



The Empirical Formulae to Determine the Astrophysical S-Factor of (α, n) Reactions for some Medium Elements

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Abstract: In the present work, the cross-section of (α, n) reactions available in the literature as a function of α -particle energies for the medium element targets such as [$^{45}\text{Sc}(\alpha, n)^{48}\text{V}$, $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{51}\text{V}(\alpha, n)^{54}\text{Mn}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{55}\text{Mn}(\alpha, n)^{58}\text{Co}$, $^{54}\text{Fe}(\alpha, n)^{57}\text{Ni}$, $^{59}\text{Co}(\alpha, n)^{62}\text{Cu}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$, $^{63}\text{Cu}(\alpha, n)^{66}\text{Ga}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$] reactions have been rearranged and interpolated for α -particle energies from near threshold up to (10 or 15 MeV) in steps of 0.050 MeV by using the MATLAB computer program in lab. system. The obtained data were used to calculate the weighted average of the cross-section in unit of (mb). The obtained weighted average of the cross-section (mb) were taken into consideration and used to calculate the astrophysical S-factors of the above reactions as a function of α -particle energies in the center-of-mass system. The polynomial expressions have been used to fit the calculated astrophysical S-factor of the studied medium elements to determine the astrophysical S-factor from the best fitting equations with minimum (CHISQ) at various α -particle energies in the center-of-mass system. Empirical formulae of a set of reactions such as $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ has been used to calculate the astrophysical S-factor $S(E)$ for each reactions as a function of α -particle energies in center-of-mass system and the atomic number (Z) of the target nucleus. Empirical formulae of a set of reactions such as $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{54}\text{Fe}(\alpha, n)^{57}\text{Ni}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ has been used to calculate the astrophysical S-factor $S(E)$ for each reactions as a function of α -particle energies in center-of-mass system and the total binding energy, $B_{\text{tot}}(A, Z)$ of the target nucleus. The results are compared with the adopted astrophysical S-factor calculating from the fitting equations are presented graphically and in numerical forms, and shown to be in a good agreement.

Keyword: Cross-section, nuclear reaction, astrophysical s-factor, sommerfeid parameter, gammo energy

I. Introduction

The nuclear astrophysical S-factor is introduced to separate the energy dependence of the coulomb barrier penetration from the cross-section [1]. In charged particle-induced reactions, the cross section for both reaction mechanisms (non-resonant and resonant) drops rapidly (on an exponential scale) at low energies owing to the effect of the Coulomb barrier, and it becomes more difficult to measure the relevant cross sections [2]. With improved experimental techniques direct measurements of $\sigma(E)$ for charged-particle-induced reactions can be extended toward lower energies, but in practice one hardly reaches the relevant stellar energy regions for quiescent stellar burning. The observed energy dependence of $\sigma(E)$, or equivalently of $S(E)$, must therefore extrapolated into the stellar energy region (essentially to zero energy). Of course, the basis for extra- potation will be improved if extremely low energy data are available. However, these extrapolated data represent only lower limits of the stellar reaction rate [2]. The cross section of (α, n) reactions for different targets have been measured as a function of alpha particle energies in the (lab.) and (C.M.) systems by several authors. Total cross sections for $^{45}\text{Sc}(\alpha, n)^{48}\text{V}$, $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{51}\text{V}(\alpha, n)^{54}\text{Mn}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{55}\text{Mn}(\alpha, n)^{58}\text{Co}$, $^{54}\text{Fe}(\alpha, n)^{57}\text{Ni}$, $^{59}\text{Co}(\alpha, n)^{62}\text{Cu}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$, $^{63}\text{Cu}(\alpha, n)^{66}\text{Ga}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ reactions have been measured by several authors mentioned by different references such as [3,4,5,6], [7,8], [5,6,9,10], [5,6,7,11], [12,13], [5,14], [5,6,15,16], [6,17,18], [5,6,17,19] and [6,17] respectively for each above reactions as a function of various alpha particle energies. The aim of the present work is to determine the astrophysical S-Factor using the modified cross sections of the above reactions. The results are compared with those published in the literature.

II. Theory

The Q – value of the reaction $X(\alpha, n)Y$, is defined as the difference between the initial and final rest mass energies [20]:

$$Q = [M_x + M_\alpha - (M_Y + M_n)] c^2 \quad (1)$$

Where (m_x , m_α , m_Y and m_n) represents the atomic masses of the target, incident particles, product nucleus and out going particle, respectively. From conservation law of energy [20]:

$$M_\alpha c^2 + T_\alpha + M_x c^2 = M_n c^2 + T_n + M_Y c^2 + T_Y \quad (2)$$

Where T is the kinetic energy of each particle.

In the laboratory system conservations of energy and momentum lead to the following equation [20]:

$$Q = T_n(1 + M_n/M_y) - T_a(1 - M_a/M_y) - (2/M_y)(M_a T_a M_n T_n)^{1/2} \cos \theta \quad (3)$$

Which is called Q - equation.

The Q- value is positive, $Q > 0$, the reaction is said to be (exoergic) or (exothermic); when Q- value is negative, $Q < 0$, the reaction is (endoergic) or (endothermic). For (exoergic) reactions, threshold energy is (zero) and for (endoergic) reactions, the threshold energy is given by [21]:

$$E_{thr} = -Q_0 \left(1 + \frac{M_a}{M_x}\right) \quad (4)$$

The binding energy can be calculated as the reduction in mass times the square of the velocity of light ($c^2 = 931.494013 \text{ MeV/u}$) [22]:

$$B_{tot}(A, Z) = [ZM_H + NM_n - \frac{A}{Z}M]c^2 \quad (5)$$

where (Z and N) are the number of protons and neutrons, (M_H , M_n and $\frac{A}{Z}M$) are the masses of hydrogen atom, neutrons and nucleus respectively

The charged particle reaction cross-sections at low energies is obtained by Breit- wigner formula [23]:

$$\sigma(E) = \frac{1}{E} S(E) e^{-G(E)} \quad (6)$$

Where $G(E)$ is given by the following equation [23]:

$$G(E) \approx \frac{\pi}{\hbar c} \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \right) \sqrt{\frac{2\mu c^2}{E}} = \sqrt{\frac{E_G}{E}} \quad (7)$$

$$\text{Or } G(E) = 2\pi\eta \quad (8)$$

$$E_G = 2\mu c^2 \left(\frac{\pi Z_1 Z_2 e^2}{\hbar c 4\pi\epsilon_0} \right)^2 \quad (9)$$

$G(E)$ is the sammerfeld parameter and (E_G) is the Gammow energy, Z_1 and Z_2 are the atomic numbers of the projectile and target nucleus, $\hbar = \frac{h}{2\pi}$ (h is the planks constant = 6.63×10^{-34} Joule. sec) (c) is the speed of light = 3×10^8 m/sec, ϵ_0 is the electric permittivity = 8.85×10^{-12} F/meter, and (μ) is the reduced mass calculated from the following equation [20]:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (10)$$

Where (m_1) and (m_2) are the atomic masses of the projectile and target nucleus, respectively.

