



Effect of Variable Thermal Conductivity & Heat Source/Sink near a Stagnation Point on a Linearly Stretching Sheet using HPM

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Abstract: Aim of the paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet. The equations of continuity, momentum and energy are transformed into ordinary differential equations and solved numerically using Similarity transformation and Homotopy Perturbation Method. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameter are presented through Tables.

Keywords: Homotopy Perturbation Method, Similarity transformation method, Steady, boundary layer, variable thermal conductivity, stretching sheet, skin-friction coefficient and Nusselt number.

I. INTRODUCTION

Study of heat transfer in boundary layer find applications in extrusion of plastic sheets, polymer, spinning of fibers, cooling of elastic sheets etc. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Liquid metals have small Prandtl number of order $0.01 \sim 0.1$ (e.g. $Pr = 0.01$ is for Bismuth, $Pr = 0.023$ for mercury etc.) and are generally used as coolants because of very large thermal conductivity.

Aim of the present paper is to investigate effects of variable thermal conductivity, heat source/sink and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer on a non-conducting stretching sheet. Linear stretching of the sheet is considered because of its simplicity in modelling of the flow and heat transfer over stretching surface and further it permits the similarity solution, which are useful in understanding the interaction of flow field with temperature field. The heat source and sink is included in the work to understand the effect of internal heat generation and absorption [Chaim (1998)].

The Homotopy Perturbation Method is a combination of the classical perturbation technique and homotopy technique, which has eliminated the limitations of the traditional perturbation methods. This technique can have full advantage of the traditional perturbation techniques. J.H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media. J.H. He, A coupling method of homotopy technique and perturbation technique for nonlinear problems. To illustrate the basic idea of the Homotopy Perturbation Method for solving nonlinear differential equations, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \quad (1)$$

Subject to boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad (2)$$

Where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, and Γ is the boundary of the domain Ω .

The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N . Equation can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \quad (3)$$

By the homotopy technique, we construct a homotopy $V(r, p) : \Omega^*(0, 1) \rightarrow R$ which satisfy

$$H(V, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad (4)$$

$$H(V, p) = L(v) - L(u_0) + p[L(u_0) + p[N(v) - f(r)]] = 0 \quad (5)$$

Where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of which satisfies the boundary conditions.

$$\left. \begin{aligned} H(V, 0) &= L(v) - L(u_0) \\ H(V, 1) &= A(v) - f(r) \end{aligned} \right\} \quad (6)$$

Thus, the changing process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In Topology, this is called deformation and $L(v) - L(u_0)$, $A(v) - f(r)$ are called homotopic. According to the HPM, we can first use

the embedding parameter p as a “small parameter,” and assume that the solution of can be written as a power series in p :

$$V = V_0 + pV_1 + p^2 V_2 + \dots \quad (7)$$

Setting $p=1$ results in the approximate solution of

$$u = \lim_{p \rightarrow 1} V = V_0 + V_1 + \dots \quad (8)$$

The series is convergent for most cases; however, the convergent rate depends upon the Nonlinear operator $A(V)$. The second derivative of $N(V)$ with respect to V must be small because the parameter may be relatively large; that is, $p \rightarrow 1$.

In this paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet.

II. FORMULATION OF THE PROBLEM

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet. It is assumed that external field is zero, the electric field owing to polarization of charges and Hall Effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x -axis is taken along the sheet. The fluid occupies the upper half plane i.e. $y > 0$. The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (9)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (10)$$

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(k^* \frac{\partial T}{\partial y} \right) + Q (T - T_\infty), \quad (11)$$

where ε -perturbation parameter, η -similarity parameter $\{ = (c/\nu)^{1/2} y \}$, η_∞ -value of η at which boundary conditions is achieved, κ -uniform thermal conductivity, κ^* -variable thermal conductivity, ν -kinematic viscosity, ρ -density of fluid, ψ -stream function, σ -electrical conductivity, θ -dimensionless temperature $\{ = (T - T_\infty) / (T_w - T_\infty) \}$, τ_w -shear stress, S -heat source/sink parameter $\{ = Q/\rho C_p c \}$, T - fluid temperature.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stokes equation modifies into Prandtl's boundary layer equation.

