



Numerical Analysis of Wave Function Controlled by OLTP and Harmonic Oscillator in BEC Experiments

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Abstract: This paper will focus on numerical analysis of wave function under the action of optical lattice trapping potential (OLTP) applied along the direction of propagation together with a modified harmonic oscillator potential. This process is usually used in experiments that lead to form Bose-Einstein condensation BEC in ultra cold gases. A Crank-Nicolson scheme is employed for solve the Gross-Pitavskii equation with special care has been taken to nonlinearity term, time interval, and space steps. The results shows that the behavior of wave function is responsive to a variety of parameters such as the frequencies ratio of trapping oscillator, Interfering of optical laser beams, chemical potential and the energy.

Keywords: Laser cooled atom, BEC atom, Trapping, condensation, Atom Laser, Quantum oscillator

I. Introduction

Bose-Einstein condensation trapped in optical lattice potentials provides a unique environment for experimental and theoretical studies of a considerable amount of physical phenomenon [1-4]. The properties of ultra-cold atoms in OPLP in one, two and three dimensions have been investigated extensively specially in near-resonant and, far-detuned optical lattices, a variety of phenomena have been investigated such as the magnetic properties of atoms in optical lattice, revivals of wave-packed oscillation, and Bloch oscillations in accelerated lattices in past fifteen years [5-10]. Artificial crystals of light, consisting of hundreds of thousands of optical micro-traps, are normally created by interfering optical laser beams. These so-called optical lattices precede as resourceful potential landscapes to trap ultra-cold quantum gases of bosons and fermions. They form influential model systems of quantum many-body systems in periodic potentials for probing nonlinear wave dynamics and strongly connected quantum phases, building fundamental quantum gates or observing Fermi surfaces in periodic potentials [6]. Optical lattices represent a fast-paced modern and interdisciplinary field of research. An optical lattice is simply a set of standing wave lasers. The electric field of these lasers can interact with atoms - the atoms observe a potential and therefore gather in the potential minima. In the case of a typical one-dimensional setup, the wavelength of the opposing lasers is chosen so that the light shift is negative. This means that the potential minima occur at the intensity maxima of the standing wave. Furthermore, the natural beam width can constrain the system to being one-dimensional. To keep the atoms from distributing over too large a distance, the lattice is superimposed with an additional trap. This trap is generated by a dipole laser beam focused at the position of the atom cloud, perpendicular to the beam axis; this creates a Gaussian intensity profile. For small excursions from the trap centre this is a near harmonic trap. Along the beam axis, the trapping frequency is too low, though: atoms could spread out many 100 μm . To close the trap in this direction, a second (and later a third) perpendicular laser beam is focused onto the atom cloud.

If one of these laser beams is now collimated after passing through the atom cloud and retro-reflected on a mirror, the intensity and thus the trap-depth at the trap centre is doubled; but now a standing wave forms, with its first node at the surface of the retro-reflecting mirror. The interference pattern extends back to the atom cloud, producing an intensity modulation with a distance of half the laser wavelength between intensity maxima. A 2D or 3D lattice is formed by also retro-reflecting the other laser beams. The standing waves intersect and lattice sites are where all standing waves have an intensity maximum. Consider the oblate traps of one standing wave as parallel planes. Then two perpendicular groups of planes intersecting with each other form an array of cigar-shaped traps in a regular 2D lattice. A third group of parallel planes divide these 2D lattice sites into spherically symmetric traps arranged in a 3D optical lattice [6-14]. In some literatures, many authors investigated the effect of gravitation [15] by adding the gravitational potential as an external interaction. In this paper, we analyse in one dimension the influences of various terms in GPE on the distribution of the wave function under the action of two kind of trapping potential applied in parallel along the axis of propagation optical lattice external trapping potential which are typically used in experiments of BEC.

