10)$$

where $\gamma = (1 - v_{e0}^2/c^2)^{1/2}$ is the relativistic gamma factor, and v_{e0} the electron streaming velocity.

In closing, it should be mentioned that we have not here discussed the generation of magnetic fields and filamentation instabilities due to colliding electron clouds, since the free-energy sources in colliding electron clouds could also drive purely growing magnetic fields. The latter could be saturated when the e-folding time is comparable to the electron gyroperiod in the excited magnetic fields. The saturated magnetic fields, in turn, can be associated with the observed magnetic fields in cosmic and laboratory plasmas.

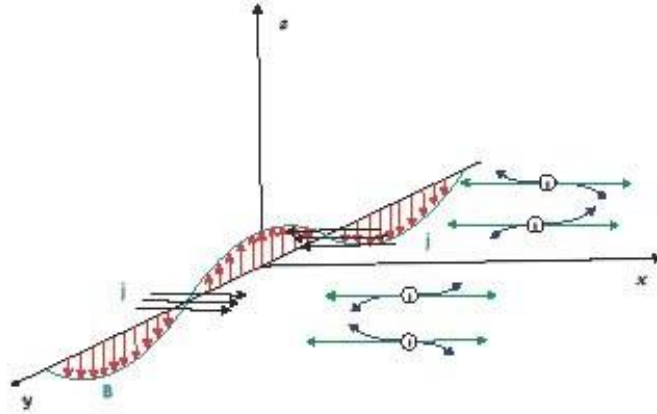


Figure 1: Physical mechanism of the Weibel instability : counter-propagation of two electron beams.

III. Plasma magnetization by a nonuniform electromagnetic beam

The ponderomotive force of a nonuniform intense electromagnetic beam has been considered as a possible alternative mechanism for generating magnetic fields in an unmagnetized plasma [11].

The ponderomotive force is the technical term for the radiation pressure that is exerted by intense laser beams or high-frequency electromagnetic (HF-EM) waves that are interacting with the plasma. In the presence of the HF-EM waves, electrons experience a jitter motion. The advection of the electron fluid and the nonlinear Lorentz force in the electric and magnetic fields of the HF-EM waves give rise to a low-frequency ponderomotive force that primarily acts on the electrons. The ponderomotive force, which is proportional to the HF-EM wave intensity, will swiftly expel the electrons from the region of the high-field intensity such that they instead pile up in the regions of low field intensity.

Let us consider the single particle motion in a given electromagnetic field in the plasma. We assume a monochromatic electromagnetic field given by

$$E_w(r, t) = E_0(r, t) \cos(\omega t), \quad (11)$$

$$\frac{\partial B_w(r, t)}{\partial t} = -\nabla \times E_w(r, t), \quad (12)$$

$$B_w(r, t) = -\frac{1}{\omega} \nabla \times E_0(r) \sin(\omega t) = B_w(r) \sin(\omega t), \quad (13)$$

where E_0 is the space dependent part of the electric field and $\omega = (k_0^2 c^2 + \omega_{pe}^2)^{1/2}$ is the angular frequency of the HF-EM waves in the plasma. The equation of motion for an electron moving in the HF-EM fields is

$$m_e \frac{dv_e}{dt} = -e[E_0(r, t) \cos(\omega t) + v_e \times B_w \sin(\omega t)] \quad (14)$$

In the weakly nonlinear limit, viz. $v_e/c \ll 1$, (the term $v_e \times B_w$ is smaller than the E_w term. Therefore due to the first order $v_e = v_1, r = r_1$) electrons would oscillate in the direction of E_w , and in this case one has to solve

$$m_e \frac{dv_1}{dt} + eE_0(r_0, t) = 0 \quad (15)$$

$$m_e \frac{dv_1}{dt} + eE_0(r_0, t) \cos(\omega t) = 0, \quad v_1 = \frac{dr_1}{dt} \quad (16)$$

The solution of equation (16) is

$$v_1 = -\frac{eE_0(r_0) \sin(\omega t)}{m_e \omega}, \quad r_1 = \frac{eE_0(r_0) \sin(\omega t)}{m_e \omega^2} \quad (17)$$

To study the equations to second order in the amplitude, we write

$$v_e = v_1 + v_2, E_0 = E_0(r_0 + r_1) \approx E_0(r_0) + (r_1 \cdot \nabla) E_0(r = r_0), B_w = B_w(r_0) \quad (18)$$

After substituting Eqs. (7) and (8) into (4), we obtain the second order equation

$$m_e \frac{dv_2}{dt} = -e[(r_1 \cdot \nabla) E_0(r_0) \cos(\omega t) + v_1 \times B_w \sin(\omega t)] \quad (19)$$

Substituting Eqs. (13) and (17) into Eq. (19) and averaging over the period $2\pi/\omega$ one can calculate the average nonlinear force

$$\langle F_{NL} \rangle = m_e \left\langle \frac{dv_2}{dt} \right\rangle = -\frac{e^2}{2m_e\omega^2} [(E_0 \cdot \nabla) E_0 + E_0 \times (\nabla \times E_0)] \quad (20)$$

