



Newtonian Heating and Mass Transfer Effects of Chemical Reactive Fluid on Natural Convection Flow Past a Vertical Surface with Radiation

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Abstract: The aim of this paper is to investigate the influence of chemical reaction and the combined effects of Newtonian heating and mass transfer on laminar boundary layer flow over an accelerated vertical plate in the presence of radiation. The governing equations are solved analytically by Laplace-transform technique. Graph results are presented for temperature, velocity, skin friction. The effects of various parameters on flow variables are illustrated graphically and the physical aspect of the problem is discussed.

Keywords: Chemical reaction; Newtonian heating; radiation; accelerated plate.

I. INTRODUCTION

In many cases in the process of free convection, chemical reaction takes place due to the presence of foreign masses (as impurities) in fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. Flow past a vertical plate with chemical reaction is analyzed by Fayez [1] and Sarada et. al. [2] under different physical conditions. Bhaben et. al. [3] analyzed chemical reaction effects on flow past a vertical plate with variable temperature.

In many practical situations where the heat transfer from the surface is taken to be proportional to the local surface temperature. Such types of proportionally condition of the heat transfer to the local surface temperature is termed as Newtonian heating. Unsteady boundary layer flow past a vertical plate with Newtonian heating is elucidated by Chaudhary et. al. [4]. Chemical reaction and mass transfer effects on flow past a surface with Newtonian heating is analyzed by Rajesh [5].

In the above mentioned studies, the effects of radiation on flow has not been considered. Actually, many processes in new engineering areas occur at high temperature and knowledge of radiative heat transfer becomes imperative for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering areas. Thermal radiation effect on flow past a vertical plate with mass transfer is elucidated by Rajput et. al. [6]. Natural convective flow past a plate in the presence of radiation is studied by Chaudhary et. al. [7]. Radiation and Newtonian heating effects on flow past a vertical plate under different physical conditions are analyzed by Narahari et. al. [8] and Das et. al. [9].

The aim of the present work is to provide an exact solution for the problem of chemically reactive fluid flow over a moving vertical plate in presence of radiation with Newtonian heating.

II. MATHEMATICAL ANALYSIS

Consider the unsteady two-dimensional flow of an incompressible and viscous fluid along an infinite vertical plate. The x' -axis is taken on the infinite plate and parallel to the free stream velocity and y' -axis normal to it.

Initially, the plate and the fluid are at same temperature T_∞' with concentration level C_∞' at all points. At time $t' > 0$. It accelerates with a velocity $\frac{U_R^3 t'}{v}$ in its own plane. At the same time, the heat transfer from

plate to the fluid is directly proportional to the local surface temperature T' and the plate concentration is changed to C_w' . It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_1 between the diffusing species and the fluid. Since the plate is infinite in extent therefore the flow variables are the functions of y' and t' only. The fluid is considered to be gray absorbing-emitting radiation but non scattering medium. The radiation heat flux in the x' -direction is considered negligible in comparison that of y' -direction. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} \quad (1)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - k_1 C' \quad (2)$$

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T'_\infty) + g \beta_c (C' - C'_\infty) \quad (3)$$

with following initial and boundary conditions

$$u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y', t' \leq 0 \quad (4)$$

$$\left. \begin{aligned} u' &= \frac{U_R^3 t'}{\nu}, \frac{\partial T'}{\partial y'} = -h_s T', C' = C'_w \text{ at } y' = 0, t' > 0 \\ u' &\rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty, t' > 0 \end{aligned} \right\}$$

The radiation heat flux term, by using the Rosseland's approximation is given by

$$q_r = -\frac{4\sigma}{3\kappa^*} \frac{\partial T'^4}{\partial y'} \quad (5)$$

where U_R is reference velocity, g is gravitational acceleration, C_p is specific heat at constant pressure, D is mass diffusivity, β is thermal expansion coefficient, β_c is concentration expansion coefficient, ρ is density, κ is thermal conductivity of fluid, κ^* is mean absorption coefficient, ν is kinematic viscosity and q_r is radiation heat flux, σ is Stefan-Boltzmann constant.

