



Free Convective Flow of a Viscous Fluid Past a Hot Vertical Plate With Constant Porosity and Span-wise Co-sinusoidal Temperature Using HPM

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Abstract A study of the free convective flow of a viscous fluid past a hot vertical porous plate is done with the assumption that the suction velocity is constant and normal to the plate and the plate temperature is span-wise co-sinusoidal. Approximate solutions of the equations of motion and energy have been obtained by the method of Regular Perturbation along with Homotopy Perturbation method. The effects of the different flow parameters affecting the mean velocity, temperature, skin-friction and rate of heat transfer have been presented through the graphs and tables.

I. Introduction

The problem of viscous free convective boundary layer flow past a hot vertical plate was discussed by schlichting [1]. Soundalgekar [2] has discussed the fully developed free convective flow of a viscous Fluid past a hot vertical porous plate with constant suction and magnetic field.

The problem of free convection turns out to be very complicated in fluid fuel nuclear reactors as the plate temperature is really not constant. Mishra and Mohapatra [3] have investigated the unsteady free convective viscous flow with magnetic field assuming the plate temperature to be a function of time i.e. periodic. Another possible plate temperature in such types of problems can be span-wise co-sinusoidal. Flow of an electrically conducting incompressible fluid past a hot vertical porous wall in the presence of transverse magnetic field with periodic temperature was discussed by Sharma & Sarangi [4].

Homotopy Perturbation Method (HPM) is a brilliant method that was introduced by He [5, 6]. These days it has been utilized by many researchers to find an analytical solution. Dehghan and Shakeri [7] studied the problem of modeling of flow in porous media. Siddiqui et al. Ganji and sadighi [8] solved nonlinear heat transfer and porous media equations by use of HPM and variational iteration method, and found a convincing agreement between these methods and exact solution. Biazar et al. [9] solved general form of porous medium equation by HPM and compared with the Adomian decomposition with a very good agreement between them. There are many other related works in [10-15].

In this Paper, our aim is to study the free convective flow of a viscous Fluid past a vertical plate with constant porosity and span-wise co-sinusoidal temperature by using Regular Perturbation along with Homotopy Perturbation method.

II. The Basic of Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following functional Equation

$$A(u) - f(r) = 0, \quad r \in \Omega; \quad (1)$$

with the following boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \quad (2)$$

where A is a functional operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω . The operator A can be decomposed into a linear part and a nonlinear one, designated as L and N respectively. Hence Eq. (1) can be written as the following form:

$$L(u) + N(u) - f(r) = 0. \quad (3)$$

Using homotopy technique, we construct a homotopy $v(r, p): \Omega \times [0, 1] \rightarrow R$ which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_i)] + p[A(v) - f(r)] = 0, \quad (4)$$

where $p \in [0, 1]$ is an embedding parameter u_i and is an initial approximation for the solution of Eq.(1) which satisfies the boundary conditions. Obviously, from Eq. (4) we will have

$$H(v, 0) = L(v) - L(u_i) = 0, \quad (5)$$

$$H(v, 1) = A(v) - f(r) = 0. \quad (6)$$

By changing the values of p from zero to unit, $v(r, p)$ change from $u_i(r)$ to $u(r)$, in topology this is called deformation, and $L(v) - L(u_i)$ and $A(v) - f(r)$ are called homotopy. Due to the fact that can be considered as a small parameter, hence we consider the solution of Eq.(4) as a power series in p as the following:

$$v = v_0 + p^1 v_1 + p^2 v_2 + \dots, \quad (7)$$

setting $p = 1$ results in an approximation solution of Eq. (1)

