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## BIANCHI TYPE V COSMOLOGICAL MODEL WITH DECAYING COSMOLOGICAL CONSTANT TERM

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**Abstract:** In this paper, spatially homogeneous and anisotropic Bianchi type V space time with perfect fluid source and time dependent cosmological constant are considered. Exact solution of Einstein field equations are presented via a suitable assumption for the decaying law of cosmological constant, i.e.,  $\Lambda \propto R^{-2}$  [Chen, W. and Wu, Y.S. Phys. Rev. D 41,695, (1990)] Some Physical properties have been discussed in detail.

**Keywords:** Cosmological models; Bianchi type V models; Hubble's parameter; Constant deceleration parameter;  $\Lambda$ -term; perfect fluid.

### I. INTRODUCTION:

Observations for the duration of last few years provided increasingly strong evidence that Bianchi type models are playing important role in cosmology. Recent Observational data by WMAP satellite indicate that the universe is very close to flat. For a flat universe, its energy density must be equal to a certain critical density, which demands a huge contribution from some unknown energy source. Thus the Observational effects like the cosmic acceleration, sudden transition, flatness of universe and many more need explanation. It is generally believed that some kind of "dark energy" is pervading the whole universe. It is a hypothesis form of energy that permeate all of space and tends to increase the rate of expansion of the universe [1]. the most recent WMAP observations indicate that dark energy accounts for 72% of the total mass energy of the universe [2].

In order to explain the homogeneity and flatness of the presently observed Universe, it is usually assumed that this has undergone a period of exponential expansion [3,4,5,6]. Mostly the expansion of the universe is described within the framework of the homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmology. The reasons for this are purely technical. The simplicity of the field equations and the existence of analytical solutions in most of the cases have justified this over simplification for the geometry of space-time. However, there are no compelling physical reasons to assume the former before the inflationary period. To drop the assumption of homogeneity would make the problem intractable, while the isotropy of the space is something that can be relaxed and leads to anisotropy. Several authors [7,8,9,10,11] have studied particular cases of anisotropic models and found that the scenario predicted by the FRW model stand essentially unchanged even when large anisotropies were present before the inflationary period. Thus, the anisotropic Bianchi models become more interesting.

In recent years, models with relic cosmological constant  $\Lambda$  have drawn considerable attention among researchers for various aspects such as the age problem, classical tests, observational constraints on  $\Lambda$ , structure formation and gravitational lenses have been discussed in the literature. It is notable here that in the absence of any interaction with matter or radiation this would force the cosmological constant to be constant, but, in the presence of the interaction with matter or radiation, a solution of Einstein's field equation and unspecified equation of covariant conservation of energy with a time varying  $\Lambda$  can be found. For these solutions, conservation of energy requires that any decrease in the energy density of the vacuum component be compensated for by a corresponding increase in the energy density of matter or radiation. Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant are studied by Bertolami [12] Ratra and Peebles[13], Dolgov[14,15,16] Sahni and Starobinsky [17], Padmanabhan [18], Vishwakarma [19]. Recent observations by Perlmutter et al. [20] and Riess et al.[21] strongly favour a significant and positive  $\Lambda$ . Their results arise from the study of more than 50 type Ia supernovae with redshifts in the range  $0.10 \leq z \leq 0.83$  and suggest Friedmann models with negative pressure matter such as a cosmological constant, domain walls or cosmic strings Vilenkin,[22], Garnavich et.al.[23] Recently, Carmeli and Kuzmenko [24] have shown that the cosmological relativity theory, Behar and Carmeli [25], predicts the value  $\Lambda = 1.934 \times 10^{-35} \text{S}^{-2}$  for the cosmological constant. This value of  $\Lambda$  is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al.;[23] Perlmutter et al.[20], Riess et al[21], Schmidt et al.[26]). The main conclusion of these works is that the expansion of the universe is accelerating.

