



Using Delta Domain technique online identification based on estimation & Different control Schemes

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Abstract: Basically we have described in this paper, about the problem of adaptive control of stable Linear Time Invariant (LTI) systems in delta operator representation is considered. Time moment method, a tool so far used for model order reduction is applied to compute online time moments from the input-output response of the plant and reference model to estimate the controller parameters adaptively. The delta operator representation is considered to derive the benefit of unification of design method to its continuous-time counterpart at higher sampling frequency. An example is included to highlight the efficacy of the design methods proposed in this work.

Keywords: Linear Time Invariant, Delta operator, Estimation of time, Adaptive control etc.

I. INTRODUCTION

An adaptive control system is one which measures the dynamic characteristics of the plant, compares them with the desired characteristics, uses the difference to vary the adjustable parameters in order to get optimal performance regardless of the environmental changes and maintain it throughout the process. As defined [1] an adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. Simply speaking, an adaptive control system consists of two closed loops. One loop is a normal feedback control with the plant and the controller and the other loop is the parameter adjustment loop. Control of fully known deterministic, linear time invariant dynamic systems have received wide attention for many decades and a lot of study and surveys have been performed. Among the different types of adaptive schemes traditionally [2] four such schemes namely self-oscillating, gain scheduling, auto tuning, model reference adaptive control (MRAC) are in wide use. Here we have used Model Reference Adaptive Control (MRAC) [3] framework to attain performance characteristics. We have gone through the existing control schemes reported in [4, 5] in delta operator formulation. Using these schemes simulation results obtained are presented.

Section II gives a brief introduction to delta operator representation. In Section III the proposed scheme of time moment estimation using delta operator is presented while in Section IV the different control schemes [6] recast using delta operator formulation are discussed. In Section V an example has been taken up and simulation results are shown. Section VI gives the conclusion from the results obtained and discuss on its further scope of work.

II. DELTA OPERATOR

The δ -operator [7, 8, 9] is defined in the time domain as

$$\delta = \frac{q - 1}{\Delta} \quad (1)$$

where Δ is the sampling period and q is the forward shift operator. Operating δ on a differential signal $x(t)$ gives.

$$\delta x(t) = \frac{x(t + \Delta) - x(t)}{\Delta} \quad (2)$$

It is straightforward to see that

$$\lim_{\Delta \rightarrow 0} \delta x(t) = \frac{d}{dt} x(t) \quad (3)$$

which demonstrates the close relationship between the discrete-time δ -operator and the continuous-time differential operator $\frac{d}{dt}$ at high sampling rates. Similar relation exists in the complex domain as well. The delta transform operator γ is defined as

$$\gamma = \frac{z - 1}{\Delta} \quad (4)$$

Where z is the complex domain transform operator for discrete-time system, like the Laplace transform operator for continuous-time system.

III. ESTIMATION OF TIME MOMENTS IN DELTA DOMAIN

As a first step towards adaptive control of an unknown plant, Sivaramakumar [10] has proposed a time moment estimation scheme. No more than the plant input and output is available for this purpose. Following his steps, we have derived the corresponding time-moments in delta domain.

Consider a plant in delta domain as $G(\gamma)$. Let its impulse response be $g(k\Delta)$. Let y be the output of G excited by u . Then

$$Y(\gamma) = G(\gamma)U(\gamma) \quad (5)$$

Since G is asymptotically stable, it permits a series expansion in terms of its time moments $\{k_i\}$ as

$$G(\gamma) = k_0 + k_1 \gamma + k_2 \gamma^2 + \dots = \sum_{k=0}^{\infty} k_i \gamma^i \quad (6)$$

The problem is to obtain estimates of $\{k_i\}$ from on-line measurements, up to the current time of u and y . Consequently, $U(\gamma)$ is defined as follows:

$$\begin{aligned} U(\gamma) &= \Delta \sum_{k=0}^{\infty} U(k\Delta)(1 + \Delta\gamma)^{-k} \\ &= \Delta[U(0) + U(\Delta) + U(2\Delta) + \dots] - \Delta^2 \gamma[U(\Delta) + 2U(2\Delta) + \dots] + \Delta^3 \gamma^2[U(\Delta) + 3U(2\Delta) + \dots] + \dots \\ (7) \\ &= \Delta \sum_{k=0}^{\infty} U(k\Delta) - \Delta \gamma \sum_{k=0}^{\infty} (k\Delta)U(k\Delta) + \Delta \gamma^2 \sum_{k=0}^{\infty} k(k+1)\Delta^2 U(k\Delta) + \dots \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Similarly, } Y(\gamma) &= \Delta \sum_{k=0}^{\infty} Y(k\Delta)(1 + \Delta\gamma)^{-k} \\ &= \Delta[Y(0) + Y(\Delta) + Y(2\Delta) + \dots] - \Delta^2 \gamma[Y(\Delta) + 2Y(2\Delta) + \dots] + \Delta^3 \gamma^2[Y(\Delta) + 3Y(2\Delta) + \dots] + \dots \\ (9) \end{aligned}$$

$$= \Delta \sum_{k=0}^{\infty} Y(k\Delta) - \Delta \gamma \sum_{k=0}^{\infty} (k\Delta)Y(k\Delta) + \Delta \gamma^2 \sum_{k=0}^{\infty} k(k+1)\Delta^2 Y(k\Delta) + \dots \quad (10)$$