It is evident that the energy of a pair of particles in their center- of -mass is related to the laboratory energy of incident particle by the relationship [20]:

$$E_{lab.} = \frac{m_1 + m_2}{m_2} E_{c.m.} \quad (11)$$

Where ($E_{lab.}$) is the energy of the incident particle in the laboratory system, ($E_{c.m.}$) is the energy of the incident particle in the center -of- mass .

The coulomb barrier inhibits nuclear reaction. For the P+P reaction the effective height E_c of this coulomb barrier is $E_c = 550 \text{ KeV}$ it is useful to remember that $e^2/4\pi\epsilon_0 = 1.44 \text{ MeVfm}$ and $R = 1.3A^{1/3} \text{ fm}$, then it is easy to show [24].

$$E_c = \frac{1.44}{1.3} \frac{Z_1 Z_2}{(A_1^{1/3} + A_2^{1/3})} \quad (12)$$

The weighted averages of the cross-section σ (mb) and the uncertainty ($\Delta \sigma$) for medium elements whose cross-section (σ) have been calculated using the following expressions [25] :

$$\sigma(\text{mb}) = \frac{\sum_i \frac{\sigma_i}{\Delta_i^2}}{\sum_i \frac{1}{\Delta_i^2}} \quad \dots\dots(13) \quad \Delta\sigma = \pm \frac{1}{\sqrt{\sum_i \frac{1}{\Delta_i^2}}} \quad (14)$$

Where (σ_i) is the cross section of i^{th} reference and (Δ_i) is the errors corresponding to each of (σ_i) .

The type of formalism has been considered in the present work is the polynomial fit expression of the general form [26]:

$$C_0 + C_1 E + C_2 E^2 + C_3 E^3 + \dots = \sum_{i=0}^N C_i E^i \quad (15)$$

Where $(C_0, C_1, C_2, C_3, \dots)$ are parameters and $(i=0,1,2,3,\dots)$ which were fitted to the adopted data the best fit is obtained with increasing order to provide the minimum value of CHISQ (χ^2), corresponding to the best fit, was deduced through [26].

$$\chi^2 = \frac{1}{(N-m)} \sum_i^N \left(\frac{Y_{\text{exp}}^i - Y_{\text{cal}}^i}{\Delta Y_{\text{exp}}^i} \right)^2 \quad (16)$$

Where (N) is the number of data points, (m) is the number of coefficients in the fitting, (Y_{exp}^i) and $(\Delta Y_{\text{exp}}^i)$ are the experimental (adopted value) of $\ln[S(E)]$ (natural logarithm of nuclear astrophysical S-factor) and its error respectively. (Y_{cal}^i) is the calculated $\ln[S(E)]$ (natural logarithm of nuclear astrophysical S-factor) using the fitting equations.

III. Data Reduction and Analysis

In calculating the Q-values, the threshold energies, the reduced masses, the coulomb barriers and the binding energies, the atomic masses of ref. [27] have been taken into consideration using the equations (1,4,10,12 and 5) respectively. The results obtained for the mentioned reactions are presented in Table (I). The Gamow Factor $G(E)$, the Sommerfeld Parameter (η) , the Gamow energy (EG) and the ratio $(E_{\text{c.m.}})/(E_{\text{lab}})$ of the above (α, n) reactions have been calculated using the equations (7,8,9 and 11) respectively. The results are presented in Table(II).

The cross-section of (α, n) reactions for medium element such as (^{45}Sc , ^{46}Ti , ^{51}V , ^{50}Cr , ^{55}Mn , ^{54}Fe , ^{59}Co , ^{62}Ni , ^{63}Cu and ^{68}Zn) available in the literatures has been taken into consideration and re-plotted, interpolated and analyzed by using the MATLAB computer program to obtain the cross-section in fine steps of (0.05MeV).

The weighted average of the modified cross-sections of i^{th} references for the above medium elements whose cross section (σ) and uncertainty of (σ) is $(\Delta\sigma)$ has been calculated by using eqs. (13) and (14) respectively, the obtained results are taken into consideration and used to calculate the astrophysical S-factor of (α, n) reactions as a function of alpha particle energies in center-of-mass system using eq. (6). The obtained formula to calculate the Astrophysical S-Factor of the mentioned reactions are presented in Table (II). The adopted astrophysical S-Factor $S(E)$ has been used as the input data, Excel computer programs (Format Trendline) Polynomial fit expressions of the form :

$$Y = C_0 + C_1 X + C_2 X^2 + \dots + C_N X^N = \sum_{i=0}^N C_i X^i \quad (17)$$

$$\text{and } C_i = \sum_{j=0}^n C_{ij} E^j \quad (18)$$

have been considered in the present work.

Subsequently, by combing the above equations (15) & (17), the following expression can be obtained:

$$Y = \sum_{i=0}^N \left(\sum_{j=0}^n C_{ij} E^j \right) X^i \quad (19)$$

where $Y=\ln[S(E)]$, is the natural logarithm of nuclear astrophysical S-factor, E is the energy of α -particle in center-of-mass system $(C_{00}, C_{01}, \dots)$ are free parameters, $(i=0,1,2,3,\dots)$, $(j=0,1,2,3,\dots)$, and X is representing the physical quantities; atomic number (Z) , natural logarithm of binding energy $\ln[B_{\text{tot}}(A, Z)]$, for each time of expression establishments.

