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Optimizing Strength Characteristics of Powder Workpieces of Fe-C-Cu System

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Abstract: In this publication is presented optimization strength properties – tensile strength and elongation, of sintered structural materials of Fe-C-Cu system. Optimization is carried out by the method of simplex lattices adduced. Are presented graphical relationships amending studied characteristics depending on the concentration of the elements in the triple system.

Keywords: optimizing; powder workpieces; strength characteristics; pseudo components.

I. Introduction

One way to optimize the resistance properties of the Fe-C powder samples is by the addition thereto of the copper to form the liquid phase at sintering and to accelerate coalescence of pores in the samples. Copper, however, in the process of sintering increases the size of the samples and for adding and must be in amounts which, in addition to intensify caking and should not significantly affect the size of the workpiece. [3,6,7]

Tasks to study the properties of multi-component systems are among the most – common in chemistry and metallurgy. In chemistry, a multi-components system is different mixtures, and in metallurgy and materials science are various alloys.

In a study of multi – systems, all factors are divided into two main categories – *interdependent* and *not interdependent*.

Interdependent factors are usually the components of the mixture - the system. The total amount of a mixture or alloy in this case is assumed to be a unit, and often at i - both components it is meant X_i . [1,5]

Obviously it is the inequality 1.

$$0 \leq X_i \leq 1 \quad \text{where: } i = 1, 2 \dots q \text{ - the number of components in the alloy.} \quad (1)$$

It is evident that the sum of all X_i must be equal to the total mixture - i.e. the sum of all X_s must be equal to 1. Consequently, variation quantity of one component is not necessarily related to the amount of change of the other components of the mixture. For this reason, these factors are called interconnected.

Not interconnected factors are those in which may be set and maintained certain value independent of each other. Then encodes them in force the condition 2.

$$-1 \leq X_i \leq 1 \quad \text{where: } i = q + 1, q + 2, q + 3 \dots m \quad (2)$$

In this case, m is the total number of factors, and $r = m - q$ is the number of not interconnected.

Taking into account their physical nature, not interconnected factors may be referred to one of the following categories: *variables technological process* - these are the magnitudes defining the way of working or generate an alloy; *components of the mixture* and *quantity of the mixture*.

In some technology components of the alloy can be divided into two groups. One of them is taken as the basic and the amount from components of which it is composed is always accepted as 100 weight units. For this group is implemented inequality 1. The amount of the components of the second group is determined at preset limits and may be varied independently of any other. For this group is implemented condition 2.

In some from the research the results depend both on the composition of the mixture and of its quantity. It can be varied independently from the composition and may therefore be referred to mutually independent factors.

The conditions in phrase 1 determined space amending the factors. This is a geometric body called simplex. For one-dimensional space simplex is determined by any two mismatching points, i.e. simplex is each segment. In two-dimensional space simplex formed of three points not located on one straight, i.e. simplex is every triangle. In three-dimensional space simplex is a triangular pyramid, etc.

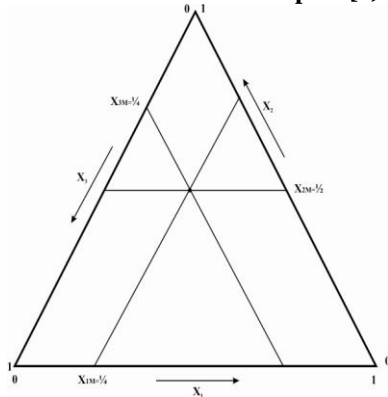
In cases where in the study participate only q mutually independent components, acting as a coordinate system is convenient to use the fins of $q-1$ dimensional simplex. This is called barycentric. In binary mixtures

barrycentric coordinate system is segment. In this case, the composition of the mixture is determined completely by the application of a coordinate values – for example X_1 , on section, since the amount of the second component is calculated uniquely according to 3.

$$X_2 = 1 - X_1 \quad (3)$$

For a ternary system baritsentrichnata coordinate system is formed by the edges of an equilateral triangle. The values of the individual factors are applied to them in a uniform scale of 0 to 1. In determining the point corresponding to the specified mix using the following rule. The rib of the simplex corresponding to the first component is applied to the set value of X_1 . This point is passed parallel to the straight lying against the tip of the thigh in which $X_1 = 1$.

Figure 1. Determination of the position of points in three-dimensional simplex [1,5]



Then, the axis X_2 is applied to the value of the second component. In the resulting point is passed straight, parallel to the rib subtended by the tip in which $X_2 = 1$. Likewise draw the straight rib parallel to the engaged against the tip of $X_3 = 1$.

This does not intersect at a point corresponding to a composition of the mixture.

