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Admission Control of Bulk Retrial Queue under Bernoulli Vacation Schedule

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Abstract: This paper examines the Bernoulli admission control policy for the two phase bulk arrival retrial queueing system wherein the customers has the option either to take second phase of service or to leave the system after completing first essential phase of service. In addition, the arrival rate of the customers depends upon the server status. After the service completion of each individual customer, the server takes vacation according to Bernoulli schedule at any phase of service. The vacation times and service times of both first essential and second optional services are assumed to be general distributed while the retrial times are exponential distributed. By using embedded markov chain technique, we have derived the distribution of the number of customers in the retrial group and in the queue. Some other system characteristics such as average orbit size, average queue size, etc. have been derived. The effects of different parameters on various performance measures are explored via numerical experiments.

Keywords: Retrial queue, Bernoulli admission, Optional phase service, Embedded markov chain, Queue size.

I. Introduction

Retrial queues can be seen in computer and telecommunication networks including local and wide area networks with the random multiple access protocols, distribution services, manufacturing plants, call centers, etc.. There has been rapid growth in the literature on the queueing models with repeated attempts which are characterized by the following feature: When an arriving customer finds that the server is busy and no waiting position is available, the customer joins a virtual pool of blocked customer called 'orbit'. The batch arrival retrial queueing model was introduced by Falin (1976) who worked on the rule that "if the server is busy at the arrival epoch, then the whole batch joins the retrial group, whereas if the server is free, then one of the arriving units starts its service and the rest join the retrial group". The detailed accounts and related references on retrial queues can be found in the reference book of Falin and Templeton (1997) and the survey papers by Artelejo (1999a, b).

In this paper, we study the **classical retrial policy**, according to which the customers retry for its service with rate $j\mu$, where j is the number of customers in the retrial group. This discipline is typical of telephone applications where the retrials are made individually by each blocked customer with rate μ . The consideration of the admission probability can be viewed as a first step to extend the existing control mechanisms for the admission of customers in the standard waiting lines to queues with repeated attempts. The motivation to study such type of queueing system comes from applications to real time systems including telephone system wherein a person makes a phone call to any institution in order to ask about the pending matter. If the pending matter is successfully solved by the interlocutor, then it is referred as first phase of service whereas, if it is not successfully solved by the interlocutor in the first contact phase, then he passes the request to another employee (i. e. second optional phase of service). It may also happen that the interlocutor can either go for a vacation with some probability σ or may receive the next call, if any, with probability $\bar{\sigma}$ ($=1-\sigma$). The customers may also arrive in batches and their arrival rate may depend upon server's status. This kind of system can be viewed as a single server state dependent retrial queue with batch arrivals, first essential service and second optional service under Bernoulli schedule vacation and Bernoulli admission.

Various scenarios have been considered in the literature, thus giving rise to various denominations for the retrial queueing systems. Aissani (2000) examined the bulk arrival retrial queue with exhaustive vacations i.e. the vacation begins only when there are no customers in the system. Choudhury (2007) considered an $M^x/G/1$ queueing system with two phases of heterogeneous service and Bernoulli vacation schedule which operate under classical retrial policy. Boualem *et al.* (2009) considered an $M/G/1$ retrial queue with vacations and derived several stochastic comparison properties in the sense of strong stochastic ordering and convex ordering. Langaris and Dimitriou (2010) have obtained the notable performance measures of a retrial queueing system with a single server providing n -phases of service in succession. The overwhelming literature is available on $M/G/1$ queueing system but only a few has included the concept of two phase service/optional

service in their study. Several researchers have put forward their efforts towards this field viz. Madan and Choudhury (2005), Wu *et al.* (2009), Wang *et al.* (2010), etc..

The Bernoulli admission control policy has also been appeared in some papers. Choudhury and Madan (2007) have obtained explicit results for the queue length and system state probabilities for the two stage batch arrival retrial queueing system under Bernoulli vacation schedule. Moreover, he has considered that the server can go for a vacation according to Bernoulli schedule, only after the completion of two stages of service in succession. Choudhury and Deka (2009) introduced the concept of Bernoulli admission mechanism for the $M^X/G/1$ retrial queue with an additional second phase of optional service and unreliable server where breakdowns occur randomly at any instant while serving the customers. Do (2010) introduced a retrial queueing problem for wireless cellular networks applying the Fractional Guard Channel (FGC) admission control policy.