The Excel computer programs were used to obtain the best fit formulae corresponding to different energy ranges from near threshold up to 10MeV or 15MeV in the center-of-mass system. In each step, the data of these ranges was excluded till an acceptable value of the coefficient of determination $R^2 \approx 1$ was reached. Thus the final expressions of the following form can be obtained to calculate the natural logarithm of nuclear astrophysical S-factor:

$$\ln S(E) = \sum_{i=0}^N \left(\sum_{j=0}^n C_{ij} E^j \right) X^i \quad (20)$$

Table (I): Q-Value, Threshold Energies (E_{thr}), Reduced Mass(μ), Coulomb Barrier of (α,n) reaction and the total Bindin Energies $B_{tot.}(\alpha,n)$ of the targets

Reactions	Q-Value (MeV)	$E_{threshold}$ (MeV)		Reduced Mass(μ) (a.m.u.)	Coulomb barrier(E_c) (MeV)	Binding Energy $B_{tot}(A,Z)$ (MeV)
		Lab. System	C.M. System			
$^{45}_{21}\text{Sc}(\alpha,n)^{48}_{23}\text{V}$	-2.239	2.438	2.239	3.675	9.044	387.841
$^{46}_{22}\text{Ti}(\alpha,n)^{49}_{24}\text{Cr}$	-4.438	4.825	4.438	3.682	9.426	398.185
$^{51}_{23}\text{V}(\alpha,n)^{54}_{25}\text{Mn}$	-2.292	2.462	2.283	3.711	9.622	445.836
$^{50}_{24}\text{Cr}(\alpha,n)^{53}_{26}\text{Fe}$	-4.96	5.357	4.96	3.706	10.09	435.041
$^{55}_{25}\text{Mn}(\alpha,n)^{58}_{27}\text{Co}$	-3.511	3.767	3.511	3.731	10.27	482.066
$^{54}_{26}\text{Fe}(\alpha,n)^{57}_{28}\text{Ni}$	-5.816	6.248	5.816	3.726	10.73	471.754
$^{59}_{27}\text{Co}(\alpha,n)^{62}_{29}\text{Cu}$	-5.076	5.421	5.076	3.748	10.91	517.303
$^{62}_{28}\text{Ni}(\alpha,n)^{65}_{30}\text{Zn}$	-6.480	6.899	6.480	3.760	11.19	545.252
$^{63}_{29}\text{Cu}(\alpha,n)^{66}_{31}\text{Ga}$	-7.501	7.978	7.501	3.763	11.54	551.374
$^{68}_{30}\text{Zn}(\alpha,n)^{71}_{32}\text{Ge}$	-5.745	6.084	5.745	3.780	11.72	595.375

Table (II): Gamow Factor $G(E)$, Sommerfeld Parameter(η) , Gamow energy (E_G) , ratio ($E_{cm.}/(E_{lab.})$) and the Relation to calculate the astrophysical S-factor for (α,n) reactions:

Reactions	Gamow Factor $G(E)$	Sommerfeld parameter η	Gamow Energy E_G (MeV)	Ratio ($E_{cm.}/E_{lab.}$)	Astrophysical S- Factor $S(E)$
$^{45}_{21}\text{Sc}(\alpha,n)^{48}_{23}\text{V}$	$\frac{79.6738}{\sqrt{E_{cm.}}}$	$12.6805 / \sqrt{E_{cm.}}$	6.35E+03	0.9182	$E_{cm.}\sigma(E)\exp(79.6738 / \sqrt{E_{cm.}})$
$^{46}_{22}\text{Ti}(\alpha,n)^{49}_{24}\text{Cr}$	$\frac{83.5419}{\sqrt{E_{cm.}}}$	$13.2961 / \sqrt{E_{cm.}}$	6.98E+03	0.9199	$E_{cm.}\sigma(E)\exp(83.5419 / \sqrt{E_{cm.}})$
$^{51}_{23}\text{V}(\alpha,n)^{54}_{25}\text{Mn}$	$\frac{87.6841}{\sqrt{E_{cm.}}}$	$13.9554 / \sqrt{E_{cm.}}$	7.69E+03	0.9272	$E_{cm.}\sigma(E)\exp(87.6841 / \sqrt{E_{cm.}})$
$^{50}_{24}\text{Cr}(\alpha,n)^{53}_{26}\text{Fe}$	$91.43 / \sqrt{E_{cm.}}$	$14.5515 / \sqrt{E_{cm.}}$	8.36E+03	0.9258	$E_{cm.}\sigma(E)\exp(91.43 / \sqrt{E_{cm.}})$
$^{55}_{25}\text{Mn}(\alpha,n)^{58}_{27}\text{Co}$	$\frac{95.5622}{\sqrt{E_{cm.}}}$	$15.2092 / \sqrt{E_{cm.}}$	9.13E+03	0.9321	$E_{cm.}\sigma(E)\exp(95.5622 / \sqrt{E_{cm.}})$
$^{54}_{26}\text{Fe}(\alpha,n)^{57}_{28}\text{Ni}$	$\frac{99.3223}{\sqrt{E_{cm.}}}$	$15.8076 / \sqrt{E_{cm.}}$	9.86E+03	0.9309	$E_{cm.}\sigma(E)\exp(99.3223 / \sqrt{E_{cm.}})$
$^{59}_{27}\text{Co}(\alpha,n)^{62}_{29}\text{Cu}$	$\frac{103.446}{\sqrt{E_{cm.}}}$	$16.4639 / \sqrt{E_{cm.}}$	1.07E+04	0.9364	$E_{cm.}\sigma(E)\exp(103.446 / \sqrt{E_{cm.}})$
$^{62}_{28}\text{Ni}(\alpha,n)^{65}_{30}\text{Zn}$	$\frac{107.442}{\sqrt{E_{cm.}}}$	$17.1 / \sqrt{E_{cm.}}$	1.15E+04	0.9393	$E_{cm.}\sigma(E)\exp(107.442 / \sqrt{E_{cm.}})$
$^{63}_{29}\text{Cu}(\alpha,n)^{66}_{31}\text{Ga}$	$\frac{111.333}{\sqrt{E_{cm.}}}$	$17.7192 / \sqrt{E_{cm.}}$	1.24E+04	0.9402	$E_{cm.}\sigma(E)\exp(111.333 / \sqrt{E_{cm.}})$
$^{68}_{30}\text{Zn}(\alpha,n)^{71}_{32}\text{Ge}$	$\frac{115.426}{\sqrt{E_{cm.}}}$	$18.3707 / \sqrt{E_{cm.}}$	1.33E+04	0.9444	$E_{cm.}\sigma(E)\exp(115.426 / \sqrt{E_{cm.}})$