Simplex grid plans apply in cases where all factors are met 1 and 2

They are suitable for determining the coefficients of the polynomial models adduced. Typical for them is that the number of points they are always equal to the number of regression coefficients.

Constructions of simplex grid plans are performed as to record all possible combinations of the expression 4.

$$0, 1/n, 2/n, \dots, (n-1)/n, 1 \quad (4)$$

When drawing up the plans simplex is necessary to observe the following main conditions:[1]

- ❑ number of indices is less than or equal to the polynomial degree - n ;
- ❑ uneven number of indices equals the number of components in the system;
- ❑ the system is composed of as many equal parts as the number of indices;
- ❑ the components are contained in so of these parts, as the number of indices

Complying with the above conditions has is being developed experimental plan and draw up a simplex lattice Of figure 2 are shown examples of simplices linear model brought the type 5 and simplex brought a second order polynomial - equality 6, which is found the greatest application in practice.[1,2,5]

$$Y = b_1.X_1 + b_2.X_2 + b_3.X_3 \quad (5)$$

$$Y = b_1.X_1 + b_2.X_2 + b_3.X_3 + b_{12}.X_1.X_2 + b_{13}.X_1.X_3 + b_{23}.X_2.X_3 \quad (6)$$

II. Technical requirements

Simplex lattices are only applicable if running conditions 1 for each component. For our sample case this is impossible since the underlying on modeling system is threefold Fe - C - Cu, and none of the components can not accept zero. In this case, the area of amendment of the components is narrowed to include only the compositions under which the properties of the alloy adopting reasonable values. Limited subzone in which experiment is carried out is selected for each particular case by technological considerations and can have various conventional forms.

If the limited subzone is simplex in planning the experiment after a preliminary transformation of the variables are used simplex grilles of Chefe. The tips of the simplex are denoted by S_1 , S_2 , and S_3 , and it needs not necessarily be correct.[1,4]

As can be seen from the figure within the hatched subzone the components cannot be changed from 0 to 1. I.e. breached be a condition 1.

As a consequence, pass to new variables called psevdokomponenti - Z_1 , Z_2 , and Z_3 , which everywhere in the limited subzone be implemented conditions 7 and 8.

Figure.2 Scheme of the simplex in linear model brought [5]

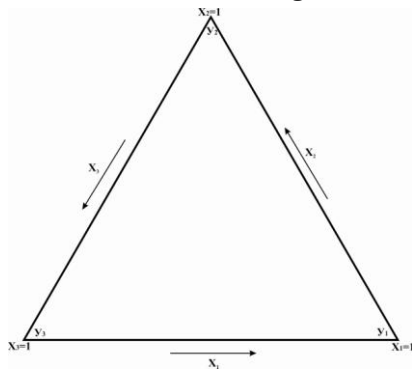
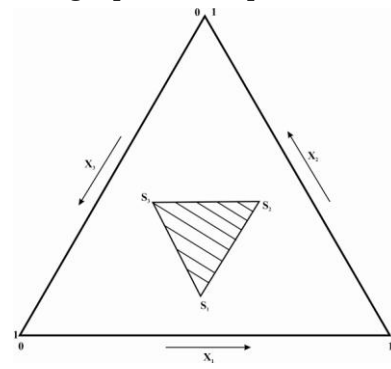


Figure 3. A schematic diagram of a simplex brought pseudo components [1,4]



$$0 < Z_i < 1 \quad (7) \quad \sum_{i=1}^q Z_i = 1 \quad (8)$$

In this regard, research in this publication are directed to optimizing the amount of copper in the triple system Fe-C-Cu, in order to achieve optimum strength characteristics without substantial changes in the size of the workpiece.

Of study are subject powder metallurgy construction materials based on iron powder ASC 100.29 to which is added 0,2 ÷ 0,8%C and 2,0÷3,0% Cu. Defined are their tensile strength and their relative elongation. Depending on the concentration of copper and carbon in the iron matrix is made of the optimization of these characteristics of the method adduced simplex gratings.

In optimizing the composition of samples of powder metallurgy system Fe-C-Cu, used brought simplex gratings plans in which optimization is performed in a restricted subregion of the simplex. In the optimization process used pseudokomponenti. The influence of iron concentration on the properties of the tested samples is indicated by X_1 , X_2 of the carbon and the copper with X_3 .

As the main parameters subject to use for testing: *relative elongation* - Y_1 and *the tensile strength* - Y_2 .

Proceeding from preliminary experiments, a state diagram Fe - C, and Fe - Cu, and by the process parameters in the process of sintering of the test system model determining the concentration of the components be varied in the range of under 9.