Using equation (5) in (1)

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty^3}{3\kappa^*} \frac{\partial^2 T'}{\partial y'^2} \quad (6)$$

Introducing the following dimensionless quantities

$$\begin{aligned} t &= \frac{t'}{t_R}, y = \frac{y'}{L_R}, u = \frac{u'}{U_R} \\ Pr &= \frac{\mu C_p}{\kappa}, Sc = \frac{\nu}{D}, \theta = \frac{T' - T'_\infty}{T'_\infty}, G = \frac{g \beta_r T'_\infty \nu}{U_R^3}, R = \frac{\kappa^* \kappa}{4\sigma T_\infty^3} \\ C &= \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gm = \frac{\nu g \beta_c (C'_w - C'_\infty)}{U_R^3}, k = \frac{\nu k_1}{U_R^2}, \Delta T = T'_w - T'_\infty, U_R = (\nu g \beta \Delta T)^{1/3}, \\ L_R &= \left(\frac{g \beta \Delta T}{\nu^2} \right)^{-1/3}, t_R = (g \beta \Delta T)^{-2/3} \nu^{1/3} \end{aligned} \quad (7)$$

where L_R is reference length, t_R is reference time, Gm is modified Grashof number, Pr is Prandtl number, Sc is Schmidt number and u is dimensionless velocity component, θ is dimensionless temperature, C is dimensionless concentration, μ is viscosity of fluid, t is time in dimensionless coordinate, R is radiation parameter and k is chemical reaction parameter.

The governing equations (1) to (3) reduce to the following non-dimensional form

$$Pr \frac{\partial \theta}{\partial t} = \left(1 + \frac{4}{3R}\right) \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - kC \quad (9)$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G\theta + Gm C \quad (10)$$

with the following initial and boundary conditions

$$u = 0, \theta = 0, C = 0 \text{ for all } y, t \leq 0 \quad (11)$$

$$\left. \begin{aligned} u &= t, \frac{\partial \theta}{\partial y} = -\gamma(1 + \theta), C = 1 \text{ at } y = 0, t > 0 \\ u &\rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty, t > 0 \end{aligned} \right\} \quad (12)$$

$\gamma = \frac{h_s v}{U_R}$ is Newtonian heating parameter. Equation (12) gives $\theta=0$ when $Y=0$ which physically means that no heating from the plate exists.

On solving equations (8) to (10) by Laplace-transform technique, we get

$$\theta = \exp(-2\gamma\eta\sqrt{t} + b^2t) \operatorname{erfc}(\eta\sqrt{a} - b\sqrt{t}) - \operatorname{erfc}(\eta\sqrt{a})$$

(13)

$$C = \frac{1}{2} \{ \exp(2\eta\sqrt{kSc t}) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{kt}) + \exp(-2\eta\sqrt{kSc t}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{kt}) \} \quad (14)$$

For $Sc \neq 1$

$$\begin{aligned} u = t & \left\{ (1+2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta \exp(-\eta^2)}{\sqrt{\pi}} \right\} \\ & + \frac{Gb}{a-1} \left\{ \frac{1}{4\sqrt{\pi t}} \exp(-\eta^2) + b \exp(b^2t - 2b\eta\sqrt{t}) \operatorname{erfc}(\eta - b\sqrt{t}) \right\} \\ & - \frac{Gb}{a-1} \left\{ \frac{1}{4\sqrt{\pi t}} \exp(-\eta^2 a) + b \exp(b^2t - 2\gamma\eta\sqrt{t}) \operatorname{erfc}(\eta\sqrt{a} - b\sqrt{t}) \right\} + \frac{Gm}{kSc} \operatorname{erfc}(\eta) \\ & - \frac{Gm}{2kSc} \left\{ \exp\left(\frac{kSc t}{1-Sc}\right) \right\} \left(\exp(2\eta\sqrt{\frac{kSc t}{1-Sc}}) \operatorname{erfc}\left(\eta + \sqrt{\frac{kSc t}{1-Sc}}\right) + \exp(-2\eta\sqrt{\frac{kSc t}{1-Sc}}) \operatorname{erfc}\left(\eta - \sqrt{\frac{kSc t}{1-Sc}}\right) \right) \\ & - \frac{Gm}{2kSc} \left(\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{kt}) + \left(\exp(-2\eta\sqrt{kSc t}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{kt}) \right) \right) \\ & + \frac{Gm}{2kSc} \exp\left(\frac{kSc t}{1-Sc}\right) \left(\exp(2\eta\sqrt{\frac{kSc t}{1-Sc}}) \operatorname{erfc}\left(\eta\sqrt{Sc} + \sqrt{\frac{kt}{1-Sc}}\right) + \exp(-2\eta\sqrt{\frac{kSc t}{1-Sc}}) \operatorname{erfc}\left(\eta\sqrt{Sc} - \sqrt{\frac{kt}{1-Sc}}\right) \right) \end{aligned}$$

$$\text{Where } b = \frac{\gamma}{\sqrt{a}}, \quad a = \frac{Pr}{1+R}, \quad \eta = \frac{y}{2\sqrt{t}} \quad (15)$$