IV. The Results and Discussion

A. The Empirical Formula:

The adopted astrophysical S-factor $S(E)$ have been used to obtain the fitting parameter by using the polynomial expressions (15), (16) and (18) by the following steps:

1. The polynomial expressions were used in eq. (15) to fit the calculated natural logarithm of astrophysical S-factor $S(E)$ of the studied medium elements to determine the adopted natural logarithm of astrophysical S-factor $S(E)$ from the best fitting with minimum CHISQ (χ^2) using eq. (16). The obtained best fitting equations of the mentioned reactions are present in equations (21, 22, 23, 24, 25, 26, 27, 28, 29 and 30) respectively for the above reactions.

$$^{45}_{21}\text{Sc}(\alpha, n)^{48}_{23}\text{V}: Y_3 = 30.3931 + 1.1955E - 0.242E^2 + 0.0085E^3 \quad (\chi^2 = 0.0019) \quad (21)$$

$$^{46}_{22}\text{Ti}(\alpha, n)^{49}_{24}\text{Cr}: Y_3 = 10.281 + 9.0981E - 1.2754E^2 + 0.0538E^3 \quad (\chi^2 = 0.0026) \quad (22)$$

$$^{51}_{23}\text{V}(\alpha, n)^{54}_{25}\text{Mn}: Y_3 = 32.4458 + 1.3088E - 0.2411E^2 + 0.0079E^3 \quad (\chi^2 = 0.0064) \quad (23)$$

$$^{50}_{24}\text{Cr}(\alpha, n)^{53}_{26}\text{Fe}: Y_2 = 27.223 + 1.9196E - 0.178E^2 \quad (\chi^2 = 0.02) \quad (24)$$

$$^{55}_{25}\text{Mn}(\alpha, n)^{58}_{27}\text{Co}: Y_3 = 31.0692 + 2.3009E - 0.3314E^2 + 0.0108E^3 \quad (\chi^2 = 0.0021) \quad (25)$$

$$^{54}_{26}\text{Fe}(\alpha, n)^{57}_{28}\text{Ni}: Y_4 = -589.17 + 332.24E - 65.59E^2 + 5.6972E^3 - 0.1843E^4 \quad (\chi^2 = 0.0036) \quad (26)$$

$$^{59}_{27}\text{Co}(\alpha, n)^{62}_{29}\text{Cu}: Y_3 = 24.37 + 4.8709E - 0.5769E^2 + 0.0184E^3 \quad (\chi^2 = 9.12E-05) \quad (27)$$

$$^{62}_{28}\text{Ni}(\alpha, n)^{65}_{30}\text{Zn}: Y_3 = -19.097 + 21.699E - 2.6707E^2 + 0.1043E^3 \quad (\chi^2 = 0.0012) \quad (28)$$

$$^{63}_{29}\text{Cu}(\alpha, n)^{66}_{31}\text{Ga}: Y_2 = 41.3253 - 0.1458E - 0.0418E^2 \quad (\chi^2 = 1.91E-04) \quad (29)$$

$$^{68}_{30}\text{Zn}(\alpha, n)^{71}_{32}\text{Ge}: Y_3 = 23.351 + 6.8694E - 0.8771E^2 + 0.0331E^3 \quad (\chi^2 = 0.0054) \quad (30)$$

2. At a fixed values of α -particle energy in center-of-mass system ($E_{\text{c.m.}}$), the variation of the natural logarithm of astrophysical S-factor with the physical parameters (atomic number Z and natural logarithm of total binding energy $B_{\text{tot}}(A, Z)$) has been fitted to the polynomial expression using eq.(15). The obtained results are taken into consideration to determine the free parameters (C_i).

3. The free parameters C_i , are plotted against the prefixed values of α -particle energy in center-of-mass system and fitted to adequate the polynomial expression where presented in eq. (18).

4. The final formula of a set of reactions have been determined by using the combination of the two polynomials to show the systematic behavior of the reactions.

B. The Empirical Formulae Relating the Astrophysical S-factor to α -Particle Energies in Center-of-Mass System (E_{α}) and the Atomic Number (Z) of the Target Nucleus:

The empirical formulae relating the astrophysical S-factor (MeV-b) with both of the α -particle energies in center-of-mass system $E_{\text{c.m.}}$ and the target atomic number (Z) were performed as follows:

1- At a fixed value of α -particle energy in center-of-mass system (7, 7.5, 8, 8.5, 9, 9.5, and 10 MeV) the variation of the natural logarithm of astrophysical S-factor with the atomic number (Z), for $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ reactions have been plotted as shown in Fig.(I), and the data have been fitted to the following polynomial expression

$$Y = \sum_{i=0}^2 C_i X^i \quad (31)$$

where $Y = \ln[S(E)]$ and $X = Z$, with free parameters C_i (C_0 , C_1 and C_2)

2- The adopted astrophysical S-factor $S(E)$ have been used as the input data as a function of target atomic number at fixed alpha particle energies in center-of-mass system using the Excel computer program to obtain the fitting expressions and then used to calculate the fitting parameters. The obtained results are presented in Table (III).

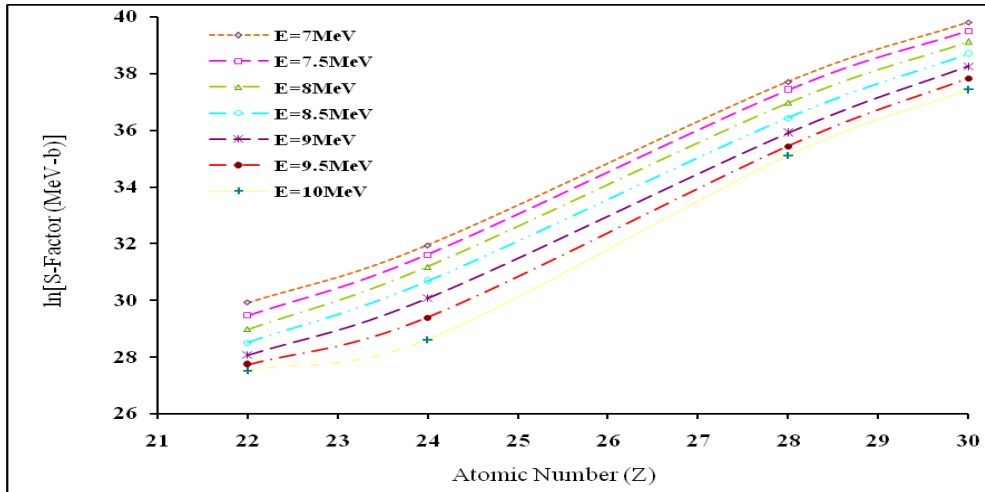


Fig.(I): The variation of natural logarithm of the astrophysical S-factor $S(E)$ with the atomic number (Z) for $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ reactions, at a fixed values of α -particle energy in center-of- mass system.