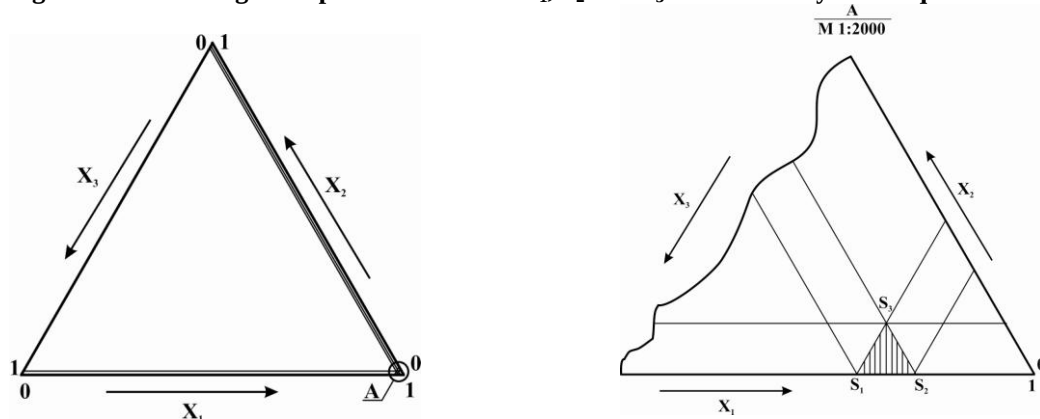
Research areas defined by the restrictions is presented in figure 4

From the figure shows that the constraints form a simplex with vertices according to the matrix 10.

$$\begin{aligned} 0,970 &\leq X_1 \leq 0,962 \\ 0,000 &\leq X_2 \leq 0,008 \\ 0,022 &\leq X_3 \leq 0,030 \end{aligned} \quad (9)$$

$$\begin{aligned} S_1 &= (0,962; 0,000; 0,030) \\ S_2 &= (0,970; 0,008; 0,022) \\ S_3 &= (0,962; 0,008; 0,030) \end{aligned} \quad (10)$$

Figure 4. Area brought simplex with vertices S_1 , S_2 and S_3 are limited by the inequalities 9.



Assumed that our study properties can be described brought a second order polynomial. Plan Schaffel in pseudo components in this area is presented in table 1.

It consists of six main points and the control point 7 is chosen so that - it's easy to switch to incomplete polynomial of the third degree in case initially selected model proves inadequate.

Table 1. Experimental plan

№ of oper	Pseudo components			Primary components			Characteristics studied	
	Z ₁	Z ₂	Z ₃	X ₁	X ₂	X ₃	Y ₁	Y ₂
1	2	3	4	5	6	7	8	9
1	1	0	0	0,970	0	0,030	4,9	291
2	0	1	0	0,970	0,008	0,022	2,6	625
3	0	0	1	0,962	0,008	0,030	1,8	693
4	0,500	0,500	0	0,970	0,004	0,026	3,3	412
5	0,500	0	0,500	0,966	0,004	0,030	2,8	438
6	0	0,500	0,500	0,966	0,008	0,026	2,1	649
7	0,333	0,333	0,333	0,9664	0,0053	0,0283	3,1	467
8	0,250	0,500	0,250	0,968	0,006	0,026	2,9	493
9	0,500	0,250	0,250	0,968	0,004	0,028	3,0	429
10	0,250	0,250	0,500	0,966	0,006	0,028	2,4	517

To implement the plan, it is necessary to calculate the initial components. For this purpose was used the matrix 11. After performing the matrix multiplication we get system of equations 12.

$$\begin{matrix} X_1 & 0,970 & 0,970 & 0,962 & Z_1 \\ X_2 & = & 0,000 & 0,008 & 0,008 & . & Z_2 \\ X_3 & & 0,030 & 0,022 & 0,030 & & Z_3 \end{matrix} \quad (11)$$

$$\begin{aligned} X_1 &= 0,970.Z_1 + 0,970.Z_2 + 0,962.Z_3 \\ X_2 &= 0,000.Z_1 + 0,008.Z_2 + 0,008.Z_3 \\ X_3 &= 0,030.Z_1 + 0,022.Z_2 + 0,030.Z_3 \end{aligned} \quad (12)$$

The values of the input variables were prepared 12 substituting Z_1 , Z_2 and Z_3 for each point of the plan. Inflict the results in columns 5, 6 and 7 of table 1.