$$u = \lim_{p \rightarrow 1} v_0 + v_1 + v_2 + \dots \quad (8)$$

III. Formulation of the Problem

Let the plate be the z^* , x^* plane and the y^* axis is normal to it and the positive direction of x^* axis be vertically upwards. Let the plate be uniformly porous and the suction velocity normal to it be v_0 . Let (u^*, v^*, w^*) be the component of velocity of the fluid at any point (x^*, y^*, z^*) . Since $v^* = v_0$ throughout w^* is independent of z^* and we assume $w^* = 0$ throughout. Let the fluid velocity parallel to the x^* axis at infinity be U_∞ and the span wise co sinusoidal plate temperature θ_w^* be

$$\theta_w^* = \theta_0 \left(1 + \epsilon \cos \frac{\pi z^*}{L} \right), \quad (9)$$

where ϵ a small positive number, L is the wave length and θ_0 is constant. Using the momentum and energy equation including the viscous dissipative term are given by

$$v_0 \frac{\partial u^*}{\partial y^*} = v \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) + g\beta\theta^* \quad (10)$$

$$\rho C v_0 \frac{\partial \theta^*}{\partial y^*} = K \left(\frac{\partial^2 \theta^*}{\partial y^{*2}} + \frac{\partial^2 \theta^*}{\partial z^{*2}} \right) + \eta_0 \left(\left(\frac{\partial u^*}{\partial y^*} \right)^2 + \left(\frac{\partial u^*}{\partial z^*} \right)^2 \right) \quad (11)$$

with corresponding boundary conditions:

$$\left. \begin{aligned} y^* = 0: u^* = 0, \theta^* &= \theta_0 \left(1 + \epsilon \frac{\cos \pi z^*}{L} \right) \\ y^* \rightarrow \infty: u^* &= U_\infty, \theta^* = 0 \end{aligned} \right\} \quad (12)$$

Introducing the following non dimensional quantities

$$u = \frac{u^*}{U_\infty} = \text{dimensionless velocity in } x^* \text{-direction}$$

$$\left. \begin{aligned} y &= \frac{y^*}{l} \\ z &= \frac{z^*}{l} \end{aligned} \right\} = \text{dimensionless space variables}$$

$$\theta = \frac{\theta^*}{\theta_0} = \text{dimensionless temperature}$$

$$P = \frac{\eta_0 C}{K} = \text{Prandtl number}$$

$$G = \text{Grashoff number}$$

$$E = \text{Eckert number}$$

we get

$$\frac{\partial u}{\partial y} = -\frac{1}{R} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - RG\theta \quad (13)$$

$$\frac{\partial \theta}{\partial y} = -\frac{1}{PR} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) - \frac{E}{R} \left(\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right) \quad (14)$$

The corresponding boundary conditions:

$$\left. \begin{aligned} y=0: u=0, \theta &= 1 + \epsilon \cos \pi z \\ y \rightarrow \infty: u=1, \theta &= 0 \end{aligned} \right\} \quad (15)$$

Assuming

$$\left. \begin{aligned} u &= U + \epsilon u_i + O(\epsilon^2) \\ \theta &= \phi + \epsilon \theta_i + O(\epsilon^2) \end{aligned} \right\} \quad (16)$$

where U and ϕ are functions of y only and represent the velocity and temperature fields respectively when the /plate temperature is constant and equal to θ_0 where u_i and θ_i are functions of both y and z . where $i \geq 1$.

Substituting the assumptions made in Eq. (13) into Eq. (10), and (11) and on comparing the coefficients like power of ϵ , we get the set of zeroth order equations

$$\frac{d^2 U}{dy^2} + R \frac{dU}{dy} + R^2 G \phi = 0, \quad (17)$$

$$\frac{d^2 \phi}{dy^2} + PR \frac{d\phi}{dy} + EP \left(\frac{dU}{dy} \right)^2 = 0, \quad (18)$$

with the boundary conditions:

$$\left. \begin{aligned} y=0: U=0, \phi &= 1, \\ y \rightarrow \infty: U=1, \phi &= 0. \end{aligned} \right\} \quad (19)$$

and the set of first order equations:

$$\left. \begin{aligned} \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + R \frac{\partial u_1}{\partial z} + R^2 G \theta_1 &= 0, \\ \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} + PR \frac{\partial \theta_1}{\partial y} + 2EP \frac{\partial U}{\partial y} \frac{\partial V}{\partial y} &= 0 \end{aligned} \right\} \quad (20)$$

with boundary conditions:

$$\left. \begin{aligned} y=0: u_1=0, \theta_1 &= \cos \pi z \\ y \rightarrow \infty: u_1=0, \theta_1 &= 0 \end{aligned} \right\} \quad (21)$$