II. Theory

The Heisenberg interpretation for the time evolution of the field operator, with effective potential is given by[12]:

$$i\hbar \frac{\partial \hat{\psi}(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) + g \hat{\psi}^\dagger(\mathbf{r}, t) \hat{\psi}(\mathbf{r}, t) \right] \hat{\psi}(\mathbf{r}, t) \quad (1)$$

$\hbar$ is the Planck constant, $\hat{\psi}(\mathbf{r}, t)$ and $\hat{\psi}^\dagger(\mathbf{r}, t)$ are the quantum field operators which creates and annihilates a particle at position $\mathbf{r}$ at time t , $V(\mathbf{r}, t)$ is the external trapping potential, g is the interaction parameter. Replacing the quantum field $\hat{\psi}(\mathbf{r}, t)$ in (1) by the classical field $\psi(\mathbf{r}, t)$. It gives rise to a nonlinear Schrodinger equation, the well-known Gross-Pitaevskii equation (GPE) “which is a self-consistent mean field nonlinear Schrodinger equation (NLSE)” [16,17]

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g |\psi(\mathbf{r}, t)|^2 \right] \psi(\mathbf{r}, t) \quad (2)$$

For the Bose-Einstein condensed system. Here The external trapping potential $V(\mathbf{r})$ is taken to be time-independent. The macroscopic wave function/order parameter is normalized to the total number of particles in the system, which is conserved over time [17], i.e.

$$\int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1 \quad (3)$$

For ideal (non-interacting) gas, all particles occupy the ground state at $T = 0K$ and $\psi(\mathbf{r}, t)$. in the GPE describes the properties of all N particles in the system. For interacting gas, owing to the inter-particle interaction, not all particles condense into the lowest energy state even at zero temperature. This phenomenon is called the quantum depletion. One can assume that the a *semi-classical* approximation is valid, this means that the broaden in Doppler shifts due to the quantum uncertainty in momentum is small compared with the natural line-width, and the spatial coherence length of the atomic wave function is small compared with the optical wavelength. In addition the internal degrees of freedom must slow down much faster than the external degrees of freedom so that one can treat the atom as a classical particle experiencing an instantaneous force. This approximation seem to be comparable with the early BEC experiment results, in that experiments, a quadratic harmonic oscillator well was used to trap the atoms. Recently more advanced and complicated traps have been applied for studying BECs in laboratories [17,18, 19, 20, 21]. In order to solve equation (2) numerically along the X-Axis one can rearrange it as follow

$$\frac{\partial \psi(x, t)}{\partial t} = \frac{i\hbar}{2m} \nabla^2 \psi(x, t) - \frac{1}{\hbar} V(\mathbf{r}) \psi(x, t) - \frac{1}{\hbar} g |\psi(x, t)|^2 \psi(x, t) \quad (4)$$

The Crank-Nicolson Scheme for equation (4) is:

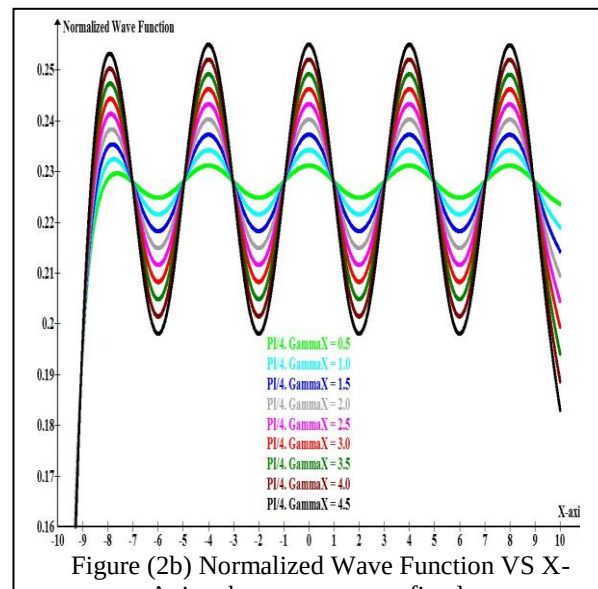
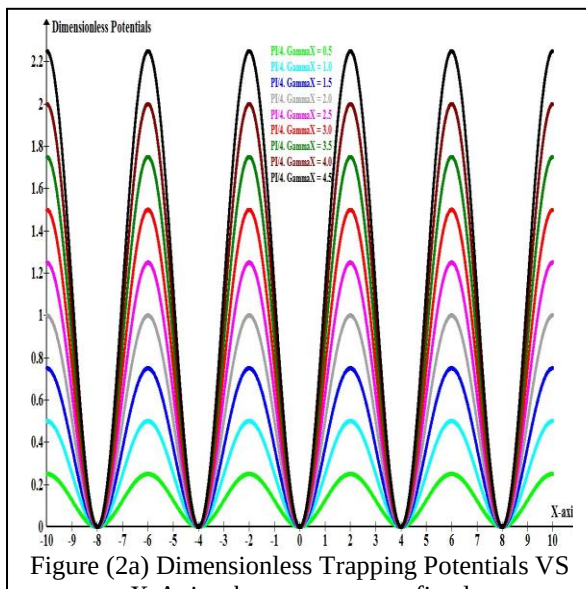
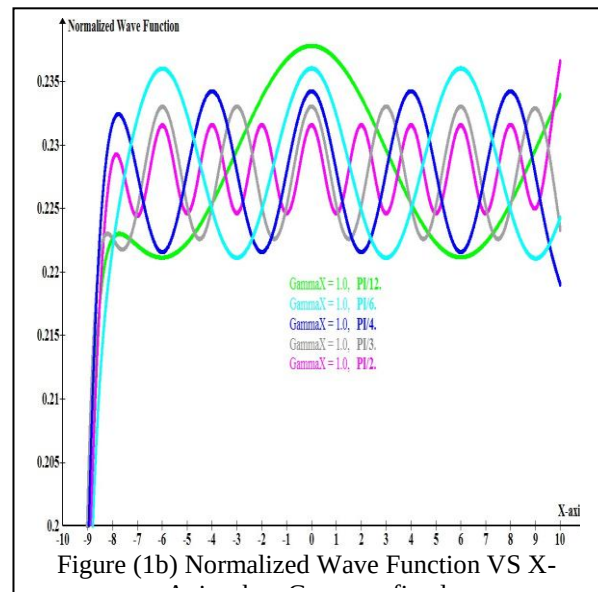
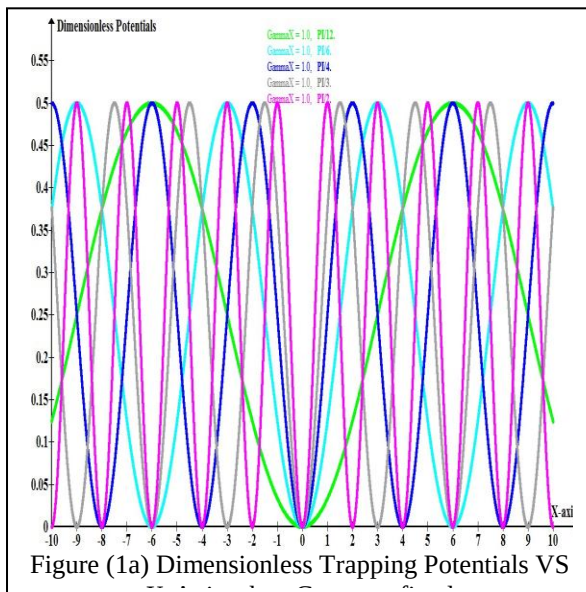
$$\frac{\psi_i^{n+1} - \psi_i^n}{k} = \frac{i\hbar}{2mh^2} [\psi_{i=1}^{n+1} - 2\psi_i^{n+1} + \psi_{i-1}^{n+1} - 2\psi_i^n + \psi_{i-1}^n] - \frac{1}{\hbar} V_i^n \psi_i^n - \frac{1}{\hbar} g |\psi_i^n|^2 \psi_i^n, \quad i = 1, 2, 3, \dots \quad (5)$$