To a quantified ratios calculated for the components in the experimental plan - columns 5, 6 and 7 has been added lubricant 0,8% Kenolube and made powder samples in the following technological characteristics: *forging effort - 600MRa; temperature of sintering - 1150 °C and duration of sintering - 60 min.*

After the sintering density of the samples is in the range of 7,0 g/cm³. The thus manufactured samples tested were measured mechanical properties and have been filled columns 8 and 9 of the experimental plan. To eliminate the effect of voids content on the results obtained for each type of model have been tested on 5 measurements in the experimental plan are done the arithmetic mean of the results.

Using the results in table 1 and equations 13 and 14 are calculated on the adapted text polynomial coefficients for the studies mechanical properties, and the results of the calculation are shown in table 2.

$$b_i = y_i \quad (13) \quad b_{ij} = 4y_{ij} - 2Y_i - 2Y_j \quad i < j \quad (14)$$

Table 2. Coefficients of the polynomial

Sampled characteristics	Coefficients of the polynomial					
	b ₁	b ₂	b ₃	b ₁₂	b ₁₃	b ₂₃
1	2	3	4	5	6	7
Y ₁	4,9	2,6	1,8	-1,8	-2,2	-0,4
Y ₂	291	625	693	-184	-216	-40

Proceeding from values in table 2 we can write the mathematical model for studying characteristics - 15 and 16.

$$Y_1 = 4,9Z_1 + 2,6Z_2 + 1,8Z_3 - 1,8Z_{12} - 2,2Z_{13} - 0,4Z_{23} \quad (15)$$

$$Y_2 = 291Z_1 + 625Z_2 + 693Z_3 - 184Z_{12} - 216Z_{13} - 40Z_{23} \quad (16)$$

For the verification of the adequacy of the model is computed evaluation of the residual dispersion in four control points - 7 ÷ 10, and calculate the residual sum of squares of these points - formula 17, and the results obtained for the surveyed characteristics are presented in table 3.

$$S_{oct}^2 = \frac{Q_{oct}^2}{\vartheta_{oct}^2} \quad (17)$$

Through the doing trials with same composition of a mixture - item 7 of table № 1 on the define evaluation of the dispersion error - S_e^2 in formula 18.

The values obtained after conduct of additional trials are presented in table 4. In the same table presents the values of mathematical expectation - m_y , calculated by formula 19.

$$S_{\varepsilon}^2 = \frac{1}{n-1} \sum_{i=1}^n (y_n - m_{\hat{y}})^2 \quad (18) \quad m_{\hat{y}_1} = \frac{1}{n} \sum_{i=1}^n y_n \quad (19)$$

Using the results of tables 3 and 4 by $F = \frac{S_{\text{oct}}^2}{S_{\varepsilon}^2}$ are defined dispersion relations for the tested functions - table 5.

Table 3. Residual sum of squares

Sampled characteristics	Residual sum of squares – S_{oct}^2						
	$\tau.7$	$\tau.8$	$\tau.9$	$\tau.10$	$Q_{\text{oct.}}$	$V_{\text{oct.}}$	S_{oct}^2
1	2	3	4	5	6	7	8
Y_1	3,008	2,513	3,024	2,337	0,1635	4	0,0408
Y_2	487,005	512,000	422,500	532,000	1028,450	4	257,113

Table 4. Evaluation of the dispersion error

Sampled characteristics	Residual sum of squares – S_{ε}^2						
	1	2	3	4	5	m_y	S_{ε}^2
1	2	3	4	5	6	7	8
Y_1	3.15	3.07	2.95	3.20	3.10	3,094	0,00893
Y_2	471,0	469,0	459,0	461,0	475,0	467,0	46,000

Table 5. Dispersion ratios

Sampled characteristics	Dispersion ratios			
	S_{oct}^2	S_{ε}^2	F_Y	$F_{\text{табл.}}$
1	2	3	4	5
Y_1	0,0408	0,00893	4,56887	6,3900
Y_2	257,113	46,000	5,58941	6,3900

Figure 5 Location of lines of equal importance to the function Y_1 – elongation, %

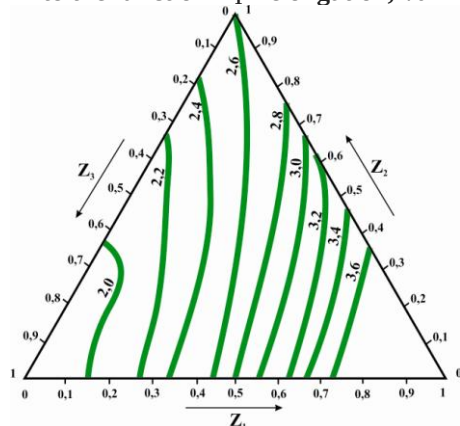


Figure 6 Position of the lines of equal significance of the function Y_2 - tensile strength, MPa