The study of Bianchi type V cosmological models create more interest as these Models contain isotropic special cases and permit arbitrary small anisotropy levels at some instantaneous cosmic time. Among different models

Bianchi type-V Universes are the natural generalizations of the open FRW model, which finally become isotropic. Roy and Prasad [27] have investigated Bianchi type V Universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have been studied by several authors, as: Farnsworth [28], Collins [29] Maartens and Nel [30], Wainwright et al. [31], Bee sham [32], Maharaj and Beesham [33], Hewitt and wainwright [34], Camci et al. [35], Meena and Bali [36], Pradhan et al. [37,38], Aydogdu and Salti [39] in different physical contexts. Christodoulakis et al. [40,41,42] have studied untitled diffuse matter Bianchi V Universe with perfect fluid and scalar field coupled to perfect fluid sources obeying a general equation of state. Following the work of Saha [43], Singh and Chaubey [44,45] have obtained the quadrature form of metric function for Bianchi type-V model with perfect fluid and viscous fluid. Several authors (Hajj-Boutros [46], Hajj-Boutros and Sfeila [47], Ram [48], Mazumder [49] and Pradhan and Kumar [50]) have investigated the solutions of EFEs for homogeneous but anisotropic models by using some different generation techniques. Bianchi spaces [ I-IX ] are useful tools in constructing models of spatially homogeneous Cosmologies (Ellis and MacCallum [51], Ryan and Shepley [52]) From these models, homogeneous Bianchi type V Universes are the natural generalization of the open FRW model which eventually isotropize. Camci et al. [53] derived a new technique for generating exact solutions of EFEs with perfect fluid for Bianchi type V space-time. Recently Bali and Singh [54], Rao et al. [55,56,57,58], Tiwari [59], Singh and Baghel [60] and Singh et al. [61] have studied Bianchi type V cosmological models n different context. Recently Singh, Ram and Zeyauddin [62] have extended the work of Singh and Kumar [63] to spatially homogeneous and totally anisotropic Bianchi type-V models with perfect fluid as source.

## II. Metric and Field equations :

We consider the homogeneous and anisotropic Bianchi type V space-time described by the line element

$$ds^2 = -dt^2 + A^2(t) dx^2 + e^{2mx} \{B^2(t)dy^2 + C^2(t)dz^2\} \quad (1)$$

We assume the cosmic matter consisting of perfect fluid represented by the energy momentum tensor

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \quad (2)$$

The Einstein field equations,

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G T_i^j + \Lambda g_i^j \quad (3)$$

$$8\pi G p - \Lambda = \frac{m^2}{A^2} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \quad (4)$$

$$8\pi G p - \Lambda = \frac{m^2}{A^2} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} \quad (5)$$

$$8\pi G p - \Lambda = \frac{m^2}{A^2} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} \quad (6)$$

$$8\pi G \rho + \Lambda = \frac{-3m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \quad (7)$$

$$0 = \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \quad (8)$$

Where A,B,C are the function of t and m is a constant and (.) dot denotes the derivative with respect to t. We define the average scale factor R as  $R^3 = ABC$

By equation (8) we get,

$$A^2 = BC$$

From equation (4) – (7), we get

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \quad (9)$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{\alpha}{R^3} \quad (10)$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{\alpha}{R^3} \quad (11)$$

where  $\alpha$  is constant.

Taking into account of the conservation equation,

$$\text{div}(T_i^j) = 0,$$

we have,

$$\dot{\rho} + (\rho + p) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (12)$$

$$\text{where, } p = \omega \rho \quad (13)$$

On integration of equation (9) – (11)

$$A = m_1 R \quad (14)$$

$$B = m_2 R \exp \left( -\alpha \int \frac{dt}{R^3} \right) \quad (15)$$

$$C = m_3 R \exp \left( +\alpha \int \frac{dt}{R^3} \right) \quad (16)$$

where  $m_1, m_2$ , and  $m_3$  are constant of integration satisfying  $m_1 m_2 m_3 = 1$ , suitable coordinate transformations constant

$m_2$  &  $m_3$  can be absorbed. Therefore  $m_2$  &  $m_3$  can be taken to be 1.  $\Rightarrow m_1 m_2 m_3 = 1$ .  
The generalized mean Hubble parameter  $H$  is given by,

$$H = \frac{\dot{R}}{R} = \frac{1}{3} (H_1 + H_2 + H_3) \quad (17)$$

where,  $H_1 = \frac{\dot{A}}{A}$ ,  $H_2 = \frac{\dot{B}}{B}$  and  $H_3 = \frac{\dot{C}}{C}$ , are Hubble directional factors.