Our aim in this paper is to generalize the work of Choudhury (2007) by incorporating (i) state dependent arrival rates, (ii) FES and SOS and (iii) choice for the server to go for a vacation in either phase of service according to Bernoulli schedule. In the next section II, we describe the model formally by stating the requisite assumptions and notations. Section III is devoted to embedded markov chain approach to study the limiting distribution of queue size at random and departure epochs and the joint distribution of the number of customers in the retrial group and the state of the server. Various queueing characteristics have been derived in section IV. Numerical results have also been provided in section V to verify the analytical results established in previous sections. In section VI, conclusion related to the investigation done have been drawn.

II. Model Description

Consider the single server retrial queueing system with batch arrivals. The primary units arrive in bulk according to Poisson process with rate λ_1 (λ_2) and λ_v when the server is busy with FES (SOS) and on vacation, respectively. In the batch arrival retrial queue, it is assumed that at every arrival epoch, a batch of k primary units arrives with probability a_k . If θ denotes the probability of admission for each individual customer, then the probability that a group of size $n \geq 0$ joins the system is defined by

$$c_0 = \sum_{k=1}^{\infty} a_k (1-\theta)^k \text{ and } c_n = \sum_{k=n}^{\infty} a_k \binom{k}{n} \theta^n (1-\theta)^{k-n}; n \geq 1 \quad (1)$$

The admitted customers follow the *Bernoulli admission control policy*, i. e. if the server is free at the arrival epoch, then one of the customers starts its service and the rest join the retrial group. On the other hand, all the admitted customers go to the retrial group when the server is busy. The discipline to access from the retrial group to the server is governed by the *classical retrial policy* (i. e. $\gamma_j = j\gamma$, where j is the number of customers in the retrial group). If the server is busy at the arrival epoch, then all the arriving units join the orbit, whereas if the server is free, then one of the unit begins its service and the others form sources of repeated attempts. The server provides preliminary first essential service (FES) to all the arriving units and second optional service (SOS) to only some customers depending upon their choice. As soon as the FES of a customer is completed, he may opt for SOS with probability p or leave the system with the complementary probability $\bar{p}$. However, after each service completion, the server either takes a vacation with probability σ_1 (σ_2) or starts a new service with the complementary probability $\bar{\sigma}_1$ ($\bar{\sigma}_2$) during FES (SOS). The decisions about taking a vacation after each service completion (either during FES or during SOS) are iid random variables independent of the input process. In addition, we denote the r^{th} factorial moments of $a(z)$ and $c(z)$ by $a^{(r)}$ and $c^{(r)}$ ($r \geq 1$), respectively; thus $c^{(r)} = \theta^r a^{(r)}$; $r \geq 1$. We also denote $c^{(1)} = E(X)$ and $c^{(2)} = E(X^2)$. The time required by a customer to complete a service cycle is called as a modified service time (denoted by G). Table 1 describes the notations used for some random variables (r. v.) along with their probability distribution function (PDF), Laplace Steiljes transform (LST) and r^{th} ($r \geq 1$) moments.

Table 1: Notations

Time	r. v.	PDF	LST	r^{th} moments
FES	S_1	$S_1(t)$	$b_1^*(s)$	$b_1^{(r)}$
SOS	S_2	$S_2(t)$	$b_2^*(s)$	$b_2^{(r)}$
Vacation during FES	V_1	$V_1(t)$	$\xi_1^*(s)$	$\xi_1^{(r)}$
Vacation during SOS	V_2	$V_2(t)$	$\xi_2^*(s)$	$\xi_2^{(r)}$
Modified service	G	$G(t)$	$G^*(s)$	$g^{(r)}$

III. Queue Size Distribution

In this section, we analyze the queueing process in order to derive the queue size distribution at random and departure epochs and the joint distribution of the number of customers in the retrial group and the state of the

server. The analysis is conducted using the embedded markov chains, generating functions, markov regenerative processes and embedded markov renewal process. First of all we describe the state of the system at time t by means of the process $Y(t) = \{C(t), M(t), \zeta(t)\}$, where $C(t)$ is equal to 0, 1, 2, or 3 according to whether the server is idle, busy with FES, busy with SOS or on vacation at time t . Let $M(t)$ denotes the number of customers in the orbit at time t . If $C(t) \in \{1, 2\}$, then $\zeta(t)$ represents the corresponding elapsed time of the service in progress. Let τ_n be the time of the n^{th} service completion epoch, i. e. the epochs at which the total service time requested by a customer expires. The sequence $N_n = N(\tau_n +)$ where $N(\tau_n)$ represents the number of customers in the system at time instant τ_n and forms a markov chain which is the embedded markov renewal process of the continuous time markov process $Y(t)$.