Assuming

$$\begin{aligned} u_1 &= v(y) \cos \pi z \\ \theta_1 &= \psi(y) \cos \pi z \end{aligned}$$

and using these assumptions into eq.(20) and eq.(21), we get

$$\frac{d^2 V}{dy^2} + R \frac{dV}{dy} - \pi^2 V + R^2 G \psi = 0 \quad (22)$$

$$\frac{d^2 \psi}{dy^2} + PR \frac{d\psi}{dy} - \pi^2 \psi + 2EP \frac{dU}{dy} \frac{dV}{dy} = 0 \quad (23)$$

with boundary conditions are

$$\left. \begin{aligned} y=0: V=0, \psi=1 \\ y=\infty: V=0, \psi=0 \end{aligned} \right\} \quad (24)$$

Equations (17),(18) and (22),(23) are non-linear coupled differential equation. To solve these equations, we introduce the following Homotopy.

$$\begin{aligned} D(U, p) &= (1-p) \left[\left(\frac{d^2 U}{dy^2} + R^2 \frac{dU}{dy} \right) - \left(\frac{d^2 U_I}{dy^2} + R^2 \frac{dU_I}{dy} \right) \right] + p \left(\frac{d^2 U}{dy^2} + R^2 \frac{dU}{dy} + R^2 G \phi \right) \\ &= 0 \end{aligned} \quad (25)$$

$$\begin{aligned} D(\phi, p) &= (1-p) \left[\left(\frac{d^2 \phi}{dy^2} + PR \frac{d\phi}{dy} \right) - \left(\frac{d^2 \phi_I}{dy^2} + PR \frac{d\phi_I}{dy} \right) \right] + p \left(\frac{d^2 \phi}{dy^2} + PR \frac{d\phi}{dy} + EP \left(\frac{dU}{dy} \right)^2 \right) \\ &= 0 \end{aligned} \quad (26)$$

$$\begin{aligned} D(V, p) &= (1-p) \left[\left(\frac{d^2 V}{dy^2} + R \frac{dV}{dy} - \pi^2 V \right) - \left(\frac{d^2 V_I}{dy^2} + R \frac{dV_I}{dy} - \pi^2 V_I \right) \right] + p \left(\frac{d^2 V}{dy^2} + R \frac{dV}{dy} - \pi^2 V + R^2 G \psi \right) \\ &= 0 \end{aligned} \quad (27)$$

$$\begin{aligned} D(\psi, p) &= (1-p) \left[\left(\frac{d^2 \psi}{dy^2} + PR \frac{d\psi}{dy} - \pi^2 \psi \right) - \left(\frac{d^2 \psi_I}{dy^2} + PR \frac{d\psi_I}{dy} - \pi^2 \psi_I \right) \right] \\ &\quad + p \left(\frac{d^2 \psi}{dy^2} + PR \frac{d\psi}{dy} - \pi^2 \psi + 2EP \frac{dU}{dy} \frac{dV}{dy} \right) = 0 \end{aligned} \quad (28)$$

Let

$$\left. \begin{aligned} U &= U_0 + pU_1 + p^2U_2 + p^3U_3 + \dots \\ \phi &= \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \dots \\ V &= V_0 + pV_1 + p^2V_2 + p^3V_3 + \dots \\ \psi &= \psi_0 + p\psi_1 + p^2\psi_2 + p^3\psi_3 + \dots \end{aligned} \right\} \quad (29)$$