Deceleration parameter  $q$  are defined as ,

$$q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 = -\frac{\dot{H}}{H^2} - 1 \quad (18)$$

and Anisotropic parameter  $\bar{A}$  is defined by,

$$\bar{A} = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 \quad (19)$$

We introduce volume expansion  $\theta$  and shear scalar  $\sigma$  as usual,

$$\theta = v_{;i}^i, \quad \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \quad (20)$$

For Bianchi type V metric, expressions for  $\theta$  and  $\sigma$  come out to be,

$$\theta = \frac{3\dot{R}}{R} \quad (21)$$

$$\text{And} \quad \sigma^2 = \frac{\alpha^2}{R^6} \quad (22)$$

Equation (4) – (8) and equation (12) can be written in terms of Hubble parameter  $H$ , shear scalar  $\sigma$  and deceleration parameter  $q$  as ,

$$\frac{m^2}{A^2} + H^2 (2q - 1) - \sigma^2 = 8\pi G p - \Lambda \quad (23)$$

$$\text{and} \quad \frac{-3m^2}{A^2} + 3H^2 - \sigma^2 = 8\pi G p + \Lambda \quad (24)$$

### III. Solution of the field equations :

In the system of equations (4) – (8) & (13), there are six equations and seven unknowns ( $A, B, C, \rho, p, \Lambda$  &  $G$ ). Therefore, one extra equation is needed to solve the system completely. For this purpose, we take a relation

$$\Lambda = \frac{a}{R^2}, \quad \text{where, } a \text{ is constant.} \quad (25)$$

The above variation law have already taken by Chen, W. et al .[64].

From equation (23) and (24), we get,

$$8\pi G (\rho - p) + 2\Lambda = -\frac{4m^2}{A^2} + 3H^2 - H^2 (2q - 1) \quad (26)$$

which leads to,

$$\dot{H} + 3H^2 - \frac{2m^2}{A^2} - 4\pi G (\rho - p) - \Lambda = 0 \quad (27)$$

For Stiff fluid ( $\rho = p$ ), Equation (27) leads to,

$$\dot{H} + 3H^2 - \frac{2m^2}{A^2} - \Lambda = 0 \quad (28)$$

using equation (25) in equation (28), we get,

$$\dot{H} + 3H^2 - \frac{2m^2}{A^2} - \frac{a}{R^2} = 0 \quad (29)$$

Substituting ( $H = \frac{\dot{R}}{R}$ ) in equation (29), we have,

$$\frac{\dot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{(2m^2 + a)}{R^2} = 0 \quad (30)$$

Integrating, equation (30) we get,

$$R = (Mt + t_1), \quad \text{where, } M = (2m^2 + a) \quad (31)$$

and  $t_1$  is constant of integration.

By using (31) in equation (9),(10) and (11), we get,

$$A(t) = m_1 (Mt + t_1) \quad (32)$$

$$B(t) = m_2 (Mt + t_1) \exp \left\{ \frac{a}{2(Mt + t_1)^2} \right\} \quad (33)$$

$$C(t) = m_3 (Mt + t_1) \exp \left\{ \frac{-a}{2(Mt + t_1)^2} \right\} \quad (34)$$

Where,  $m_1, m_2$  and  $m_3$  are constants.

Using equations (32) ,(33) and (34), we get directional Hubble's factor as ;

$$H_1 = m (Mt + t_1)^{-1} \quad (35)$$

$$H_2 = m (Mt + t_1)^{-1} - \alpha (Mt + t_1)^{-3} \quad (36)$$

$$H_3 = m (Mt + t_1)^{-1} + \alpha (Mt + t_1)^{-3} \quad (37)$$

After the suitable transformation, the metric (1) assumes the following form,

$$ds^2 = -dt^2 + ((Mt + t_1)^2 dx^2 + ((Mt + t_1)^2 \exp \{2mx + \frac{\alpha}{((Mt + t_1)^2)}\} dy^2 + ((Mt + t_1)^2 \exp \{2mx - \frac{\alpha}{((Mt + t_1)^2)}\} dz^2 \quad (38)$$