The boundary conditions are.

$$\left. \begin{aligned} y = 0: \quad u = u_w(x) = c \quad v = 0, \quad T = T_w \\ y \rightarrow \infty: \quad u = U(x) = bx, \quad T = T_\infty \end{aligned} \right\} \quad (12)$$

Introducing the stream function $\psi(x, y)$ as defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (13)$$

the similarity variable $\eta = (c/\nu)^{1/2} y$ and

$$\Psi(x, y) = (c\nu)^{1/2} x f(\eta), \quad (14)$$

into the equations (3) and (5), we get

$$f''' + ff'' - (f')^2 + \lambda^2 = 0, \quad (15)$$

And

$$(1+\varepsilon) \theta'' + \varepsilon (\theta')^2 + \text{Pr} \theta' f + \text{Pr} S \theta = 0. \quad (16)$$

The governing boundary layer and thermal boundary layer equations (15) and (16) with the boundary conditions (12) are solved using Homotopy Perturbation Method.

Equations (15) and (16) are non-linear coupled differential equation. To solve these equations, we introduce the following Homotopy.

$$D(f, p) = (1-p) \left[\left(\frac{d^3 f}{d\eta^3} \right) - \left(\frac{d^3 f_I}{d\eta^3} \right) \right] + p \left[\frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta} \right)^2 + \lambda^2 \right] = 0 \quad (17)$$

$$D(\theta, p) = (1-p) \left[\left(\frac{\partial^2 \theta}{\partial \eta^2} + P_r S \theta \right) - \left(\frac{\partial^2 \theta_I}{\partial \eta^2} + P_r S \theta_I \right) \right] + p \left[(1+\varepsilon \theta) \frac{\partial^2 \theta}{\partial \eta^2} + \varepsilon \left(\frac{\partial \theta}{\partial \eta} \right)^2 + P_r \frac{\partial \theta}{\partial \eta} f + P_r S \theta \right] = 0 \quad (18)$$

With the following assumption

$$f = f_0 + p f_1 + p^2 f_2 + \dots \quad (19)$$

$$\theta = \theta_0 + p \theta_1 + p^2 \theta_2 + \dots \quad (20)$$

Using equation (19),(20) into equation (10) and (11) and on comparing the like powers of p , we get the zeroth order equation,

$$\left[\left(\frac{d^3 f_0}{d\eta^3} \right) - \left(\frac{d^3 f_I}{d\eta^3} \right) \right] + \left[\frac{d^3 f_0}{d\eta^3} + f_0 \frac{d^2 f_0}{d\eta^2} - \left(\frac{df_0}{d\eta} \right)^2 + \lambda^2 \right] = 0, \quad (21)$$

$$\left[\left(\frac{\partial^2 \theta_0}{\partial \eta^2} + P_r S \theta_0 \right) - \left(\frac{\partial^2 \theta_I}{\partial \eta^2} + P_r S \theta_I \right) \right] + \left[(1 + \varepsilon \theta_0) \frac{\partial^2 \theta_0}{\partial \eta^2} + \varepsilon \left(\frac{\partial \theta_0}{\partial \eta} \right)^2 + P_r \frac{\partial \theta_0}{\partial \eta} f + P_r S \theta_0 \right] = 0, \quad (22)$$

with the corresponding boundary conditions are of zeroth order equations are:

$$\eta = 0: f_0 = 0, f'_0 = 1, \theta_0 = 1; \quad \left. \begin{array}{l} \eta = \infty: f'_0 = \lambda, \quad \theta_0 = 0 : \end{array} \right\} \quad (23)$$

$$\frac{d^3 f_1}{d\eta^3} + (e^{-\eta} - \lambda e^{-\eta}) + (\lambda \eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1)(\lambda e^{-\eta} - e^{-\eta}) - (\lambda + e^{-\eta} - \lambda e^{-\eta})(\lambda + e^{-\eta} - \lambda e^{-\eta}) + \lambda^2 = 0, \quad (24)$$

$$\frac{\partial^2 \theta_I}{\partial \eta^2} + P_r S \theta_I + e^{-\eta}(1 + \varepsilon e^{-\eta}) + \varepsilon e^{-2\eta} - P_r e^{-\eta}(\lambda \eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1) - P_r S e^{-\eta} = 0. \quad (25)$$

With the corresponding boundary conditions are of first order equations are:

$$\eta = 0: f_0 = 0, f'_0 = 0, \theta_0 = 0 \quad \left. \begin{array}{l} \eta = \infty: f'_0 = 0, \quad \theta_0 = 0; \end{array} \right\} \quad (26)$$