Where k is the time interval and h is the space step. This scheme is unconditionally stable, time reversible, conserve the total particle number but it is not time transverse-invariant. A comparism tests with fully implicit and fully explicit finite difference methods are carried out but not include in this paper. Reader can refer to references [22], and [23] for a mathematical analysis of finite differences methods for Schrodinger equations in semi-classical regimes. In this work, we will analysis the wave function under the action of a typical optical lattice trapping potentials which are widely used in current experiments $V_{olpt}(x) = S_x E_x \sin^2(\hat{q}_x x)$, where $\hat{q}_x = 2\pi/\lambda_x$ is the angular frequency of the laser beam, with wavelength λ_x , that creates the stationary 2D periodic lattice, $E_r = (\hbar^2 \hat{q}_x^2)/2m$ is the recoil energy, and S_x is a dimensionless parameter characterizing the intensity of the laser beam. The optical lattice potential has periodicity $T_x = \pi/\hat{q}_x = \lambda_x/2$ along the x-axis. The choices for the scaling parameters t_0 and x_0 , the dimensionless potential $V(x)$, the energy unit $E_0 = \hbar/t_0 \rightarrow E_0 = \hbar^2/mr_0^2$, and the interaction parameter $g = 4\pi a_s N/r_0$ for external optical lattice trapping potentials are reads as follow: $t_0 = 1/\omega_r$, $r_0 = \sqrt{\hbar/m\omega_r}$, $k_r = 2\pi^2 r_0^2 S_x / \lambda_x^2$, $q_x = 2\pi r_0 / \lambda_x$.

III. Result and Discussion

First one can assume the atoms are tightly confined in two directions and can be successfully described by one-dimension by Appling optical Lattice potentials over lapping the harmonic potential along the x-axis. The time interval used in this solution is 0.00020 and the space step is 0.002500. The most factors which affect this numerical solution are the stability since a constant amplification in one time step turns into an exponential amplification over time. In addition to this classical stability requirement, we would also like that the norm of the system is unchanged. In the present case this corresponds to conservation of the particle number and that the energy is unchanged. These considerations from the physical properties of the system some time do not fulfill the norm and energy preservation properties. The careful adjustments between the time interval and space step will reflect that the physical properties of this system is satisfied and the result of this numerical solution can be explained satisfactory. The distributions of optical lattice potential over lapping harmonic oscillators for different value of q-factor (15° , 30° , 45° , and 60°) and fixed value of frequency ratio is shown in figure (1a). One can conclude from this figures that the shape of distributions of the potential are not affected

by the values of q-factor, it preserve the sine wave like distributions but the distance between two adjacent peaks of this distributions are increases as the q-factor of the OLTP decreases. Of course by changing OLTP will reflect definitely on the distribution of the normalized wave function along the x-Axis this shown in figure (1b). It seem from this figure that the distribution of the normalized wave function along the x-axis become more broaden and that is in harmony with the distribution of the OLTP, moreover the value of the normalized wave function increase as the q-factor decrease. This picture of the distributions of the wave function and the over all trapping potential will not sustained when the distribution of the OLTP kept unchanged while the the distribution of the harmonic oscillator alter by changing the value of the frequency ratio. Figure (2a) and (2b) shows the distributions of the trapping potentials and the distribution of the wave wave function along the X-axis for different values of frequencies ratio that are (0.5,1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, and 4.5). It's worth to say that these distributions have been computed after 20000 runs. The relation between the normalized wave function along the z-direction and trapping potential along y-direction and the axis of propagation x-axis is shown in figure (3) with a contour levels of the wave function apply over the surface view. At the centre of the trapping potential when $x = 0$, and $q = \pi/4$ there is nonlinear relation between the energy and the chemical potential as shown in figure (4). Moreover a linear relation is recorded between the normalized wave function and chemical potential as shown in figure (5). As well as this linear relation between the normalized wave function and the energy is recorded at $x = 0$, and $q = \pi/4$ as shown in figure (6). As a conclusion one can say that each term in Gross–Pitaevskii equation play minor and/or major role in analysis the wave function of the Bose-Einstein condensation and by careful handling these terms will bring the computational values to a satisfactory experimental one.