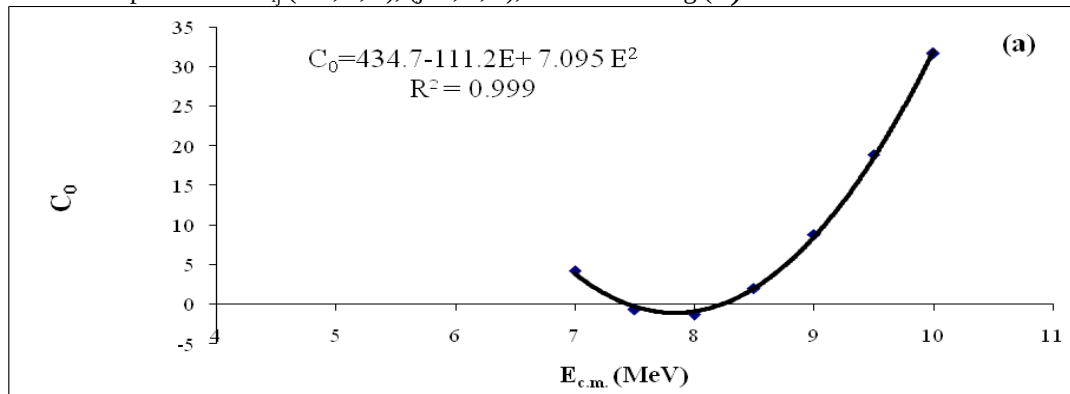
Table (III): Free parameters C_i (C_0 , C_1 and C_2) as a function of α -particle energy in center-of-mass system E_α .

E_α (MeV)	C_0	C_1	C_2
7	4.2499	1.0735	0.0039
7.5	-0.6966	1.416	-0.0024
8	-1.3201	1.4213	-0.0023
8.5	1.9916	1.1201	0.0036
9	8.8509	0.543	0.0147
9.5	18.87	-0.2792	0.0306
10	31.662	-1.316	0.0506

3- The obtained free parameters C_i (C_0 , C_1 and C_2), as shown in Table (III) are plotted against with the prefixed values of α -particle energy in center-of-mass system (7, 7.5, 8, 8.5, 9, 9.5, and 10 MeV) as shown in Fig.(II), and then the obtained free parameters C_i have been fitted to the following polynomial expression:

$$C_i = \sum_{ij} C_{ij} E^j \quad (32)$$

with the free parameters C_{ij} ($i=0, 1, 2$), ($j=0, 1, 2$), as shown in Fig.(II)



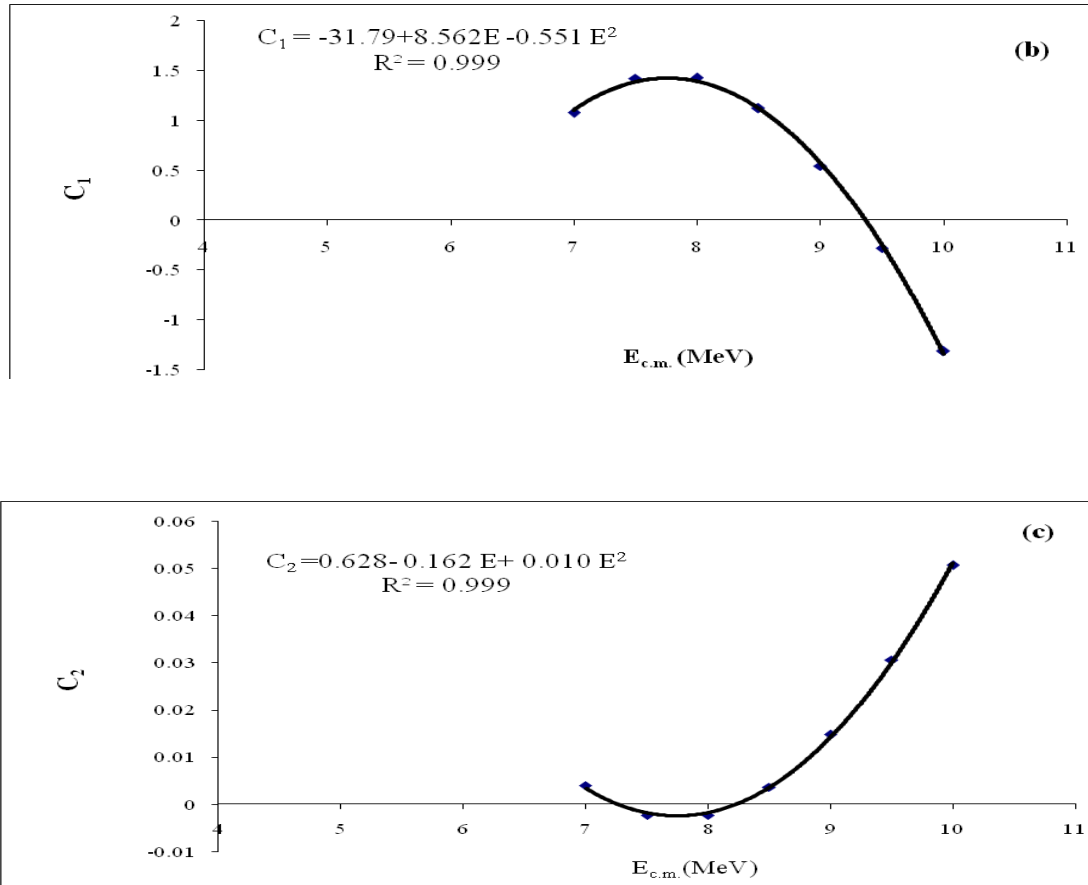


Fig. (II): Plots of C_i coefficients against α -particle energy in center-of-mass system. (a) for C_0 , (b) for C_1 , (c) for C_2 . The solid line represents the fitted curve through the data. The fitted expressions for each coefficient are also presented.