Solving equations with corresponding boundary conditions, the following functions can be obtained successively, by summing up the results, and $p \rightarrow 1$ we write the $f(\eta)$, $\theta(\eta)$, profile as:

$$f(\eta) = \lambda \eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1 + (\eta e^{-\eta} + 4e^{-\eta} + 3\eta - 4), \quad (27)$$

$$\theta(\eta) = e^{-\eta} + \frac{A_2 e^{-\eta}}{(A_1)^2 + 1} + \frac{A_3 e^{-2\eta}}{(A_1)^2 + 4} + \frac{A_4 e^{-\eta}}{(A_1)^2} \left(\eta - \frac{\eta - 2}{(A_1)^2} \right) - \left(\frac{2A_4}{(A_1)^4} + \frac{A_3}{(A_1)^2 + 4} + \frac{A_2}{(A_1)^2 + 1} \right) e^{-\eta}. \quad (28)$$

$$\text{where } A_1 = \sqrt{P_r S} : A_2 = P_r S + P_r - \lambda P_r - 1 : A_3 = \lambda P_r - P_r - 2 : A_4 = \lambda P_r. \quad (29)$$

SKIN-FRICTION: Skin-friction coefficient at the sheet is given by

$$C_f = x f''(0). \quad (30)$$

NUSSELT NUMBER: The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = -\theta'(0) \quad (31)$$

Conclusion: It is observed from Table 1 as L increases, the numerical values of $f''(0)$ also increase. It is noted from Table 2 that the numerical values of $-\theta'(0)$ increase when λ increases and $-\theta'(0)$ decreases when ε increases. The skin-friction coefficient and Nusselt number are presented by equations (30) and (31) and they are directly proportional to $f''(0)$ and $-\theta'(0)$ respectively. The effects of ε , Pr and S on Nusselt number have been presented through Table 3 respectively.

Table 1

λ	$f''(0)$	λ	$f''(0)$
0.0	-1	0.1	-1.0800
0.01	-1.0098	1.0	0.0004
0.05	-1.0450	2.0	2.0175

Table 2

λ	$-\theta'(0)$	ε	$-\theta'(0)$
0.1	.81235	0.0	0.223558
0.5	.13629	0.05	0.215792
2.0	.24133	0.1	0.204672

Table 3

ε	S	Pr	Nu
0	0	0.5	2.010357
0.1	0	0.5	3.542315
0	-0.1	1	2.565423
0.1	-0.1	1	2.845633