The concept of modified service time was first introduced by Keilson and Servi (1986) for GI/G/1 queueing system and subsequently by Keilson and Servi (1987, 1989) and others for M/G/1 queueing system. Let B_n be the number of customers arriving during the n^{th} modified service time. Let U_n represents the number of customers admitted to join the system if the n^{th} customer in service proceeds from a batch arrival. Then we observe that the sequence $\{N_n\}_{n=1}^{\infty}$ satisfies the state equation

$$(N_n / N_{n-1} = j) = \begin{cases} j - 1 + B_n \text{ with probability } \frac{j\gamma}{\lambda' + j\lambda} \\ j - 1 + B_n + U_n \text{ with probability } \frac{\lambda}{\lambda' + j\gamma} \end{cases} \quad (2)$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_v$; $\lambda' = \lambda_1' + \lambda_2' + \lambda_v'$; $\lambda_l' = \lambda_l(1 - c_0)$ ($l=1, 2$) and $\lambda_v' = \lambda_v(1 - c_0)$.

Now we proceed to derive the queue size distribution at departure epoch and the joint distribution of the number of customers in the retrial group and the server's state, as follows:

III.A Limiting distribution of queue size at departure epoch

By using the classical criteria based on mean drifts (see Artelejo and Gomez-Corral, 1997 and Sennott et al., 1983), it can be proved that the sequence $\{N_n\}_{n=1}^{\infty}$ is positive recurrent to guarantee that the limiting probabilities

$\pi_j = \lim_{n \rightarrow \infty} P\{N_n = j\}$, $j \geq 0$; exist and are positive. Let $h_j^{(1)}$, $h_j^{(2)}$, $g_j^{(1)}$ and $g_j^{(2)}$ be the probabilities that total number of 'j' customers arrive during the time intervals $\{S_1\}$, $\{S_1 + V_1\}$, $\{S_1 + S_2\}$ and $\{S_1 + S_2 + V_2\}$, respectively. These probabilities are defined as

$$h_j^{(1)} = h_{1,j}; \quad h_j^{(2)} = \sum_{u=0}^j h_{1,u} h_{2,j-u}; \quad g_j^{(1)} = \sum_{u=0}^j h_u^{(1)} m_{j-u}^{(1)} \quad \text{and} \quad g_j^{(2)} = \sum_{u=0}^j h_u^{(2)} m_{j-u}^{(2)}.$$

$$\text{Also, for } l \in \{1, 2\}, \quad h_{l,j} = \begin{cases} \int_0^{\infty} e^{-\lambda_l' t} dS_l(t), & \text{if } j = 0 \\ \sum_{k=1}^{\infty} \int_0^{\infty} \frac{e^{-\lambda_l' t} (\lambda_l t)^k c_j^{(k)} dS_l(t)}{k!}, & \text{if } j \geq 1 \end{cases} \quad \text{and} \quad m_j^{(l)} = \begin{cases} \int_0^{\infty} e^{-\lambda_v' t} dV_l(t), & \text{if } j = 0 \\ \sum_{k=1}^{\infty} \int_0^{\infty} \frac{e^{-\lambda_v' t} (\lambda_v t)^k c_j^{(k)} dV_l(t)}{k!}, & \text{if } j \geq 1 \end{cases}$$

Here $\{c_j^{(k)}\}_{j=0}^{\infty}$ denote the k^{th} convolution of the sequence $\{c_j\}_{j=0}^{\infty}$.