The combination of the two polynomials takes the form of the following formula from energy range (7 to 10 MeV) in center-of-mass system:

$$Y = \sum_{i=0}^2 \left(\sum_{j=0}^2 C_{ij} E^j \right) X^i \quad (33)$$

$$Y = \sum_{i=0}^2 (C_{i0} E^0 + C_{i1} E^1 + C_{i2} E^2) X^i$$

$$Y = C_{00} E^0 X^0 + C_{01} E^1 X^0 + C_{02} E^2 X^0 + C_{10} E^0 X^1 + C_{11} E^1 X^1 + C_{12} E^2 X^1 + C_{20} E^0 X^2 + C_{21} E^1 X^2 + C_{22} E^2 X^2 \quad (34)$$

where $Y = \ln[S(E)]$, $X = Z$ and $(C_{00}, C_{01}, C_{02}$ and $C_{01}, \dots)$ are free parameters as follows:

$$\begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 434.76 & -111.22 & 7.0953 \\ -31.797 & 8.562 & -0.5517 \\ 0.6281 & -0.1629 & 0.0105 \end{bmatrix}, \begin{bmatrix} R^2 = 0.999 \\ R^2 = 0.999 \\ R^2 = 0.999 \end{bmatrix}$$

The obtained formula of a set of reactions such as $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ has been used to calculate the astrophysical S-factor $S(E)$ for each of the above reactions and compared with the adopted astrophysical S-factor calculated from the fitting expressions and shown to be in a good agreement and the comparison of the two results are shown in Table (IV).

Table (IV): The Comparison between polynomial fitting expression (best fitting) of the adopted astrophysical S-Factor of (α,n) reactions with those calculated from eq. (34).

$E_{c.m.}$ (MeV)	$^{46}_{22}\text{Ti}(\alpha,n)^{49}_{24}\text{Cr}$		$^{50}_{24}\text{Cr}(\alpha,n)^{53}_{26}\text{Fe}$	
	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (best fitting)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (Formula)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (best fitting)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (Formula)
7	29.9265±1.0265	29.2843	31.9382±0.3737	31.7033
7.25	29.706±1.0189	29.0015	31.784±0.3719	31.4654
7.5	29.4724±1.0109	28.7238	31.6075±0.3698	31.2152
7.75	29.2306±1.0026	28.451	31.4088±0.3675	30.9529
8	28.9858±0.9942	28.1832	31.1878±0.3649	30.6784
8.25	28.7430±0.9859	27.9204	30.9446±0.3621	30.3917
8.5	28.5071±0.9778	27.6626	30.6791±0.3589	30.0928
8.75	28.2834±0.9701	27.4097	30.3914±0.3556	29.7818
9	28.0767±0.963	27.1619	30.0814±0.352	29.4585
9.25	27.8922±0.9567	26.9190	29.7492±0.3481	29.1231
9.5	27.7349±0.9513	26.6812	29.3947±0.3439	28.7754
9.75	27.6098±0.9470	26.4483	29.0180±0.3395	28.4156
10	27.5220±0.9440	26.2204	28.6190±0.3348	28.0436

Table (IV): Continued

$E_{c.m.}$ (MeV)	$^{62}_{28}\text{Ni}(\alpha,n)^{65}_{30}\text{Zn}$		$^{68}_{30}\text{Zn}(\alpha,n)^{71}_{32}\text{Ge}$	
	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (best fitting)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (Formula)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (best fitting)	$\text{Ln}[\text{S-Factor}(\text{MeV-b})]$ (Formula)
7	37.7066±1.8589	36.5965	39.8122±1.0192	39.0707
7.25	37.5885±1.8531	36.3685	39.6653±1.0154	38.8079
7.5	37.4202±1.8448	36.1255	39.4987±1.0112	38.5444
7.75	37.2114±1.8345	35.8675	39.3156±1.0065	38.2801
8	36.9718±1.8227	35.5944	39.119±1.0014	38.0152
8.25	36.7113±1.8099	35.3063	38.9121±0.9961	37.7495
8.5	36.4397±1.7965	35.0031	38.6980±0.9907	37.4832
8.75	36.1666±1.7830	34.6849	38.4797±0.9851	37.2161
9	35.9020±1.770	34.3517	38.2604±0.9795	36.9483
9.25	35.6555±1.7578	34.0034	38.0432±0.9739	36.6798
9.5	35.4370±1.7470	33.6401	37.8311±0.9685	36.4106
9.75	35.2563±1.7381	33.2618	37.6274±0.9633	36.1406
10	35.123±1.7316	32.8684	37.4350±0.9583	35.870

C. The Empirical Formulae Relating Astrophysical S-factor to α -particle Energies (in Center-of-Mass System E_{α}) and the Binding Energy $B_{\text{tot}}(A,Z)$:

The empirical formulae relating the astrophysical S-factor (MeV-b) with both of the α -particle energies in center-of-mass system E_{α} and the total nuclear binding energy $B_{\text{tot}}(A,Z)$ were performed as follows:

1- At a fixed value of α -particle energies in center-of-mass system (7, 7.5, 8, 8.5, 9 and 9.5 MeV) the variation of the natural logarithm of astrophysical S-factor with the total nuclear binding energy $B_{\text{tot}}(A,Z)$, for $^{46}\text{Ti}(\alpha,n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha,n)^{53}\text{Fe}$, $^{54}\text{Fe}(\alpha,n)^{57}\text{Ni}$, $^{62}\text{Ni}(\alpha,n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha,n)^{71}\text{Ge}$ reactions are plotted as shown in Fig. (III), and the data have been fitted to the following expression :

$$Y = C_0 + C_1 \ln X \quad (35)$$

Where $Y=\ln[S(E)]$, $X=\ln[B_{\text{tot}}(A,Z)]$, with free parameters (C_0 and C_1)

