



Lowest Quantum Cost Technique of Reversible Logic Gate

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Abstract: An important feature of quantum computing is reversibility, because evolution of a quantum system is described by a unitary operator and hence it is reversible. Quantum cost is another important factor of any reversible circuit. The lowest quantum cost indicates the lowest price of reversible circuit. By lowest quantum cost can make a quantum computer. In this paper we show how to find the lowest quantum cost and show an advanced Toffoli Gate that contains lowest quantum cost.

Index Terms: Advanced Toffoli Gate, Quantum cost, Quantum computer, reversible logic gates Introduction

I. Introduction

Reversible logic design differs significantly from the traditional conventional logic design. In reversible logic circuit, the number of input lines should be equal to the number of output lines with one to one mapping[1]. The main challenges are reducing the hardware complexity, delay, quantum cost, power dissipation, garbage outputs. Energy loss in digital designing is an important factor for marking the level of performance. While some portion of the energy is dissipated due to non-ideality of switches, the other results from the equality of materials used. Though heat loss has been dramatically reduced by complex integration levels and high end fabrication processes, the part of reducing energy dissipation in digital circuits is still left undone.

It is markable that the reversible logic shows its effect and its applications in promising computer paradigms with applications in emerging technologies such as Quantum computer, Quantum Dot Cellular Automata, optical computing etc.[2,3,4,5,6,7]. Bennet [8] shows that if the system is reversible then zero power dissipation is possible. Demonstrated by R. Landauer in 1960s, the energy dissipation for every bit of information exchange associated with even high-tech systems designed with traditional and irreversible logic was numerically equal to $kT \ln 2$; where k is described as the Boltzmann constant and T as the absolute temperature [8].

Quantum cost, the cost of a circuit, refers to the overall expenses, pertaining to the use of Reversible Logic Gates for synthesizing a given logical function, incurred in designing of the circuit in terms of the cost of the primitive Reversible Logic Gates used. The quantum cost of the circuit is determined by calculating the already known cost of the primitive gates required for realizing the Arithmetic and Logic circuit units. This paper presents 3*3 reversible gate namely Advanced Toffoli gate, that is, it has 3-input lines and 3-output lines. The main attractive feature of this gate is quantum cost. It has a 3 quantum cost which is less than 5 quantum cost. It works under some conditions and application may be same as traditional conventional Toffoli gate.

II. Basic Reversible gate

A. Toffoli Gate[9]

It has a quantum cost of five and described by: Input=(A,B,C): output=(P=A, Q=B, R=AB ⊕ C);

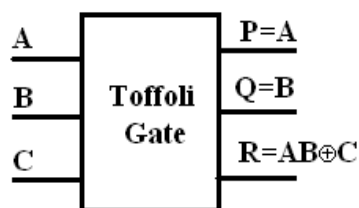


Fig 1.a Block diagram

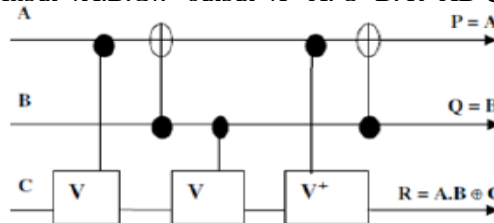


Fig 1.b Quantum implementation of Toffoli Gate

B. Peres Gate[10]

It has a Quantum Cost of four and described by: Input=(A,B,C); Output=(P=A, Q=A $\oplus$ B, R=AB $\oplus$ C)

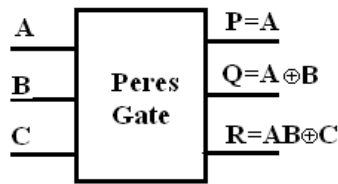


Fig 2.a Block diagram

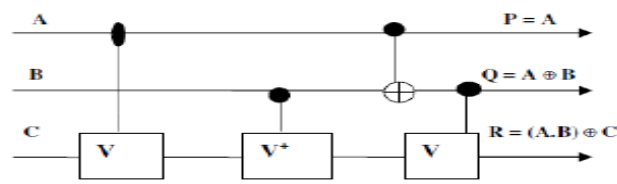


Fig 2.b Quantum implementation of Peres Gate

C. Feynman Gate[11]

It has a Quantum Cost of one and described by: Input=(A,B); output=(P=A, Q=A $\oplus$ B)

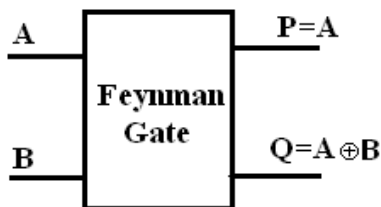


Fig 3.a Block diagram

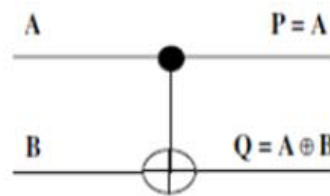


Fig 3.b Quantum implementation of Feynman Gate

III. The Internal mechanism of proposed 3*3 reversible Advanced Toffoli Gate

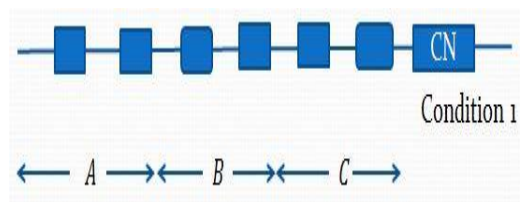
From Mersenne prime[12] we get, $2^n - 1$ where n number of inputs & 2 indicates composed double not gate.