In Eq. (20), the first term in right-hand side is the force which causes the electrons to move in a linear trajectory and the $E_w \times B_w$ force (second term into the right-hand side) acting on the electron and distorts the linear motion. By using the identity

$$E_0 \times (\nabla \times E_0) = \frac{1}{2} \nabla E_0^2 - (E_0 \cdot \nabla) E_0 \quad (21)$$

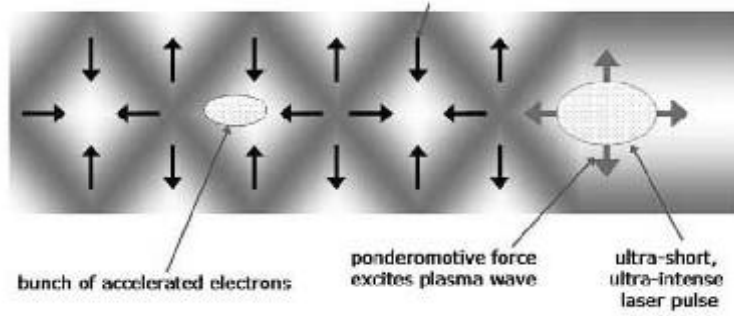


Figure 2: The physical mechanism of the ponderomotive force associated with the high-power laser beams.

and multiplying Eq. (20) with the electron number density n_e , one can obtain the ponderomotive force per unit volume for an isotropic collisionless plasma in the presence of a nonuniform HF-EM beam. The result is

$$F_p = n_e F_{NL} = \frac{n_e e^2}{4 m_e \omega^2} \nabla |E_0|^2 = -\frac{\omega_p^2}{16\pi\omega^2} \nabla |E_0|^2 \quad (22)$$

The ponderomotive force F_p is an important source of nonlinearity, which has important consequences for many nonlinear phenomena, such as stimulated scattering instabilities of EM waves, and the plasma magnetization by a nonuniform EM beam. The figure 2 depicts the physical mechanism of the EM ponderomotive force.

From the balance between the ponderomotive force of the EM waves, as well as the electrostatic and Lorentz forces on the electrons, we have in the steady state

$$\frac{e^2}{2m_e\omega^2} \nabla |E_0|^2 = -e(E_s + u_{es} \times B_s) \quad (23)$$

where we have assumed that the EM wave ponderomotive force is much stronger than the electron pressure gradient. The electric and magnetic fields associated with the slow plasma motion are denoted by E_s and B_s , respectively.

The electrons are coupled to the ions by the slowly varying electric field E_s . Thus, eliminating the electric field by using $E_s = -u_{is} \times B_s$, where u_{is} is the ion fluid velocity, as well as $\hat{z} \cdot (\nabla \times B_s) = \mu_0 j_s$, we readily obtain [11] from the radial component of Eq. (23)

$$\frac{\Omega}{\omega_{pe}} = \frac{e}{m_e \omega r c} \left(-\int_0^r dr_1 r_1^2 \frac{\partial |E_0|^2}{\partial r_1} \right)^{1/2} \quad (24)$$

Where $\Omega = eB_\theta/m_e$ and B_θ is the azimuthal component of B_s .

Considering an electromagnetic beam with the Gaussian intensity distribution, i.e. $|E_0|^2 = W_0^2 \exp(-r^2/r_0^2)$, where r_0 is the effective beam radius, we have [11] from Eq. (24)

$$\frac{\Omega}{\omega_{pe}} = \frac{r_0 e W_0}{m_e \omega r c} \left[1 - \left(1 + \frac{r^2}{r_0^2} \right) \exp(-r^2/r_0^2) \right]^{1/2} \quad (25)$$

The generated magnetic field is thus proportional to r for $r \ll r_0$ and to r^{-1} for $r \gg r_0$, whereas in the region $r \approx r_0$, Ω is of the order of $\omega_{pe} e W_0 / m_e \omega c$. It turns out that for $e W_0 / m_e \omega c \sim 0.1$, Eq. (15) yields $\Omega \sim \omega_{pe}$ at $r = r_0$. Thus, the magnitude of B_θ is of the order of 320 mega-Gauss for a plasma with $n_0 \approx 10^{20} \text{ cm}^{-3}$.

IV. Conclusion

We have here presented a review of possible mechanisms for generating seed magnetic fields in plasmas. Especially, we have described the physics of the Weibel instability, and the ponderomotive force of non-uniform

intense electromagnetic beams as sources for the seed magnetic fields in plasmas. Each mechanism has the potential of generating both non-stationary and stationary magnetic fields, the strength of which critically depends on the plasma conditions (e.g. the electron temperature and density gradients and the intensity of the electromagnetic beams). In conclusion, we note that the seed magnetic fields could scatter high energy photons and affect the propagation of EM beams, as well as the cross-field transport of charged particles in plasmas.

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