III. SKIN-FRICTION

From velocity field, skin-friction at the plate in non dimensional form is expressed as:

$$\begin{aligned} \tau = - \left(\frac{\partial u}{\partial y} \right)_{y=0} & \\ & \frac{2t}{\sqrt{\pi t}} + \frac{Gb}{a-1} \left\{ \frac{b}{\sqrt{\pi t}} + b^2 \exp(b^2t) \operatorname{erfc}(-b\sqrt{t}) \right\} \\ & - \frac{Gb}{a-1} \left\{ \frac{b\sqrt{a}}{\sqrt{\pi t}} + b^2 \exp(b^2t) \operatorname{erfc}(-b\sqrt{t}) \right\} \\ & - \frac{Gm}{kSc} \left(\exp\left(\frac{kSc t}{1-Sc}\right) \operatorname{erf}\left(\sqrt{\frac{kSc t}{1-Sc}}\right) \sqrt{\frac{kSc}{1-Sc}} - \frac{Gm}{kSc} \sqrt{kSc} \operatorname{erf} \sqrt{kt} \right) \\ & + \frac{Gm}{kSc} \left(\exp\left(\frac{kSc t}{1-Sc}\right) \operatorname{erf}\left(\sqrt{\frac{kt}{1-Sc}}\right) \sqrt{\frac{kSc}{1-Sc}} \right) \end{aligned} \quad (16)$$

IV. DISCUSSION

Figure 1 elucidates the effects of Pr on temperature profile. It is observed that temperature is maximum at the plate then tend to zero far away from the plate. Further, thickness of thermal boundary layer decreases as Pr increases. This is due to the fact that thermal conductivity of fluid decreases with increasing Pr , resulting a decrease in thermal boundary layer thickness. It is also seen that it decreases steeply for $Pr = 7$ than that of $Pr = 0.71$.

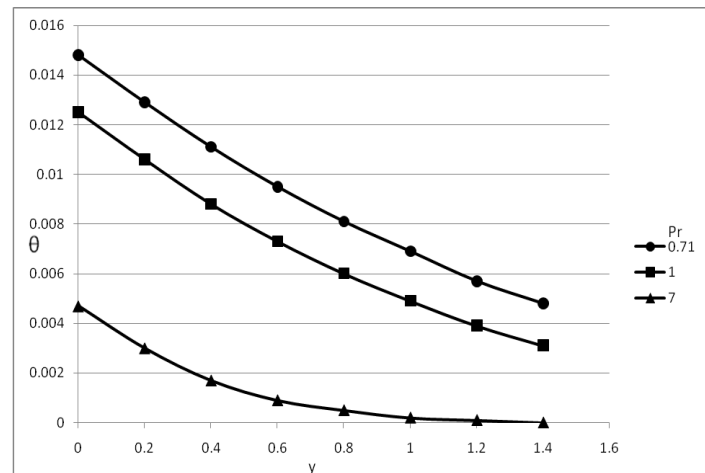


Figure 1: Temperature profile $Y=0.01$, $R=5$, $t=0.2$

Figure 2 illustrates the influences of Y and t on the velocity against η for $Pr=0.71$. It is noticed that at the plate, fluid velocity is equal to time then it increases and attains maximum velocity in the vicinity of the plate ($\eta < 1$) after that it decreases and vanish far away from the plate. Moreover, with an increase in Y the velocity increases. Figure 3 elucidates the effects of G , Gm on velocity profile. It is noticed that thickness of velocity boundary layer increases with η then attains its maximum value then decreases to asymptotic value. It is observed that there is an increase in velocity with an increase in the value of G and Gm . The reason is that the values of Grashof number and modified Grashof number has the tendency to increase the mass buoyancy effect. The increase in velocity due to increase in G and Gm is more near the plate than away from the plate.