Now if we consider n=3 then

$$2^3 - 1 = 7 \text{ not gate}$$

But we have to composed every two bits and make a single composed double not gate. Then we get three composed

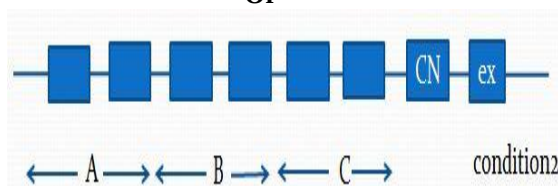
double not gate and one control not gate. We can write the following style:



By OR relation we can

$$\text{write, } 2^1_{+2} 1_{+2} 1_{+1}$$

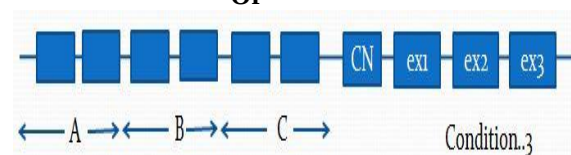
Or



By OR relation we can

$$\text{write, } 2^1_{+2} 1_{+2} 1_{+2} 0$$

Or



By OR relation we

$$\text{can write, } 2^1_{+2} 1_{+2} 1_{+2} 0$$

Quantum cost of composed double not gate is zero [13] and it has been used three composed double not gate and single control not gate and also used extra not gate for showing same result as traditional Toffoli Gate. It performs three bit OR operation and has been used three composed double not gate and so it's quantum cost is 3. Extra composed not gate does not effect on its internal function or does not effect on it's quantum cost because $2^1+2^1+2^1+1$, $2^1+2^1+2^1+2^0$ and $2^1+2^1+2^1+4^0$ are contains same value. The internal function will be used the following technique. Now, check for first condition. If

$$A=0, B=1, C=0; A=1, B=0, C=0 \text{ and } A=1, B=1, C=1$$

Then the output will be zero for 1st condition. These output are same as traditional Toffoli gate for 1st condition. If

$$A=0, B=0, C=1; A=0, B=1, C=1; A=1, B=0, C=1 \text{ and } A=1, B=1, C=0.$$

Then the output will be 1 for 2nd condition. These output are same as traditional Toffoli gate for 2nd condition. And if

$$A=0, B=0, C=0.$$

Then the output will be zero for 3rd condition. This output is same as traditional Toffoli gate for 3rd condition.

Our proposed Advanced Toffoli Gate will perform operation for above three conditions with minimum quantum cost than traditional Toffoli Gate. It's Block diagram is seem to be same as traditional Toffoli gate but internal function is differ from traditional Toffoli Gate.

IV. The proposed 3*3 reversible Advanced Toffoli Gate

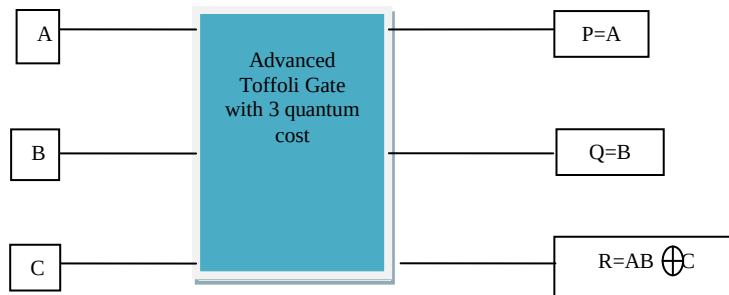


Fig.4: proposed 3*3 reversible Advanced Toffoli Gate

V. Comparison between Advanced Toffoli Gate and Traditional Toffoli Gate results

There is a comparison enlisted in a table between Advanced Toffoli Gate and Traditional Toffoli Gate results

Sl.	Advanced Toffoli Gate	Traditional Toffoli Gate
1.	It has been used 3 quantum cost.	It has been used 5 quantum cost.
2	It has been used 3 composed double not gate and 1 control not gate.	It has been used 5 gates.
3	It performs 3 times OR operation. Eg. $2^1+2^1+2^1+1$.	It performs 4 times AND operation and finally 1 time X-OR operation. Eg. $0 \text{ AND } 0=0$, $0 \text{ AND } 1=0$, $1 \text{ AND } 0=0$, $1 \text{ AND } 1=1$ and X-OR with these results.
4	OR operation performs less power dissipation. Eg. For two bits we can consider $0 \text{ OR } 0=0$, $0 \text{ OR } 1=1$, $1 \text{ OR } 0=1$ and $1 \text{ OR } 1=1$.	AND operation performs more power dissipation. Eg. For two bits we can consider $0 \text{ AND } 0=0$, $0 \text{ AND } 1=0$, $1 \text{ AND } 0=0$ and $1 \text{ AND } 1=1$.

Table 1: Comparison between Advanced Toffoli Gate and Traditional Toffoli Gate results

From the comparison defined in table we can easily express that Advanced Toffoli Gate shows better performance than traditional Toffoli Gate.

VI. Conclusion

This paper highly indicates how to reduce quantum cost under some satisfied conditions. Low quantum cost circuit will help to build up quantum computer in future

VII. References

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