The one step transition matrix $P = (p_{ij})$, associated with $\{N_n\}_{n=1}^{\infty}$ has elements

$$p_{ij} = \begin{cases} \frac{j\gamma}{\lambda' + j\gamma} [\bar{p}\bar{\sigma}_1 h_0^{(1)} + \bar{p}\sigma_1 g_0^{(1)} + p\bar{\sigma}_2 h_0^{(2)} + p\sigma_2 g_0^{(2)}] & \text{if } i \geq 1, j = i - 1 \\ \frac{\lambda}{\lambda' + j\gamma} \sum_{n=1}^{j-i+1} c_n [\bar{p}\bar{\sigma}_1 h_{j-i+1-n}^{(1)} + \bar{p}\sigma_1 g_{j-i+1-n}^{(1)} + p\bar{\sigma}_2 h_{j-i+1-n}^{(2)} + p\sigma_2 g_{j-i+1-n}^{(2)}] & \\ \quad + \frac{j\gamma}{\lambda' + j\gamma} [\bar{p}\bar{\sigma}_1 h_{j-i+1}^{(1)} + \bar{p}\sigma_1 g_{j-i+1}^{(1)} + p\bar{\sigma}_2 h_{j-i+1}^{(2)} + p\sigma_2 g_{j-i+1}^{(2)}], & \text{if } 0 \leq i \leq j \end{cases} \quad (3)$$

Using $\pi = \pi P$, the steady state departure point probabilities are given by

$$\pi_j = \sum_{n=0}^j \frac{\lambda \pi_n}{\lambda' + n\gamma} \sum_{m=1}^{j-n+1} c_m [\bar{p}\bar{\sigma}_1 h_{j-n+1-m}^{(1)} + \bar{p}\sigma_1 g_{j-n+1-m}^{(1)} + p\bar{\sigma}_2 h_{j-n+1-m}^{(2)} + p\sigma_2 g_{j-n+1-m}^{(2)}] + \sum_{m=1}^{j+1} \frac{m\gamma\pi_m}{(\lambda' + m\gamma)} [\bar{p}\bar{\sigma}_1 h_{j-m+1}^{(1)} + \bar{p}\sigma_1 g_{j-m+1}^{(1)} + p\bar{\sigma}_2 h_{j-m+1}^{(2)} + p\sigma_2 g_{j-m+1}^{(2)}]; j \geq 0 \quad (4)$$

We define the following generating functions:

$$\pi(z) = \sum_{j=0}^{\infty} \pi_j z^j; \phi(z) = \sum_{j=0}^{\infty} \frac{\pi_j z^j}{(\lambda' + j\gamma)}; H_i(z) = \sum_{j=0}^{\infty} h_j^{(i)} z^j \text{ and } G_i(z) = \sum_{j=0}^{\infty} g_j^i z^j \text{ (i=1,2).}$$

$$H_1(z) = b_1^*(\lambda_1(1-c(z))); H_2(z) = b_1^*(\lambda_1(1-c(z)))b_2^*(\lambda_2(1-c(z)));$$

$$G_1(z) = b_1^*(\lambda_1(1-c(z)))\xi_1^*(\lambda_v(1-c(z))); G_2(z) = b_1^*(\lambda_1(1-c(z)))b_2^*(\lambda_2(1-c(z)))\xi_2^*(\lambda_v(1-c(z)))$$

On the other hand, we also have the relationship

$$\pi(z) = \lambda' \phi(z) + \gamma z \phi'(z) \quad (5)$$

Theorem 1. The marginal generating function for the stationary queue size distribution of the embedded markov chain is obtained as

$$\pi(z) = \frac{(1-\rho)[1-c(z)]R(z)}{[R(z)-z]} \times \exp \left\{ \frac{1}{\gamma} \int_1^z \frac{[\lambda'y - \lambda(c(y)-c_0)R(y)]}{[R(y)-y]} \frac{dy}{y} \right\} \quad (6)$$

where

$$R(z) = [\bar{p}\bar{\sigma}_1 H_1(z) + \bar{p}\sigma_1 G_1(z) + p\bar{\sigma}_2 H_2(z) + p\sigma_2 G_2(z)] = [\bar{p}(\bar{\sigma}_1 + \sigma_1 \xi_1^*(\alpha_v))b_1^*(\alpha_1) + p(\bar{\sigma}_2 + \sigma_2 \xi_2^*(\alpha_v))b_1^*(\alpha_1)b_2^*(\alpha_2)];$$

$$\alpha_i = \lambda_i(1-c(z)); \text{ for } i \in \{1,2\} \text{ and } \alpha_v = \lambda_v(1-c(z))$$

Proof: Multiplying eq. (4) by appropriate powers of z and then sum it over j for j=0,1,2,...; and after doing some algebraic manipulations using eq. (5), we get eq. (6).