Let corresponding boundary conditions are

$$\left. \begin{aligned} y=0: U_0=0, \phi_0=1, V_0=0, \psi_0=1 \\ y=\infty: U_0=1, \phi_0=0, V_0=0, \psi_0=0 \end{aligned} \right\} \quad (30)$$

Using the assumption made in Eq. (29) into Eq. of (25),(26),(27),(28) and on Comparing like powers of p^0 , we get zeroth order of Eq. of (25),(26),(27),(28)

$$\left(\frac{d^2 U_0}{dy^2} + R^2 \frac{dU_0}{dy} + R^2 G \phi_0 \right) = \left(\frac{d^2 U_I}{dy^2} + R^2 \frac{dU_I}{dy} + R^2 G \phi_I \right) \quad (31)$$

$$\left(\frac{d^2 \phi_0}{dy^2} + PR \frac{d\phi_0}{dy} \right) = \left(\frac{d^2 \phi_I}{dy^2} + PR \frac{d\phi_I}{dy} \right) \quad (32)$$

$$\left(\frac{d^2 V_0}{dy^2} + R \frac{dV_0}{dy} - \pi^2 V_0 \right) = \left(\frac{d^2 V_I}{dy^2} + R \frac{dV_I}{dy} - \pi^2 V_I \right) \quad (33)$$

$$\left(\frac{d^2 \psi_0}{dy^2} + PR \frac{d\psi_0}{dy} - \pi^2 \psi_0 \right) = \left(\frac{d^2 \psi_I}{dy^2} + PR \frac{d\psi_I}{dy} - \pi^2 \psi_I \right) \quad (34)$$

Let corresponding boundary conditions are:

$$\left. \begin{aligned} y=0: U_1=0, \phi_1=0, V_1=0, \psi_1=0 \\ y=\infty: U_1=0, \phi_1=0, V_1=0, \psi_1=0 \end{aligned} \right\} \quad (35)$$

Using the assumption made in Eq. (29) into Eq. of (25),(26),(27),(28) and on Comparing like powers of p^1 , we get First order Eq. of (25),(26),(27),(28)

$$\begin{aligned} \left(\frac{d^2 U_1}{dy^2} + R^2 \frac{dU_1}{dy} \right) - \left(\frac{d^2 U_0}{dy^2} + R^2 \frac{dU_0}{dy} \right) + \left(\frac{d^2 U_I}{dy^2} + R^2 \frac{dU_I}{dy} \right) + \left(\frac{d^2 U_0}{dy^2} + R^2 \frac{dU_0}{dy} + R^2 G \phi_0 \right) \\ = 0 \end{aligned} \quad (36)$$

$$\begin{aligned} \left(\frac{d^2 \phi_1}{dy^2} + PR \frac{d\phi_1}{dy} \right) - \left(\frac{d^2 \phi_0}{dy^2} + PR \frac{d\phi_0}{dy} \right) + \left(\frac{d^2 \phi_I}{dy^2} + PR \frac{d\phi_I}{dy} \right) + \left(\frac{d^2 \phi_0}{dy^2} + PR \frac{d\phi_0}{dy} + EP \left(\frac{dU_0}{dy} \right)^2 \right) \\ = 0 \end{aligned} \quad (37)$$

$$\left(\frac{d^2V_1}{dy^2} + R\frac{dV_1}{dy} - \pi^2V_1\right) - \left(\frac{d^2V_0}{dy^2} + R\frac{dV_0}{dy} - \pi^2V_0\right) + \left(\frac{d^2V_I}{dy^2} + R\frac{dV_I}{dy} - \pi^2V_I\right) + \left(\frac{d^2V_0}{dy^2} + R\frac{dV_0}{dy} - \pi^2V_0 + R^2G\psi_0\right) = 0 \quad (38)$$

$$\left(\frac{d^2\psi_1}{dy^2} + PR\frac{d\psi_1}{dy} - \pi^2\psi_1\right) - \left(\frac{d^2\psi_0}{dy^2} + PR\frac{d\psi_0}{dy} - \pi^2\psi_0\right) + \left(\frac{d^2\psi_I}{dy^2} + PR\frac{d\psi_I}{dy} - \pi^2\psi_I\right) + \left(\frac{d^2\psi_0}{dy^2} + PR\frac{d\psi_0}{dy} - \pi^2\psi_0 + 2EP\frac{dU}{dy}\frac{dV}{dy}\right) = 0 \quad (39)$$