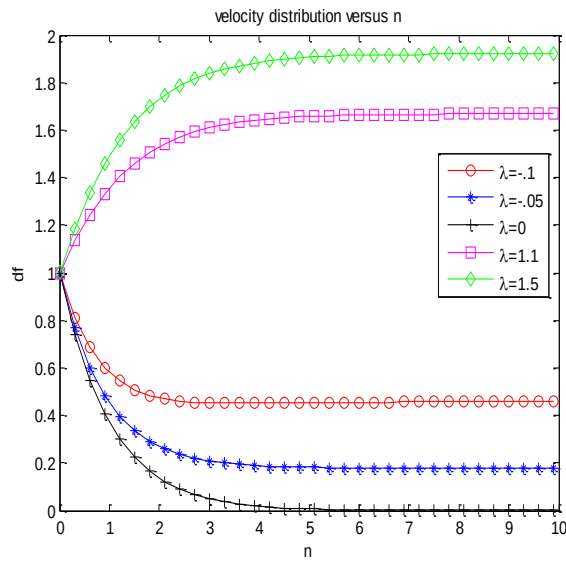


Fig 1

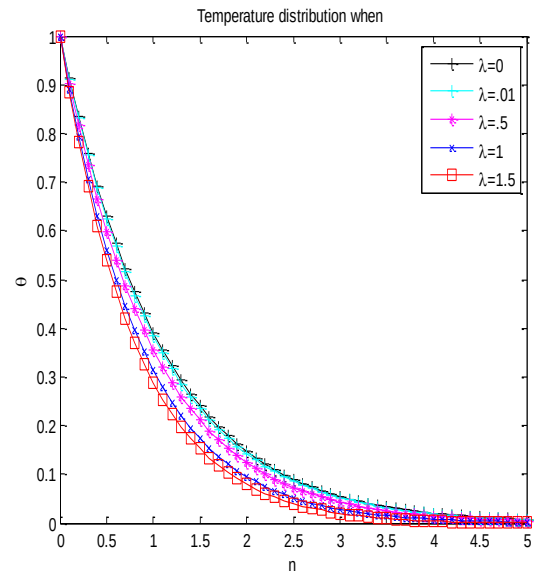


Fig 2

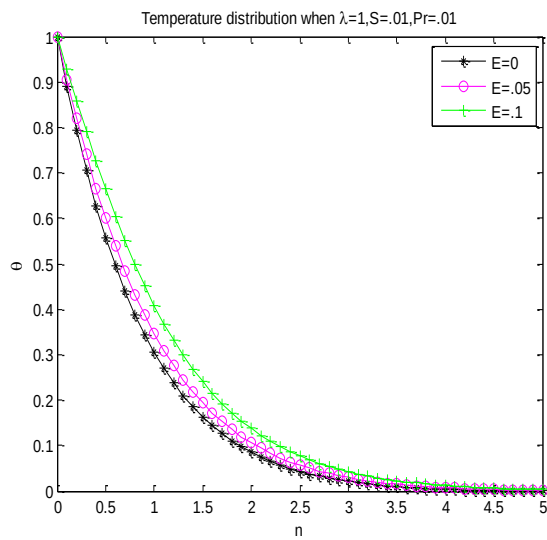


Fig 3

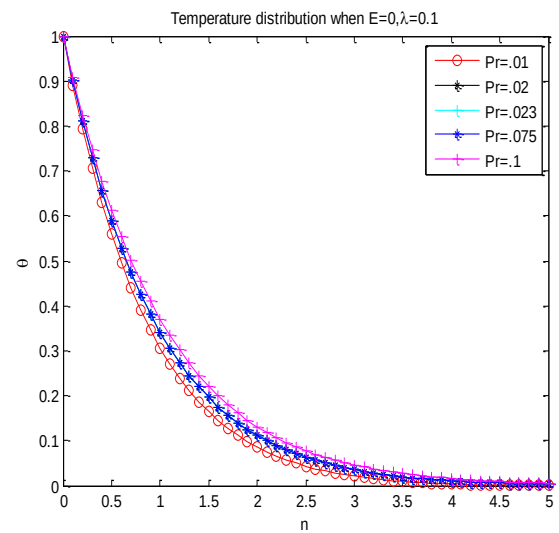


Fig 4

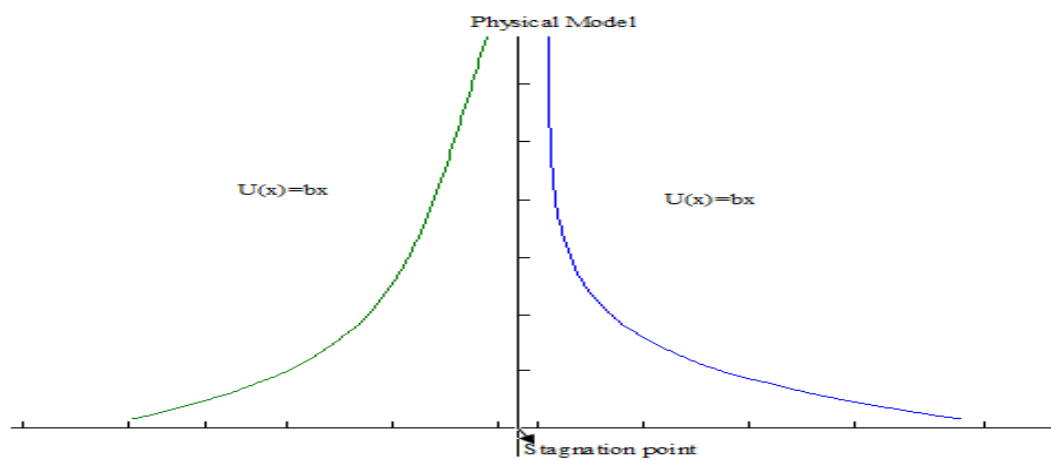


Fig 5

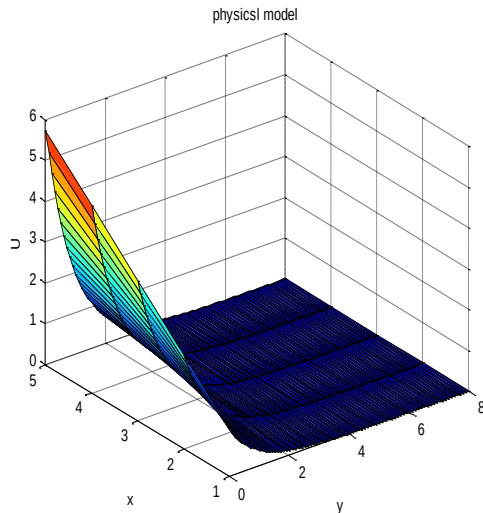


Fig 6

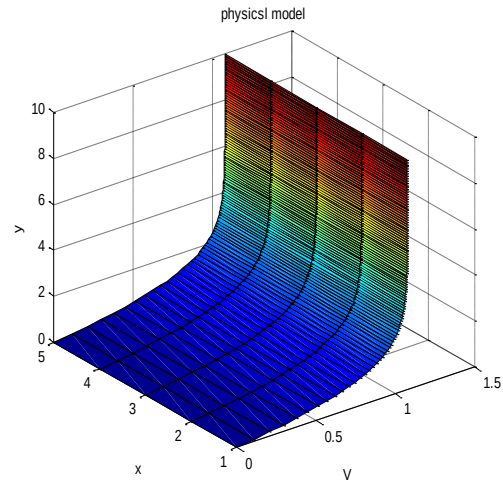


Fig 7

From figure 1, we observe that as λ increases, value of f'' also increases. From figure 2 it is observed that when λ increase simultaneously θ also increases. In figure 3, λ , S and Pr are constant but when ε increases θ will also increased. It is observed in figure 4, s , ε and λ are constant, when Pr increases, θ will also increase. Figure 5 is a physical model which becomes clearer from figure, 6 and 7.

References

- [1] Arunachalam, M. and N.R. Rajappa (1978). Forced convection in liquid metals with variable thermal conductivity and capacity. *Acta Mechanica* 31, 25-31.
- [2] Bansal, J.L. (1977). *Viscous Fluid Dynamics*. Oxford & IBH Pub. Co., New Delhi.
- [3] Chakrabarti, A. and A.S. Gupta (1979). Hydromagnetic flow and heat transfer over a stretching sheet. *Quarterly Journal of Applied Mathematics* 37, 73-78.
- [4] Bansal, J.L. (1994). *Magnetofluidynamics of Viscous Fluids*. Jaipur Pub. House, Jaipur, India.
- [5] Chen, C.H. (1998). Laminar mixed convection adjacent to vertical, continuously stretching sheet. *Heat and Mass Transfer* 33, 471-476.