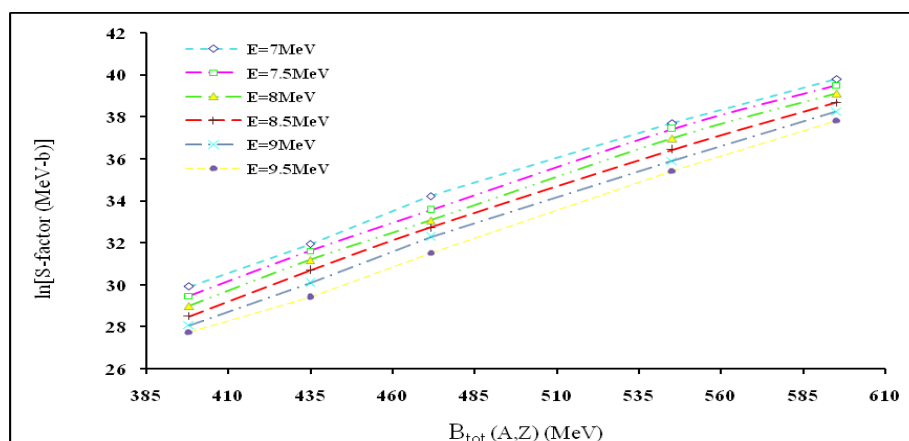


Fig.(III): The variation of natural logarithm of astrophysical S-factor $S(E)$ for $^{46}\text{Ti}(\alpha,n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha,n)^{54}\text{Fe}$, $^{54}\text{Fe}(\alpha,n)^{57}\text{Ni}$, $^{62}\text{Ni}(\alpha,n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha,n)^{71}\text{Ge}$ reactions with the target nuclear binding energy $B_{\text{tot}}(A,Z)$ at a fixed values of α -particle energy in center-of-mass system.

2- The adopted astrophysical $S(E)$ have been used as the input data as a function of target nuclear binding energy at fixed value of α -particle energies in center-of-mass system using the Excel computer programs. The obtained results are presented in Table (V).

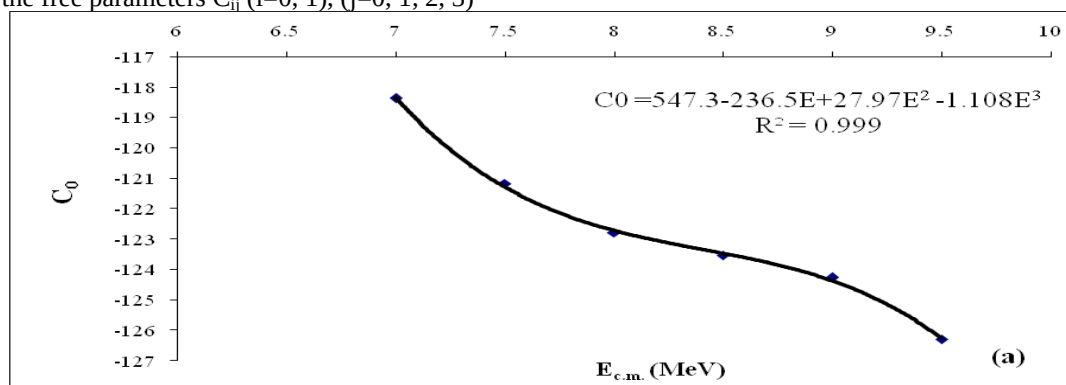
Table (V): Free parameters C_i (C_0 and C_1) as a function of α -particle energy in center-of-mass system E_α .

$E_\alpha(\text{MeV})$	C_0	C_1
7	-118.37	24.765
7.5	-121.18	25.153
8	-122.78	25.339
8.5	-123.53	25.387
9	-124.27	25.427
9.5	-126.29	25.655

3- The obtained free parameters (C_0 and C_1), as shown in Table (V) are plotted against with the prefixed values of α -particle energy in center-of-mass system (7, 7.5, 8, 8.5, 9 and 9.5MeV) as shown in Fig.(IV), and then the obtained free parameters C_i (C_0 , C_1) have been fitted to the following polynomial expression

$$C_i = \sum_{j=0}^3 C_{ij} E^j \quad (36)$$

with the free parameters C_{ij} ($i=0, 1$), ($j=0, 1, 2, 3$)



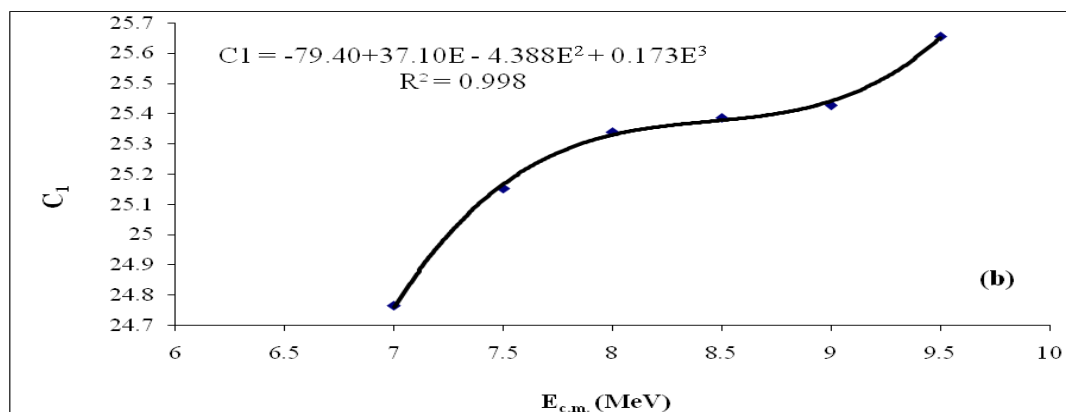


Fig. (IV): Plots of C_i coefficients against α -particle energy. (a) for C_0 , (b) for C_1 . The solid line represents the fitted curve through the data. The fitted expressions for each coefficient are also presented.