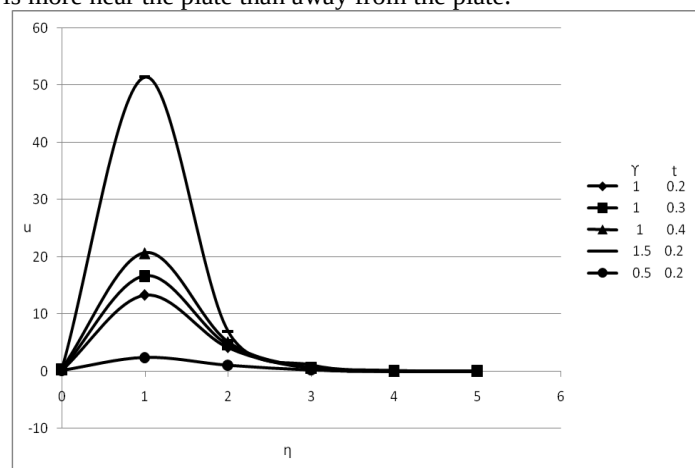


Figure 2: Velocity profile for $Pr=0.71$, $R=1$, $k=0.2$, $G=5$, $Gm=2$, $Sc=0.22$

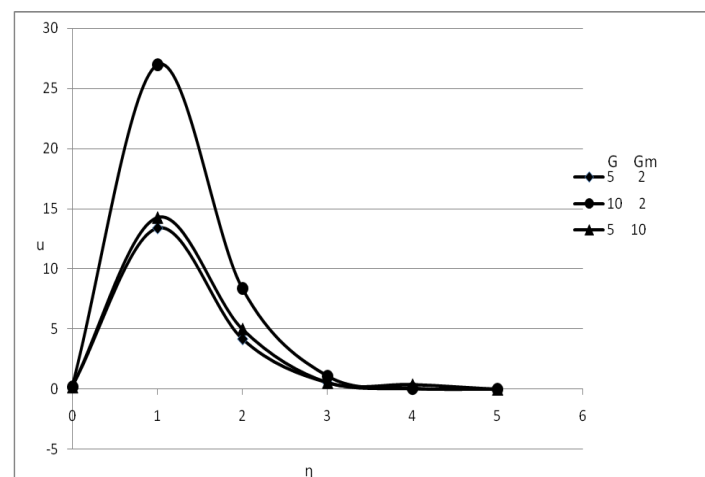


Figure 3: Velocity profile for $Pr=0.71$, $R=1$, $Y=1$, $t=0.2$, $k=0.2$, $Sc=0.22$

Skin- friction for different parameters against time t is presented in Figure 4. For smaller values of time the maximum value of magnitude of skin friction occurs and then it decreases rapidly with an increase in t . Moreover, the Figure reveals that magnitude of skin-friction increases with an increase of Gm and γ . It is observed from figure that magnitude of skin friction is lower for water vapor ($Sc=0.60$) in comparison to for Hydrogen gas ($Sc=0.22$). Physically, it is correct since an increase in Sc serves to increase momentum boundary layer thickness. Moreover, magnitude of skin friction decreases with an increase in the value of chemical reaction parameter k , the reason is that increasing value of chemical reaction parameter reflects decrease in kinematic viscosity or viscosity of fluid which results decrease in the value of magnitude of skin friction.

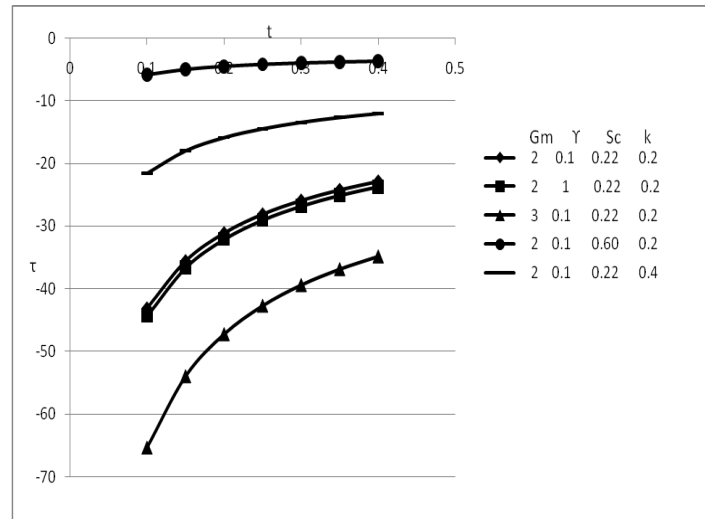


Figure 4: Skin-friction for $R=1, G=5, Pr=7$

V.

The results of the flow problem indicates:

1. Increasing Prandtl Number the temperature decreases.
3. Fluid velocity increases with an increase in Grashof number, modified Grashof number, time and Newtonian heating Parameter.
4. There is a fall in the value of skin friction with an increase in Schmidt number, chemical reaction parameter and rise with an increase in value of modified Grashof number and Newtonian heating parameter.

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