- [6] J.H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media, *Comput. Method Appl. Mech. Engrg.*, 167 (1998) 57-68.
- [7] J.H. He, A coupling method of homotopy technique and perturbation technique for nonlinear problems, *Int. J. Nonlinear Mech.*, 35 (2000) 37- 43.
- [8] Chamka, A.J. and A.R.A. Khaled (2000). Similarity solution for hydromagnetic mixed convection and mass transfer for Hiemenz flow through porous media. *Int. Journal of Numerical Methods for Heat and Fluid Flow* 10, 94-115.
- [9] Sharma, P.R and U. Mishra (2001). Steady MHD flow through horizontal channel: lower being a stretching sheet and upper being a permeable plate bounded by porous medium. *Bull. Pure Appl. Sciences, India* 20E, 175-181.
- [10] Biazar J., Ayati Z. and Ebrahimi H. (2009) "Homotopy Perturbation Method for General Form of Porous Medium Equation," *Journal of Porous Media*, 12, 1121-1127.
- [11] Rafei M., Vaseghi J. and Ganji D. (2007) "Application of Homotopy-Perturbation Method for Systems of Nonlinear Momentum and Heat Transfer Equations," *Heat Transfer Research*, 38, 361-379
- [12] Jafari H., Zabihi M. and Saidy M. (2008) "Application of Homotopy Perturbation Method for Solving Gas Dynamics Equation," *Applied Mathematical Sciences*, 2, 2393-2396
- [13] Ganj D. and Esmailpour M. (2008) "A Study on Generalized Couette Flow by He's Methods and Comparison with the Numerical Solution," *World Applied Sciences Journal*, 4, 470-478