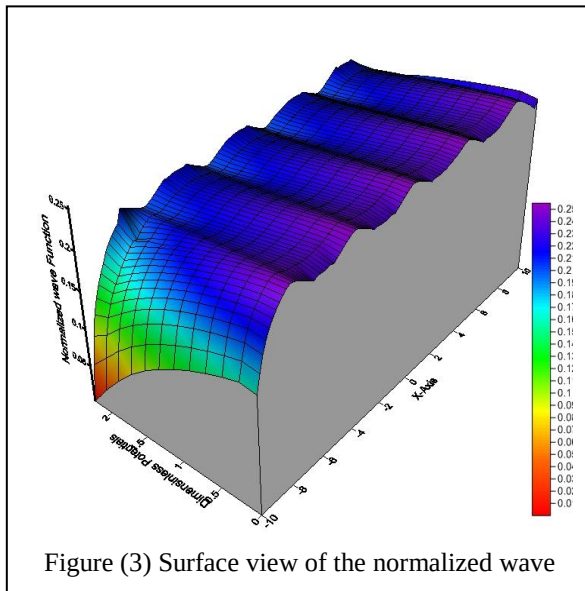


Figure (3) Surface view of the normalized wave

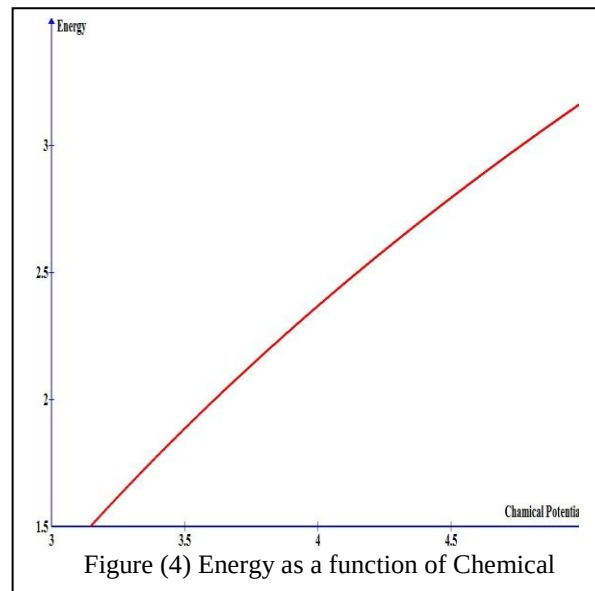


Figure (4) Energy as a function of Chemical

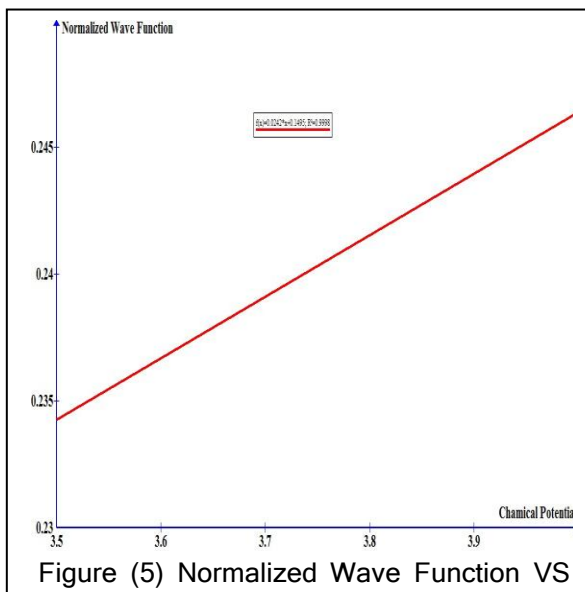


Figure (5) Normalized Wave Function VS

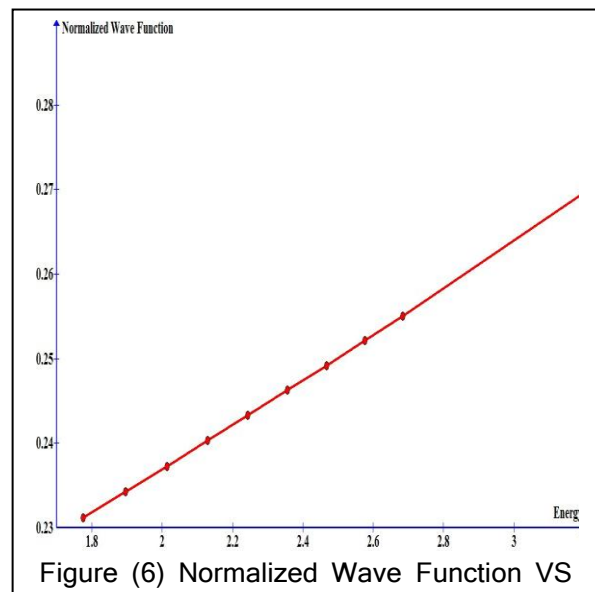


Figure (6) Normalized Wave Function VS

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