Solutions of (29) to (31) and (36) to (39) under the corresponding boundary conditions are

$$U(y) = 1 - e^{-y} + \left(\frac{1 - R^2 - R^2G}{R^2 - 1}\right)e^{-R^2y} - e^{-y}\left(\frac{1 - R^2 - R^2G}{R^2 - 1}\right) \quad (40)$$

$$\phi(y) = \left(1 - \frac{EP}{2(PR-2)}\right)e^{-PRy} + \frac{EPe^{-2y}}{2(PR-2)} \quad (41)$$

$$V(y) = e^{-y} - \left(1 + \frac{\pi^2 - R^2G + R - 1}{1 - R - \pi^2}\right)e^{-A_2y} + e^{-y}\left(\frac{\pi^2 - R^2G + R - 1}{1 - R - \pi^2}\right) \quad (42)$$

$$\begin{aligned} \psi(y) = e^{-B_2y} - 2EPB_3e^{-B_2y} & \left[\frac{R^2G}{A_2((1+A_2)^2 - (1+A_2)PR - \pi^2) - \frac{R^2G}{4-2PR-\pi^2}} - \frac{R^2G}{1-R^2-R^2G} \right. \\ & + \frac{R^2A_2((A_2+R^2)^2 - (A_2+R^2)PR - \pi^2) - \frac{R^2((1+R^2)^2 - (1+R^2)PR - \pi^2)}{1-R^2-R^2G}}{R^2A_2((A_2+R^2)^2 - (A_2+R^2)PR - \pi^2) - \frac{R^2((1+R^2)^2 - (1+R^2)PR - \pi^2)}{1-R^2-R^2G}} \\ & + 2EPB_3 \left[\frac{R^2Ge^{-(1+A_2)y}}{A_2((1+A_2)^2 - (1+A_2)PR - \pi^2) - \frac{R^2G}{4-2PR-\pi^2}} - \frac{R^2Ge^{-2y}}{1-R^2-R^2G} \right. \\ & + \frac{(1-R^2-R^2G)e^{-(A_2+R^2)y}}{R^2A_2((A_2+R^2)^2 - (A_2+R^2)PR - \pi^2) - \frac{R^2((1+R^2)^2 - (1+R^2)PR - \pi^2)}{1-R^2-R^2G}} \\ & \left. \left. - \frac{R^2((1+R^2)^2 - (1+R^2)PR - \pi^2)}{R^2A_2((A_2+R^2)^2 - (A_2+R^2)PR - \pi^2) - \frac{R^2((1+R^2)^2 - (1+R^2)PR - \pi^2)}{1-R^2-R^2G}} \right] \right] \quad (43) \end{aligned}$$

We get the analytical solution by use of HPM.

Results and discussion

Knowing the velocity and temperature fields we can calculate the skin friction coefficient τ and the rate of heat transfer Nu in the dimensionless form in the following way :

$$\tau = \frac{\partial u}{\partial y}\bigg|_{y=0} = U'(0) + \epsilon V'(0)\cos\pi z \quad (44)$$

$$Nu = -\frac{\partial \theta}{\partial y}\bigg|_{y=0} = -\phi'(0) + \epsilon \psi'(0)\cos\pi z \quad (48)$$

Table 1 values of skin friction coefficient at the plate

R	G	Z	ϵ	τ
1	5	0.1	0.001	1.1292
1.1	5	0.1	0.001	2.4545
1	5.1	0.1	0.001	1.1318
1	5	0.2	0.001	1.1099
1.1	5.1	0.2	0.002	2.4546

It is observed that skin coefficient increases on increasing R and ϵ and decreases on increasing G and Z.

