



Optimization with Economic Order Quantity in a Furniture industry: A Multiple–Item Joint Replenishment with Shortage

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Abstract: To minimize fixed costs associated with ordering, alternating demands, lead time and transportation uncertainty encourages excessive ordering quantity which can build an excessive inventory that occupied necessary space which costs an organization a lot. There are several works in associated with such problems that found economic order size, frequency, reorder point considering single item order with or without discount, with or without shortage, multiple item order with or without discount. But there is no significant work over multiple items ordering considering shortage. The objective of this paper is to determine order size, reorder point, ordering frequency, and time between orders considering shortage that an organization faced each year, in this case in NADIA Furniture Ltd. it is about 4.1% for plain particle boards. In this paper lead time uncertainty isn't taking into account.

Keywords: Inventory, EOQ, Multiple items, PPB, Shortage component;

I. Introduction

The Each time an order received of plain particle board, category A material of serial no. 1, has a discrepancy with quantity of the items ordered in a lot which causes production loss of a significant amount each year at NADIA Furniture Ltd. This paper developed a deterministic model for multi-item joint replenishment considering shortage. For single item ordering with backorder and unit discount, Jose and Leguna [1] developed an algorithm regarding those factors. Wu and Lin [2] formed a single item inventory model for quantity receiving discrepancy with lead time variation. Allowing backlogging the impact of different values of the unit backorder cost, setup cost, and unit cost to the make-or-buy decision is studied specifically in chiu's [3] mathematical analysis. Optimal solutions with complete backlogging and without backlogging are established for deteriorative items in Uthayakumar and Rameswari [4] analysis. Heuristic algorithms developed by Cha and Moon [5] for quantity discount of joint replenishment problem assuming constant demand pattern. As there is no work regarding the specific problem of shortage consideration in joint replenishment this paper determines the quantities of individual items within the group regarding shortage assuming constant demand. A deterministic model is developed; and each parameter such as setup cost S , annual value of all items in a group A_v and for item I a_i , inventory carrying charge I , shortage percentage etc. are determined with detail computations considering all possible measures.

II. Page Joint Replenishment Model Considering Shortage

Similar to the general joint replenishment problem under a deterministic demand condition, the following assumptions are made:

1. The demand rate for each item is constant and deterministic.
2. The replenishment lead time is of known duration.
3. Discounts are not considered.
4. The entire order quantity is delivered at the same time.
5. The price of each item is independent on the magnitude of the replenishment of each item.
6. The inventory holding cost for each item is known and constant, independent of the price of each item.
7. Annual demand value of all items is considered instead of quantity.

The model determines the economical order quantities for a group of items by minimizing the total cost of inventories, setups, and shortages. To discuss the model we introduce the following notation,

S = fixed cost of placing an order for a group of items
 S_i = item-dependent marginal cost of placing an order associated with an additional item i
 A_v = annual value of all items in the group ordered
 a_{vi} = annual value of item i in the group ordered
 C_i = unit cost of item i
 D_i = annual demand for item i in number of units

I = inventory carrying charge expressed as a decimal

Q_v = total value of all items ordered during a cycle

Q_{vi} = value of item i ordered during a cycle

Q_i = quantity of item i ordered during a cycle

N = number of cycles per year

T = time between orders in years

Π = shortage expressed in decimal

B_v = backorder cost per cycle

The following relationships are established-

$$A_v = \sum a_{vi}, Q_v = \sum Q_{vi}, a_{vi} = N * Q_{vi}, D_i = a_{vi}/C_i, T = 1/N = Q_v/A_v = Q_{vi}/a_{vi}$$

Placing of an order involve both fixed and variable set up costs i.e $(S+Si)$, now if backorder value of B_v is allowed with ordered items value of Q_v per cycle, then the average backorder could be $B_v/2$. So that the total relevant cost (TRC) = Annual average cost of inventory carried (a) + annual average backorder cost (b)+ total cost of placing orders per year (c).

Where,

$$\begin{aligned} a &= \frac{Q_v - B_v}{2} * I * \frac{Q_v - B_v}{Q_v} \dots \dots i \\ b &= \frac{B_v}{2} * \pi * \frac{B_v}{Q_v} \dots \dots ii \\ c &= \frac{Q_v}{B_v} * (S + Si) \dots \dots iii \end{aligned}$$

$$TRC(Q_v, B_v) = \frac{Q_v - B_v}{2} * I * \frac{Q_v - B_v}{Q_v} + \frac{B_v}{2} * \pi * \frac{B_v}{Q_v} + \frac{Q_v}{A_v} * (S + Si) \dots \dots iv$$

Here $(\frac{Q_v - B_v}{Q_v})$ represents the time on each cycle when positive inventory exists and $\frac{B_v}{Q_v}$ gives the time when negative inventory exists. The optimal values for Q_v and B_v is obtained by setting the first derivative of TRC (Q_v, B_v) to zero.