III.B Joint distribution of the number of customers in the retrial group and the state of the server

We are now interested in studying the limiting behaviour of the process $Y(t) = \{C(t), N(t), \zeta(t)\}$ as $t \rightarrow \infty$. Denote the stationary probabilities by

$$P_{i,j} = \lim_{t \rightarrow \infty} P\{(C(t), N(t)) = (i, j)\}, (i, j) \in \{0, 1, 2, 3\} \times Z^+.$$

Since the arrival stream is Poisson, it follows from Burke's theorem (see Cooper, 1981) that the stationary probabilities $\{P_{ij}\}$ exist and are positive under the same conditions of the limiting probabilities $\{\pi_j\}_{j=0}^{\infty}$ of the embedded markov chain $\psi_n(2, j)$; i. e. iff $\rho < 1$. As $\{Y(t); t \geq 0\}$ is a markov regenerative process with the embedded markov renewal process $\{N_n\}_{n=1}^{\infty}$, so we may use some classical results established in Cinlar (1975).

Denote $\psi_n(i, j)$ as the expected amount of time spent by the process $Y(t); t \geq 0$ in the state (i,j) during an interval between two successive total completion epochs given that at the beginning of this interval, the number of customers in orbit was n. Denote m_n as the expectation of the interval between two successive total completion epochs given that at the beginning of this interval, the number of customers in orbit was n. Then

$$P_{i,j} = \frac{\sum_{n=0}^{\infty} \pi_n \psi_n(i, j)}{\sum_{n=0}^{\infty} \pi_n m_n}; \quad 0 \leq i \leq 1, j \geq 0 \quad (7)$$

$$\text{Also, } \sum_{n=0}^{\infty} \psi_n \pi_n = [\lambda c^{(1)}]^{-1}.$$

We observe that

$$m_n = \frac{1}{\lambda' + n\gamma} + b_1^{(1)} + pb_2^{(1)} + \bar{p}\sigma_1 \xi_1^{(1)} + p\sigma_2 \xi_2^{(1)} \quad (8)$$

Define probability generating functions as

$$K_l(z) = \sum_{j=0}^{\infty} k_{l,j} z^j; \quad D_l(z) = \sum_{j=0}^{\infty} d_j^{(l)} z^j; \quad P_l(z) = \sum_{j=0}^{\infty} P_{l,j} z^j, \text{ for } l \in \{1,2\} \text{ and } c(z) = \sum_{j=0}^{\infty} c_j z^j$$

Then we note that for $l \in \{1,2\}$

$$K_l(z) = \frac{[1 - b_l^*(\alpha_l)]}{\alpha_l} \text{ and } D_l(z) = \frac{[1 - \xi_l^*(\alpha_v)]}{\alpha_v}$$

where, $\alpha_l = \lambda_l(1 - c(z))$ for $l \in \{1,2\}$ and $\alpha_v = \lambda_v(1 - c(z))$.

Theorem 1: The marginal distribution of the number of customers in the orbit is given by

$$Q(z) = \frac{P_0(z)[\lambda_1 \lambda_2 \lambda_v B_1(z) + \lambda \lambda_2 \lambda_v B_2(z) + \lambda \lambda_1 \lambda_v p B_3(z) + \lambda \lambda_1 \lambda_2 \bar{p} \sigma_1 B_4(z) + \lambda \lambda_1 \lambda_2 p \sigma_2 B_5(z)]}{\lambda_1 \lambda_2 \lambda_v B_1(z)} \quad (9)$$

where

$$B_1(z) = [R(z) - z]; \quad B_2(z) = [1 - b_1^*(\alpha_1)]; \quad B_3(z) = b_1^*(\alpha_1)[1 - b_2^*(\alpha_2)]; \quad B_4(z) = b_1^*(\alpha_1)[1 - \xi_1^*(\alpha_v)]; \\ B_5(z) = b_1^*(\alpha_1)b_2^*(\alpha_2)[1 - \xi_2^*(\alpha_v)]$$