The combination of the expressions and polynomial expressions from energy ranges (7 to 9.5 MeV) takes the form :

$$Y = \sum_{j=0}^3 C_{0j} E^j + \sum_{j=0}^3 C_{1j} E^j X \quad (37)$$

$$Y = C_{00}E^0 + C_{01}E^1 + C_{02}E^2 + C_{03}E^3 + C_{10}E^0X + C_{11}E^1X + C_{12}E^2X + C_{13}E^3X \quad (38)$$

Where $X = \ln[B_{\text{tot}}(A, Z)]$, $Y = \ln[S(E)]$ and $(C_{00}, C_{01}$ and $C_{02})$ are free parameters as follow:

$$\begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} \\ C_{10} & C_{11} & C_{12} & C_{13} \end{bmatrix} = \begin{bmatrix} 547.31 & -236.57 & 27.973 & -1.1089 \\ -79.401 & 37.109 & -4.3889 & 0.1733 \end{bmatrix}, \begin{matrix} R^2 = 0.999 \\ R^2 = 0.998 \end{matrix}$$

The obtained formula at a set of reactions such as $^{46}\text{Ti}(\alpha, n)^{49}\text{Cr}$, $^{50}\text{Cr}(\alpha, n)^{53}\text{Fe}$, $^{54}\text{Fe}(\alpha, n)^{57}\text{Ni}$, $^{62}\text{Ni}(\alpha, n)^{65}\text{Zn}$ and $^{68}\text{Zn}(\alpha, n)^{71}\text{Ge}$ has been used to calculate the astrophysical S-factor $S(E)$ for each of the above reactions and compared with the adopted astrophysical S-factor calculated from the fitting expressions and the comparison of the two results are shown in Table (VI).

Table (VI): The Comparison between polynomial fitting expression (best fitting) of the adopted astrophysical S-Factor of (α, n) reactions with those calculated from the equation (38)

$E_{\alpha, m.}$ (MeV)	$^{46}_{22}\text{Ti}(\alpha, n)^{49}_{24}\text{Cr}$		$^{50}_{24}\text{Cr}(\alpha, n)^{53}_{26}\text{Fe}$		$^{54}_{26}\text{Fe}(\alpha, n)^{57}_{28}\text{Ni}$	
	$\ln[S\text{-Factor(MeV-b)}]$ (best fitting)	$\ln[S\text{-Factor}]$ (MeV-b) (Formula)	$\ln[S\text{-Factor(MeV-b)}]$ (best fitting)	$\ln[S\text{-Factor}]$ (MeV-b) (Formula)	$\ln[S\text{-Factor(MeV-b)}]$ (best fitting)	$\ln[S\text{-Factor}]$ (MeV-b) (Formula)
7	29.9265±1.0265	29.8073	31.9382±0.3737	31.9981	34.2353±2.2698	34.0031
7.25	29.706±1.0189	29.5350	31.7840±0.3719	31.7471	33.8869±2.2467	33.7716
7.5	29.4724±1.0109	29.2809	31.6075±0.3698	31.5074	33.5620±2.2252	33.5451
7.75	29.2306±1.0026	29.0381	31.4088±0.3675	31.2737	33.2852±2.2068	33.3197
8	28.9858±0.9942	28.80	31.1878±0.3649	31.0407	33.0636±2.1921	33.0913
8.25	28.7430±0.9859	28.560	30.9446±0.3621	30.8032	32.887±2.1804	32.8561
8.5	28.5071±0.9778	28.3113	30.6791±0.3589	30.5559	32.7279±2.1699	32.6101
8.75	28.2834±0.9701	28.0473	30.3914±0.3556	30.2936	32.5418±2.1575	32.3494
9	28.0767±0.9630	27.7612	30.0814±0.3520	30.0110	32.2665±2.1393	32.070
9.25	27.8922±0.9567	27.4464	29.7492±0.3481	29.7029	31.8229±2.1099	31.7681
9.5	27.7349±0.9513	27.0961	29.3947±0.3439	29.3640	31.1143±2.0629	31.4396

Table (VII): Continued

E _{c.m.} (MeV)	⁶² ₂₈ Ni(α,n) ⁶⁵ ₃₀ Zn		⁶⁸ ₃₀ Zn(α,n) ⁷¹ ₃₂ Ge	
	Ln[S-Factor(MeV-b)] (best fitting)	Ln [S - F a c t o r] (MeV-b) (Formula)	Ln[S-Factor(MeV-b)] (best fitting)	Ln [S - F a c t o r] (MeV-b) (Formula)
7	37.7066±1.8589	37.5863	39.8122±1.0192	39.7627
7.25	37.5885±1.8531	37.3897	39.6653±1.0154	39.5873
7.5	37.4202±1.8448	37.1869	39.4987±1.0112	39.3988
7.75	37.2114±1.8345	36.9762	39.3156±1.0065	39.1971
8	36.9718±1.8227	36.7561	39.1190±1.0014	38.982
8.25	36.7113±1.8099	36.5250	38.9121±0.9961	38.7534
8.5	36.4397±1.7965	36.2813	38.6980±0.9907	38.5111
8.75	36.1666±1.7830	36.0235	38.4797±0.9851	38.255
9	35.9020± 1.770	35.7499	38.2604±0.9795	37.9849
9.25	35.6555±1.7578	35.4589	38.0432±0.9739	37.7007
9.5	35.4370±1.7470	35.1491	37.8311±0.9685	37.4021

V. Conclusions

Through out this work, the following points are concluded:

- 1- The astrophysical S-factor was increase and decrease unregularly by increasing the α -particle energy in center-of-mass system, while beyond the coulomb barrier astrophysical S-factor decrease regularly.
- 2- The astrophysical S-factor increased with increasing (A) target nuclei at fixed α -particle energy in center-of-mass system.
- 3- The established formulae certify that increasing of the α -particle energy leads to decreasing of the astrophysical S-factor because when the α -particle energy increase, the cross section decreases, subsequently the astrophysical S-factor decreases.
- 4- The calculated astrophysical S-factor for the even neutron number(N) for ⁴⁶Ti(α,n)⁴⁹Cr , ⁵⁰Cr(α,n)⁵³Fe, ⁶²Ni (α,n) ⁶⁵Zn and ⁶⁸Zn(α,n) ⁷¹Ge reactions behaves an increasing trend with the target atomic number (Z).
- 5- The calculated astrophysical S-factor for even-even nuclei of the reactions ⁴⁶Ti(α,n)⁴⁹Cr , ⁵⁰Cr (α,n)⁵³Fe , ⁵⁴Fe(α,n) ⁵⁷Ni , ⁶²Ni (α,n) ⁶⁵Zn and ⁶⁸Zn(α,n) ⁷¹Ge reveals an increasing trend with the binding energy B_{tot}(A,Z) of the targets in the above reactions.

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