Table 2 Values of Nusslet number at the plate

R	G	P	E	ϵ	Z	Nu
1.1	5	0.71	0.2	0.001	0.1	1.0633
1.1	5.1	0.71	0.2	0.001	0.1	1.0639
1.3	5.1	0.71	0.2	0.001	0.1	1.2207
1.1	5	0.71	0.3	0.001	0.1	1.0353
1.1	5	0.71	0.2	0.001	0.2	1.0105

It has been observed from table2 that Nusslet number increases when increases R and decreases when G decreases. And Nusslet number decreases when E and Z increases.

Table 3 (V')

R\G	5	10	20	R\G	5	10	20
1	1.3583	2.7166	5.4331	1	0.80	1.58	2.93
3	15.094	30.1879	60.3759	3	6.95	14.26	20.23
5	49.7033	99.4066	198.8100	5	17.64	32.32	40.70
	HPM				Regular Perturbation		

Table 4 (ψ')

R\G	5	10	20	R\G	5	10	20
1	3.5333	3.5286	3.5435	1	3.52	3.51	3.50
3	4.3883	4.3804	4.0163	3	4.37	4.19	3.97
5	5.3888	5.1427	4.9668	5	5.34	4.76	4.28
	HPM				Regular Perturbation		

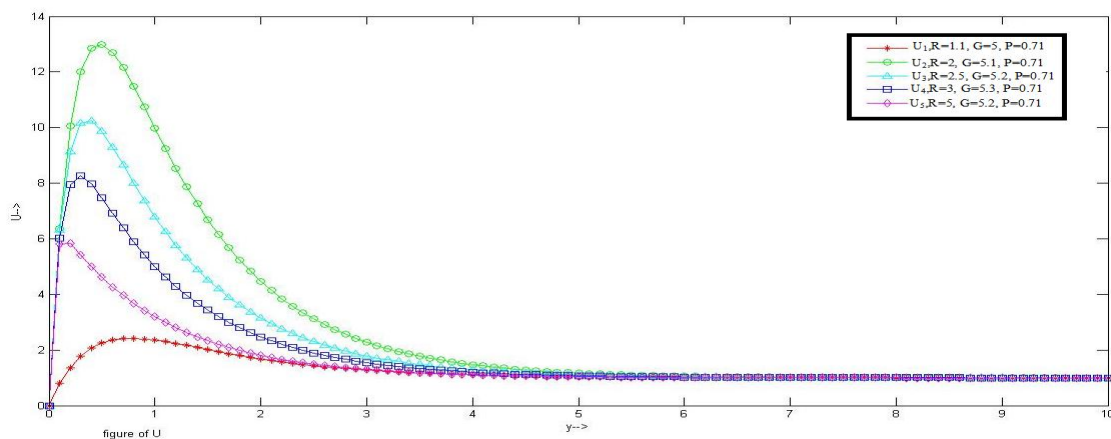


Figure 1 zeroth order velocity distribution versus y

It has been observed from figure 1 that the factor U increase when either G or R increase. and U decreases when P or E increases.