The economical value of items to be ordered is solved as-

$$Q_v = \sqrt{\frac{2(S + Si)A_v}{I}} \sqrt{\frac{I + \pi}{\pi}} \dots \dots 1$$

The order value of each item in the group can be determined by multiplying this value with the proportion of the annual usage value of the individual item i to the total usage value of items in a group-

$$Q_{vi} = Q_v (a_{vi}/A_v) \dots \dots 2$$

The quantity of item i ordered during each cycle can be-

$$Q_i = Q_{vi}/C_i \dots \dots 3$$

Here the time between orders (T) can also be calculated using this model.

$$\begin{aligned} T &= Q_v/A_v = \sqrt{\frac{2(S + Si)A_v}{I}} \sqrt{\frac{I + \pi}{\pi}} \\ T &= \sqrt{\frac{2(S + Si)A_v}{I * A_v}} \sqrt{\frac{I + \pi}{\pi}} \dots \dots 4 \end{aligned}$$

Thus the number of cycles per year can be determined, $N = 1/T \dots \dots 5$

Above equations are used to calculate economic order values and quantities for each item in a group in each cycle with no. of cycle and time interval between cycles of a multi-item joint replenishment with shortage.

III. Calculations

Data from different relevant section of NADIA Furniture Ltd. are collected to determine EOQ and order frequency. After summarizing data for year 2011-2012 fiscal year the values of different parameters computed as follows-

Table 1 Setup Cost (S+Si)

Contribution of purchase dept. Per annum in BDT	1,059,600
Others cost per annum associated with in BDT	22,945
(1)+(2), Total cost in BDT	1,082,545
No. of orders	33
Cost per order, (3)/(4)	32,804

Here the cost of purchase department concludes percentage contribution of purchase department, personnel and the other costs like electricity bill, telephone bill, computer, internet etc.

Table 2 Inventory Carrying Charge

Description	Per annum cost in BDT
(1) Warehouse space	1,920,000.00
(2) Taxes	288,000.00
(3) Insurance	2,036,367.84
(4) Material handling	1,073,396.00
(5) Prime Rate 12%	9,868,909.80
(6) Total carrying cost per year, (1)...(5)	15,186,673.64
(7) Full capacity of warehouse in BDT	82,240,914.99
(8) Inventory carrying charge, I, (6)/(7)	0.18

Table 3 Annual Value of all Items

Description	Group	a_{vi}
pp board 9mmx4x8ft	PPB	843,276
pp board 12mmx4x8 ft	PPB	1,586,409
pp board 16mmx4x8 ft	PPB	327,596,739
pp board 18mmx4x8 ft	PPB	27,923,150
pp board 25mmx4x8 ft	PPB	22,000,610
$A_v (= \sum a_{vi})$		379,950,183

Annual value of each item (a_{vi}) is determined by multiplying the demand of each item with respective price.

Table 4 Shortage in Decimal

(1) Average Production monthly	180,000,000
(2) Production annually, (1)*12	2,160,000,000
shortage days per year	15
production loss of 15 days	88,767,123.3
shortage/ loss in decimal, π, (2)/ (4)	0.041

Using the values of different notations in equation 1 from table 1, 2, 3 and 4 the economical value of items to be ordered is computed. The data are summarized in table 5 with value of each order in BDT and time interval between orders in days.

Table 5 Summarized data

Table/ Eqn.	Notation	Value
table 4	π	0.041
table 1	S	32,804.00
table 3	A_v	379,947,850.30
table 2	I	0.18
eqn. 1	Q_v	27,321,739.81
eqn. 4	T	0.07
eqn. 5	N	14
	T in days	22.44

Above table enlisted all values to compute the order value and quantity of each item to be ordered in a cycle. From equation 2 and 3 item wise economic order value and quantity is determined, shown in table 6, below.

Table 6 Item wise Economic Order Value & Quantity

Description	D _i	Unit	C _i	a _{vi}	Q _{vi}	Q _i
pp board 9mmx4x8ft	886	BDT	951	843,276	60,639	64
pp board 12mmx4x8 ft	1,184	BDT	1,340	1,586,409	113,499	85
pp board 16mmx4x8 ft	215,132	BDT	1,523	327,596,739	23,557,214	15,470
pp board 18mmx4x8 ft	15,083	BDT	1,851	27,923,150	2,007,931	1,085
pp board 25mmx4x8 ft	9,071	BDT	2,425	22,000,610	1,582,046	652

By using computed economic order value and other parameters optimum cost of inventory carrying, order setup cost and backorder cost can be determined from equation I, II & III that are equal in value at trade off situation or optimal condition. The cost is typically of BDT 456,187.

IV. Results

Joint replenishment quantity of different sizes of plain particle board in an order for each cycle should be-

Table 7 Results

Description	Q _i
pp board 9mmx4x8ft	64
pp board 12mmx4x8 ft	85
pp board 16mmx4x8 ft	15,470
pp board 18mmx4x8 ft	1,085
pp board 25mmx4x8 ft	652
Total quantity in Pcs	17,355

Time interval between consecutive orders should be 23 days and the number of orders per year should be 14.

V. Conclusions and Discussions

This paper studied an inventory model (deterministic) with backorders and shortage encounter per year. This situation appears in practice when the orders received from vendors with certain percentage of shortage each year that hamper production significantly. The paper considers system of multi-item joint replenishment with shortage. The optimal policy is obtained through optimization procedure of minimizing total relevant cost. But the model doesn't consider quantity discount because for this particular material (PPB) quantity discount is insignificant. The results obtain from this model can vary as if demand value/ quantity is no longer deterministic and constant with time because the model is not probabilistic in nature. Obtained results can also vary a little due to some of the collected data may not exactly be as it was in that time frame (in this case 2011-2012 fiscal year). Future orders can be made but should vary the order quantities and frequency with varied forecast and demand.

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