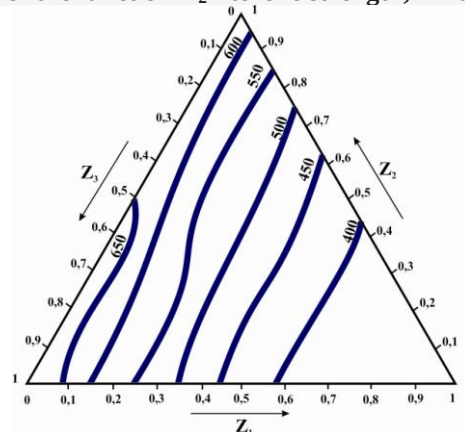


Figure 7. Location of the lines surveyed properties in simplex space

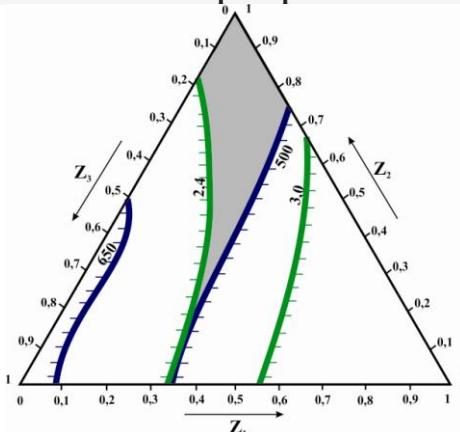
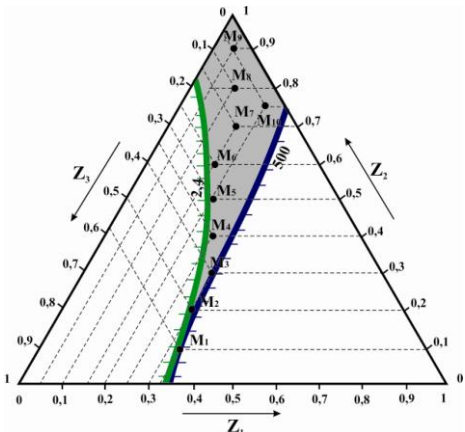


Figure 8. Optimal areas of simplex space



Whit the number of degrees of freedom $v_{\text{ost.}} = 4$ and $v_e = 4$, and a significance level $\alpha = 0,05$, the tables for the distribution the criteria of Fisher recognize the F table = 6.39.

From the obtained results in table 5 it is seen that the calculated dispersion relations for the two functions we tested are less than the tabulated values of the criteria of Fisher. It can therefore be concluded that the developed by us mathematical models are adequate.

Using mathematical models are calculated points simplex gratings surveyed us functions Y_1 and Y_2 . Graphical interpretation of the results of the functions Y_1 and Y_2 is presented respectively in figure 5 and 6. Figure 7 shows the summary of the two simplex tested functions.

The aggregate simplex using the system 12 define quantitative ratios of the optimal compositions system Fe - C - Cu responsible concentrations of point M- figure 8.

III. Conclusion

Of the examination and the results can be formulated following - important conclusions:

- Was held optimization system components Fe-C-Cu method adduced simplex gratings plans:
 - ✓ the experimental and literature data defined concentration ranges of variation for the optimized system components - Fe $0,970 \div 0,962$; Cu - $0,022 \div 0,030$; C $0,001 \div 0,008$;
 - ✓ some are manageable parameters optimization - elongation - Y_1 and tensile strength - Y_2 ;
 - ✓ the optimization is carried out by bringing it a second order polynomial, and the plan of Schaffel for pseudokomponenti used 6 major and a control point;
 - ✓ developed mathematical models for studying mechanical properties and proven their adequacy:

$$Y_1 = 4,9Z_1 + 2,6Z_2 + 1,8Z_3 - 1,8Z_{12} - 2,2Z_{13} - 0,4Z_{23}$$

$$Y_2 = 291Z_1 + 625Z_2 + 693Z_3 - 184Z_{12} - 216Z_{13} - 40Z_{23}$$
- Developed graphical relationships "composition- properties" for optimized components for control functions.
- Developed summary diagram "composition - properties" for optimized components to control functions and the optimal zone of sets of mechanical properties locked within the range of tensile strength $500 \div 650 \text{ MPa}$ and elongation $2,4 \div 3,0\%$.
- In the optimal zone specified in p.M in the 10 points are calculated actual concentrations of the components of the studied system and found that they can vary in optimal range, respectively: $96,55 \div 96,95\%$ Fe, $0,55 \div 0,75\%$ C and $2,3 \div 2,9\%$ Cu.

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