Proof: The marginal generating functions for the idle, busy with FES, busy with SOS and vacation states of the server, respectively are obtained as

$$P_0(z) = \lambda c^{(1)} \phi(z) \quad (10)$$

$$P_1(z) = \frac{\lambda^2 c^{(1)} [1 - H_1(z)] \phi(z)}{\lambda_1 [R(z) - z]} = \frac{\lambda [1 - b_1^*(\alpha_1)] P_0(z)}{\lambda_1 [R(z) - z]} \quad (11)$$

$$P_2(z) = \frac{\lambda^2 p c^{(1)} H_1(z) K_2(z) \phi(z)}{\lambda_2 [R(z) - z]} = \frac{\lambda p b_1^*(\alpha_1) [1 - b_2^*(\alpha_2)] P_0(z)}{\lambda_2 [R(z) - z]} \quad (12)$$

$$P_3(z) = \frac{\lambda^2 c_{[1]} [\bar{p} \sigma_1 H_1(z) M_1(z) + p \sigma_2 H_2(z) M_2(z)] \phi(z)}{\lambda_v [R(z) - z]} \\ = \frac{\lambda P_0(z) [\bar{p} \sigma_1 b_1^*(\alpha_1) [1 - \xi_1^*(\alpha_v)] + p \sigma_2 b_1^*(\alpha_1) b_2^*(\alpha_2) [1 - \xi_2^*(\alpha_v)]]}{\lambda_v [R(z) - z]} \quad (13)$$

$$Q(z) = P_0(z) + P_1(z) + P_2(z) + P_3(z) \quad (14)$$

Theorem 2: The stationary distribution of the total number of customers in the system is given by

$$P(z) = \frac{P_0(z) [A_1(z) b_1^*(\alpha_1) + A_2(z) b_1^*(\alpha_1) b_2^*(\alpha_2) + \lambda_2 \lambda_v (\lambda - \lambda_1) z]}{\lambda_1 \lambda_2 \lambda_v [R(z) - z]} \quad (15)$$

where

$$A_1(z) = [\lambda_1 \lambda_2 \bar{p} (\bar{\sigma}_1 \lambda_v + \sigma_1 \lambda z) + \lambda \lambda_v z (\lambda_1 p - \lambda_2) + \lambda_1 \lambda_2 \bar{p} \sigma_1 (\lambda_v - \lambda z) \xi_1^*(\alpha_v)]$$

$$A_2(z) = [\lambda_1 \lambda_2 \lambda_v p (\bar{\sigma}_2 + \sigma_2 \xi_2^*(\alpha_v)) + \lambda \lambda_1 \lambda_2 p \sigma_2 z (1 - \xi_2^*(\alpha_v)) - \lambda \lambda_1 \lambda_v p z]$$

Proof: The stationary distribution of the total number of customers in the system is

$$P_j = P_{0,j} + (1 - \delta_{j,0}) (P_{1,j-1} + P_{2,j-1} + P_{3,j-1}); \quad j \geq 0$$

Then the corresponding generating function $P(z)$ can be easily obtained by multiplying above eq. with appropriate powers of z and then summing over j for $j=0,1,2,\dots$. Now using $P(z) = P_0(z) + z[P_1(z) + P_2(z) + P_3(z)]$, we obtain eq. (15).

IV. Performance Measures

In this section, we derive some performance indices such as system state probabilities, partial moments, queue size, etc. for the system under steady-state, which are as follows:

❖ The first partial moments of orbit size for $i \in \{0,1,2,3\}$ are given by

$$M_i = \sum_{j=1}^{\infty} j P_{i,j} \text{ for } i \in \{0,1,2,3\}.$$

Thus

$$M_0 = \frac{[\lambda c^{(1)} - \lambda'(1-\rho)]}{\gamma} \quad (16)$$

$$M_1 = \frac{\lambda c^{(1)} b_1^{(1)} M_0}{1-\rho} + \frac{\lambda \lambda_1 (c^{(1)})^2 b_1^{(2)}}{2} + \frac{\lambda c^{(2)} b_1^{(1)}}{2(1-\rho)} + \frac{\lambda (c^{(1)})^3 b_1^{(1)}}{2(1-\rho)} \times [\lambda_1^2 b_1^{(2)} + \lambda_2^2 p b_2^{(2)} + \lambda_v^2 T_1 + 2\{T_2 + T_3 + T_4\}] \quad (17)$$