Therefore, the average generalized Hubble's parameter is given by,

$$H = \frac{M}{(Mt + t_1)} \quad (39)$$

And the physical quantities  $\theta$ ,  $\bar{A}$ ,  $\sigma$ ,  $q$ ,  $\rho$ ,  $p$  and  $\Lambda$  are as follows;

$$\theta = \frac{3M}{(Mt + t_1)}, \quad (40)$$

$$\bar{A} = \frac{2}{3} \frac{M^2 \alpha^2}{(Mt + t_1)^4}, \quad (41)$$

$$q = 0 \quad (43)$$

$$\rho = \frac{\alpha}{(Mt + t_1)^6}, \quad (44)$$

$$p = \frac{\alpha}{(Mt + t_1)^6} \quad (45)$$

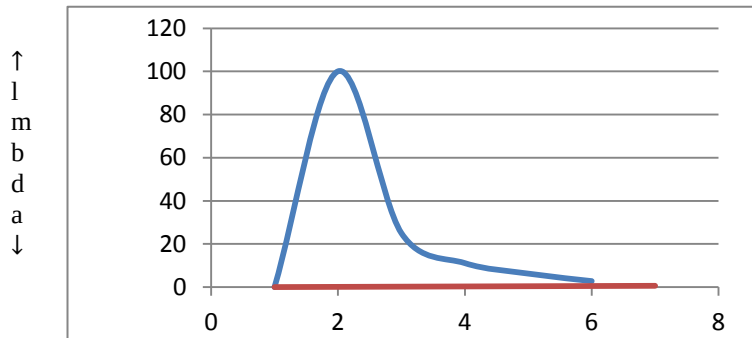
$$\Lambda = \frac{\alpha}{(Mt + t_1)^2} \quad (46)$$

which leads to,

$$G = - \left[ \frac{\alpha(Mt + t_1)^4}{8\pi\alpha} + \frac{\alpha}{8\pi} \right] \quad (47)$$

#### IV. Discussion:

For the model (38), we observe that, the spatial Volume  $V$  is zero at  $t = \frac{-t_1}{M}$  and expansion scalar  $\theta$  is infinite at  $t = \frac{-t_1}{M}$ , which shows that the universe starts evolving with zero volume and infinite rate of expansion at  $t = \frac{-t_1}{M}$ . Initially at  $t = \frac{-t_1}{M}$ , the space time exhibits a 'point type' singularity. And  $q = 0$  indicates that, the universe expand uniformly. As  $t = \frac{-t_1}{M}$ , physical parameters  $\theta$ ,  $\bar{A}$ ,  $\sigma$ ,  $\rho$ ,  $p$  and  $\Lambda$  are all infinite. The Gravitational constant  $G$  becomes negative during the whole span of evolution. The negative gravitational constant has been discussed by Vishwakarma [65]. At  $t \rightarrow \infty$  the spatial volume  $V$  becomes infinitely large. All parameters  $\theta$ ,  $\bar{A}$ ,  $\sigma$ ,  $\rho$ ,  $p$ ,  $\Lambda$ ,  $H_1$ ,  $H_2$  and  $H_3$  tend to zero. The ratio  $\frac{\sigma}{\theta} \rightarrow 0$  as  $t \rightarrow \infty$ , which shows that the model approaches isotropy for large value of  $T$ . We see that,  $\Lambda$  is positive and also  $\Lambda \sim \frac{1}{t^2}$ , i.e.,  $\Lambda$  is decreasing function of time [66].



We observe that, at the time of early universe the cosmological constant ( $\Lambda$ ) is negative and it increases rapidly during a very short period of time, at  $t = 0.1$ , the value of cosmological constant becomes positive and get its maximum value at  $t = 0.5$ , then it decreases as time increases [fig 1]. The value of  $\Lambda$  is small and positive at late time, which is supported by recent type Ia supernovae observations [20,21,26].

#### V. Conclusion:

In this paper, spatially homogeneous and anisotropic Bianchi type V space time with perfect fluid source and time dependent cosmological constant are considered. Exact solution of Einstein field equations are presented via a suitable assumption for the decaying law of cosmological constant, i.e.,  $\Lambda \propto R^{-2}$  [64]. we have observed that, The model starts with big bang at  $t = \frac{-t_1}{M}$  and the expansion in the model decreases as time increases, in the presence of perfect fluid. The energy density  $\rho \rightarrow \infty$  as  $t \rightarrow 0$  and  $\rho \rightarrow 0$  as  $t \rightarrow \infty$ . And since,  $\frac{\sigma}{\theta} \rightarrow 0$  at  $t \rightarrow \infty$ , hence the model isotropizes for large value of  $T$ . In this paper, we present a new solution to the Einstein's field equations for the orthogonal Bianchi type - V space time in the presence of a perfect fluid source and time dependent cosmological constant.

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