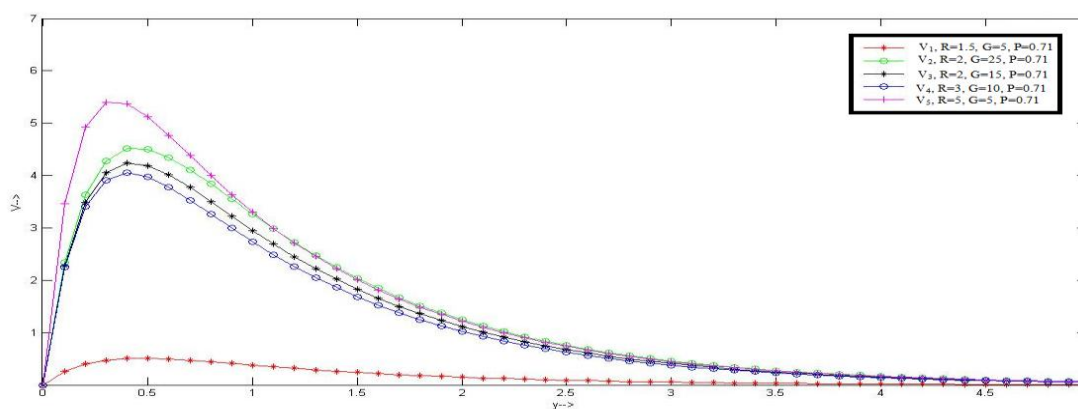


Figure 2 first order velocity distribution versus y

It has been observed from figure 2 that the factor V increases when either G or R increase .and V decreases when P or E increases.

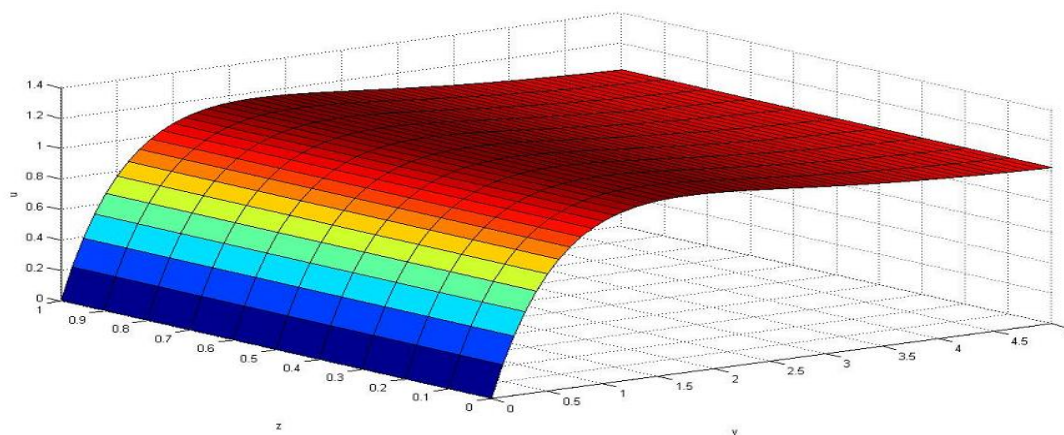


Figure 3 Velocity distribution

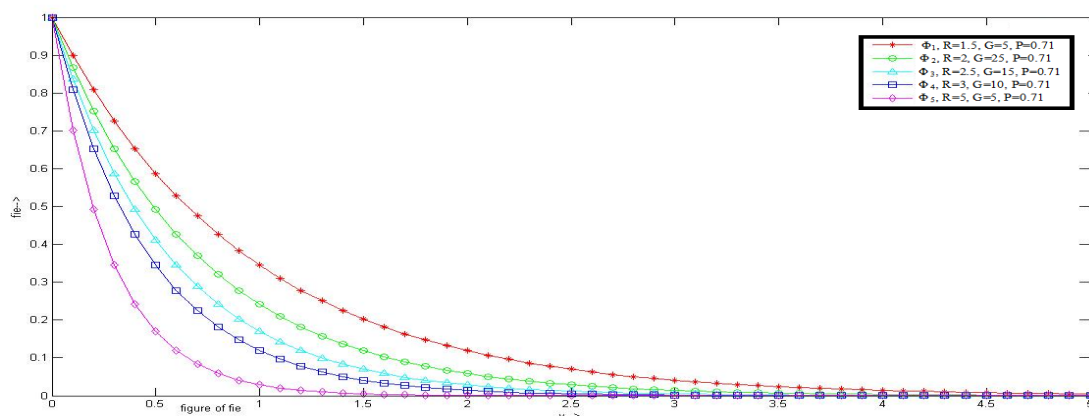


Figure 4 Zeroth order temperature distribution versus y

It is observed that ϕ decrease with increasing values of P or R whereas it increases when G or E increases.