$$M_2 = \frac{\lambda p c^{(1)} b_2^{(1)} M_0}{1-\rho} + \frac{\lambda p (c^{(1)})^2 \{\lambda_2 b_2^{(2)} + 2\lambda_4 b_1^{(1)} b_2^{(1)}\}}{2} + \frac{p \lambda c^{(2)} b_2^{(1)}}{2(1-\rho)} + \frac{\lambda p (c^{(1)})^3 b_2^{(1)}}{2(1-\rho)} \times [\lambda_1^2 b_1^{(2)} + \lambda_2^2 p b_2^{(2)} + \lambda_v^2 T_1 + 2\{T_2 + T_3 + T_4\}] \quad (18)$$

$$M_3 = \bar{p} \left[\frac{\lambda \sigma_1 c^{(1)} \xi_1^{(1)} M_0}{1-\rho} + \frac{\lambda \sigma_1 (c^{(1)})^2 \{\lambda_2 \xi_1^{(2)} + 2\lambda_4 b_1^{(1)} \xi_1^{(1)}\}}{2} + \frac{\lambda \sigma_1 c^{(2)} \xi_1^{(1)}}{2(1-\rho)} \right] + \frac{\lambda \bar{p} \sigma_1 (c^{(1)})^3 \xi_1^{(1)}}{2(1-\rho)} \times [\lambda_1^2 b_1^{(2)} + \lambda_2^2 p b_2^{(2)} + \lambda_v^2 T_1 + 2\{T_2 + T_3 + T_4\}] + p \left[\frac{\lambda \sigma_2 c^{(1)} \xi_2^{(1)} M_0}{1-\rho} + \frac{\lambda \sigma_2 (c^{(1)})^2 \{\lambda_v \xi_2^{(2)} + 2(\lambda_4 b_1^{(1)} + \lambda_2 b_2^{(1)}) \xi_2^{(1)}\}}{2} + \frac{\lambda \sigma_2 c^{(2)} \xi_2^{(1)}}{2(1-\rho)} \right] + \frac{\lambda p \sigma_2 (c^{(1)})^3 \xi_2^{(1)}}{2(1-\rho)} \times [\lambda_1^2 b_1^{(2)} + \lambda_2^2 p b_2^{(2)} + \lambda_v^2 T_1 + 2\{T_2 + T_3 + T_4\}] \quad (19)$$

where

$$T_1 = \{\bar{p} \sigma_1 \xi_1^{(2)} + p \sigma_2 \xi_2^{(2)}\}, \quad T_2 = \lambda_1 \lambda_2 p b_1^{(1)} b_2^{(1)} \\ T_3 = p \sigma_2 \lambda_v (\lambda_4 b_1^{(1)} + \lambda_2 b_2^{(1)}) \xi_2^{(1)}, \quad T_4 = \lambda_1 \lambda_v \bar{p} \sigma_1 b_1^{(1)} \xi_1^{(1)}.$$

Now, the expected number of customers in the retrial orbit is obtained as

$$L_O = P'(1) = M_0 + M_1 + M_2 + M_3 \quad (20)$$

- ❖ The expected number of customers in the system at a random epoch and at a departure epoch, respectively are given by

$$L = \rho + L_O \quad \text{and} \quad L_D = L + \frac{c^{(2)}}{2c^{(1)}} \quad (21)$$

V. Numerical Results

To demonstrate the tractability of analytical results, we provide some numerical results in this section. Numerical results are displayed in table 2 and fig. 1(a-b) to examine the effects of some system parameters such as arrival rate (λ), service rates (μ_1, μ_2), vacation rates (v_1, v_2), probabilities p and σ_1 on the average system size (L). The vacation times during FES (SOS) follows exponential (gamma) distribution with rate v_1 (v_2) and the batch size is geometric distributed with parameter $q=0.5$. We set default parameters as $\lambda=0.8$, $\lambda_1=0.2\lambda$, $\lambda_2=0.3\lambda$, $\lambda_3=0.5\lambda$, $\mu_1=1.5$, $\mu_2=2$, $v_1=v_2=0.2$, $\gamma=5$, $p=0.4$, $\sigma_1=0.3$, $\sigma_2=0.2$ and $\theta=0.2$ for table 2 and $\lambda=0.6$, $\lambda_1=0.2\lambda$, $\lambda_2=0.3\lambda$, $\lambda_3=0.5\lambda$, $\mu_1=1.5$, $\mu_2=2$, $v_1=v_2=0.2$, $\gamma=5$, $p=0.4$, $\sigma_1=0.3$, $\sigma_2=0.2$ and $\theta=0.2$ for fig. 1(a-b).