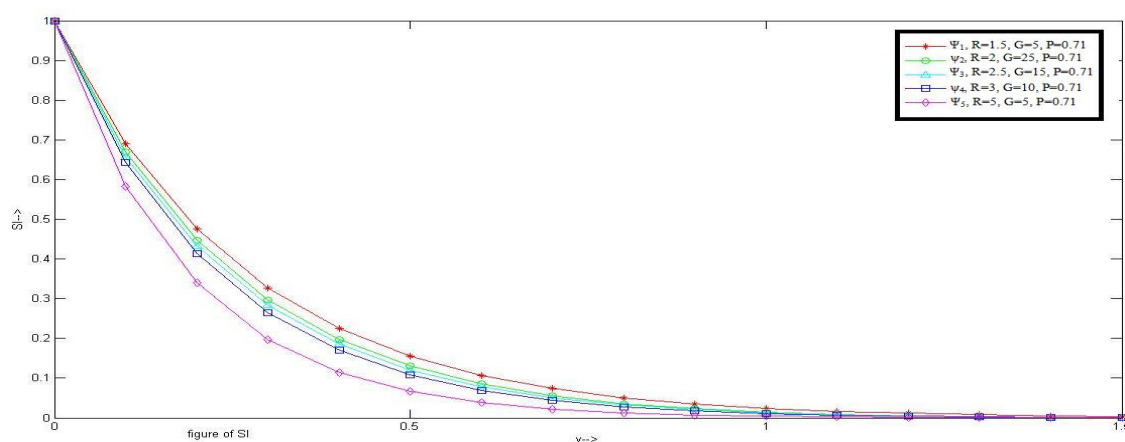


Figure 5 First order temperature distributions versus y

It is observed that ψ decrease with increasing values of P or R whereas it increases when G or E increases.

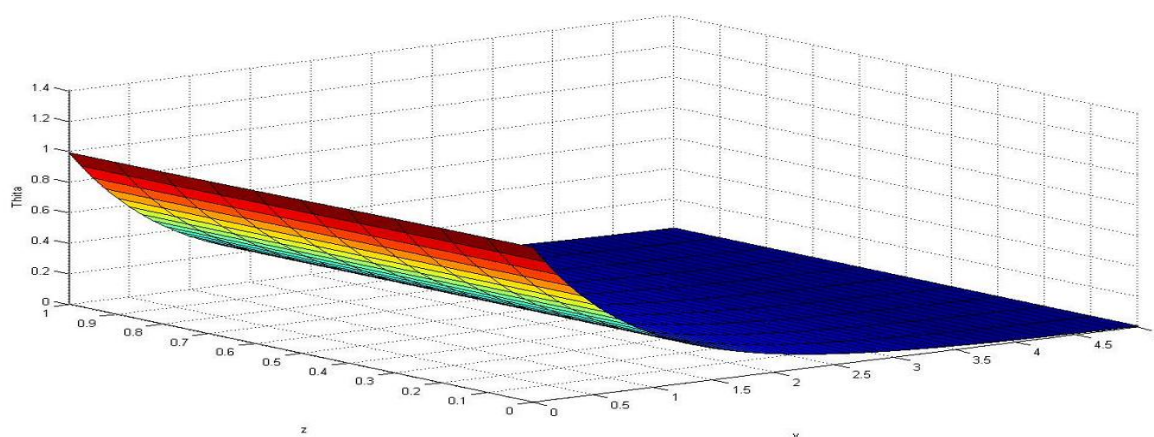


Figure 6 Temperature distribution

Conclusion

A free convective liquid of a viscous liquid past a hot vertical porous plate is presented under the assumption that the suction velocity is constant and normal to the plate and the plate temperature is span wise co-sinusoidal. solved analytically by use of HPM. The results show a suitable agreement between HPM and regular perturbation

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