Table 2 demonstrates the effect of parameters λ , μ_1 , μ_2 on the expected number of customers in different states of the server for different values of Bernoulli vacation probability σ_1 . It can be easily observed that for

λ	μ_1	v_1	$\sigma_1=1$				$\sigma_1=2$			
			M_0	M_1	M_2	M_3	M_0	M_1	M_2	M_3
.2	1.5	0.2	0.02	0.18	0.05	0.33	0.02	0.19	0.05	0.44
.4	1.5	0.2	0.05	0.54	0.16	1.12	0.06	0.60	0.17	1.55
.6	1.5	0.2	0.09	1.24	0.36	2.54	0.09	1.51	0.44	3.71
0.8	1	0.2	0.13	4.54	0.86	5.26	0.13	5.43	1.21	8.68
0.8	1.2	0.2	0.13	3.49	0.81	5.07	0.13	4.27	1.12	8.23
0.8	1.4	0.2	0.13	2.83	0.77	4.95	0.13	3.51	1.06	7.95
0.8	1.5	.2	0.13	2.58	0.76	4.91	0.13	3.51	1.04	7.84
0.8	1.5	.4	0.12	2.20	0.64	3.70	0.12	2.46	0.72	4.55
0.8	1.5	.6	0.12	2.11	0.62	3.42	0.12	2.26	0.66	3.89

Table 2: Effect of λ , μ_1 and v_1 on the expected number of customers in different states of the server

different values of σ_1 , the moments M_0 , M_1 , M_2 and M_3 increase on increasing both arrival rate (λ) whereas it decrease sharply with the increase in the service rate (μ_1).

Fig. 1(a-b) demonstrates the effect of various critical system parameters on average system size (L) for $M^x/\gamma/1$ and $M^x/E_2/1$ models shown by smooth and dotted curves, respectively. Fig. 1(a-b) illustrates the effect of vacation rates (v_1 , v_2) on the average system size (L) for different values of p . From these figs, it is quite visible that L decreases first sharply then gradually on increasing both v_1 and v_2 . This situation can be easily seen in many congestion problems where the server can go for single or multiple vacations according to Bernoulli schedule.

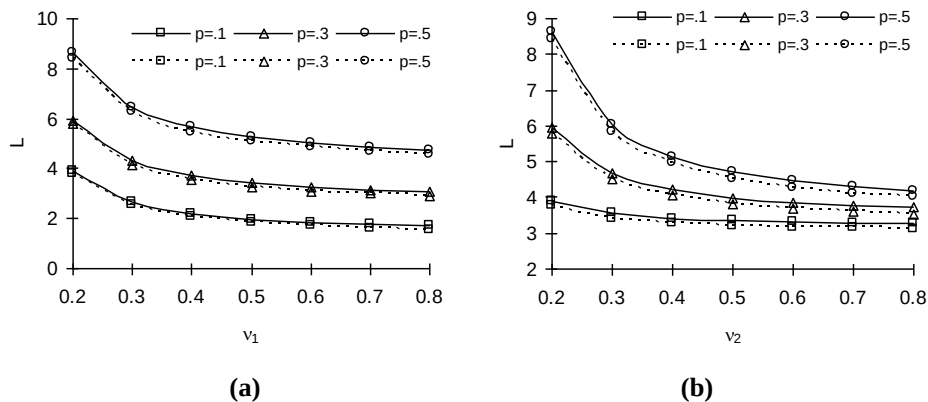


Fig. 1: Effect of (a) v_1 and (b) v_2 on L for different values of p .

VI. Conclusion

In this paper, a state dependent single server batch arrival retrial queueing system with FES and SOS under Bernoulli schedule vacation and Bernoulli admission mechanism has been investigated. The motivation for this model comes from wide range applications in many real time systems for example in computer and communication network where messages are processed in two stages by a single server and Bernoulli admission control policy may be followed. The Bernoulli admission mechanism can also be viewed as a device to model congestion situations where a proportion of the arriving packets are corrupted and consequently, they must be deleted. Other application areas wherein our model can be useful, are Local Area Networks (LAN), CSMA/CD protocol, client-server communication, telephone systems and electronic mail services on Internet, and many more.

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