Equating $Y(\gamma) = G(\gamma)U(\gamma)$

We get,

$$k_0 = \frac{\sum_{k=0}^{\infty} Y(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)} \quad (11)$$

$$k_1 = \frac{-\Delta \left[\sum_{k=0}^{\infty} kY(k\Delta) - k_0 \sum_{k=0}^{\infty} kU(k\Delta) \right]}{\sum_{k=0}^{\infty} U(k\Delta)} \quad (12)$$

$$k_2 = \frac{\Delta^2 \sum_{k=0}^{\infty} k(k+1)Y(k\Delta) - k_0 \Delta^2 \sum_{k=0}^{\infty} k(k+1)U(k\Delta) + k_1 \Delta \sum_{k=0}^{\infty} kU(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)} \quad (13)$$

IV. CONTROL SCHEMES

(a). Plant Command Modifier Scheme in Delta Domain

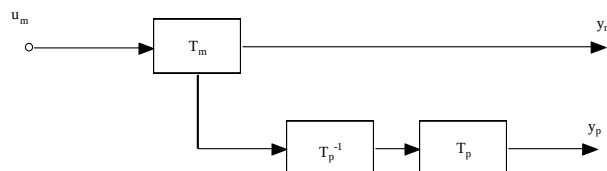


Figure 1. Basis for PCMS

The plant command modifier scheme as proposed in [6] has been modified in this section in the delta domain with the goal to study the control scheme and the application of the online estimation scheme.

The basis on which this PCMS is built is shown in the figure1. Suppose T_p^{-1} is available. Then u_p could have been obtained as

$$u_p = (T_p^{-1} T_m) u_m \quad (14)$$

to get $y_p = y_m$. Obviously, T_p^{-1} will be unstable in the event of T_p having non-minimum phase zero(s). To overcome this problem, a method involving time moments has been proposed.

Let $\{k_{i,p}\}$ denote the time moments of the unknown plant T_p . Then

$$T_p(\gamma) = \sum_{i=0}^{\infty} k_{i,p} \gamma^i \quad (15)$$

Regardless of the number of its zero in excess over its poles, T_p^{-1} permits an expansion in terms of its time moments $\{q_i\}$ as

$$T_p^{-1}(\gamma) = \sum_{i=0}^{\infty} q_{i,p} \gamma^i \quad (16)$$

$$F = \begin{bmatrix} f_0 & 0 & 0 & \dots & 0 \\ f_1 & f_0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{\eta} & f_{\eta-1} & \dots & \dots & f_0 \end{bmatrix}$$

$$\text{Now, } T_p T_p^{-1} = 1 \text{ leads to } \sum_{i=0}^{\infty} k_{i,p} \gamma^i \sum_{i=0}^{\infty} q_{i,p} \gamma^i = 1 \quad (17)$$

Collecting the coefficients of like powers of γ yields

$$k_{0,p} q_0 + (k_{1,p} q_0 + k_{0,p} q_1) \gamma + (k_{2,p} q_0 + k_{1,p} q_1 + k_{0,p} q_2) \gamma^2 + \dots = 1,$$

and hence

$$\begin{aligned} k_{0,p} q_0 &= 1, \\ k_{1,p} q_0 + k_{0,p} q_1 &= 0, \\ k_{2,p} q_0 + k_{1,p} q_1 + k_{0,p} q_2 &= 0, \end{aligned}$$

and so on. This is same as

$$\begin{aligned} k_{0,p} q_0 &= 1, \\ \sum_{j=0}^i k_{i-j,p} q_j &= 0; \quad i = 1, 2, \dots \end{aligned} \quad (18)$$

(b). Padé-adapted Plant Command Modifier Scheme in Delta Domain

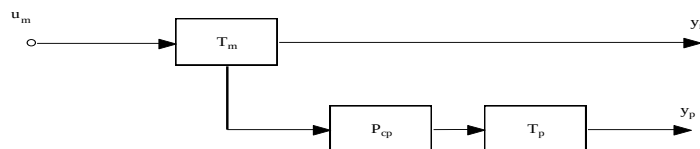


Figure 2. Basis for PPCMS

Here $P_c(\gamma) = f_0 + f_1 \gamma + \dots + f_{\eta} \gamma^{\eta}$

With

$$f_0 = \frac{1}{\hat{k}_{0,p}}, \quad f_i = - \frac{\sum_{j=0}^{i-1} \hat{k}_{i-j,p} f_j}{\hat{k}_{0,p}}; \quad i = 1, 2, \dots, \eta \quad (19)$$

We now proceed to analyze the implications of this representation with $\mu \geq \nu$

$$\frac{b_0 + b_1 \gamma + \dots + b_{\nu} \gamma^{\nu}}{a_0 + a_1 \gamma + \dots + a_{\mu} \gamma^{\mu}}$$

$$P_c(s) = f_0 + f_1 \gamma + \dots + f_{\eta} \gamma^{\eta} = \text{through } \gamma^{\mu+\nu}$$

Without loss of generality, $A(\gamma)$ can be chosen monic, i.e. $a_{\mu}=1$ set $\mu=\nu=\eta$. Cross multiplying and equating the coefficients of like powers of γ leads to

$$Fa = b, \quad (20)$$

Where

$$a = [a_0 \ a_1 \ \dots \ a_{\eta-1}]^T \quad \text{and} \quad b = [b_0 \ b_1 \ \dots \ b_{\eta-1}]^T$$

The identities obtained by equating the coefficients of γ^i , $i = \eta+1, \eta+2, \dots, 2\eta$ have been neglected. The consequence is that with the compensator taking the form

$$P_{cp} = B/A \quad (21)$$

(c). Plant Time Moment Controller Scheme in Delta Domain

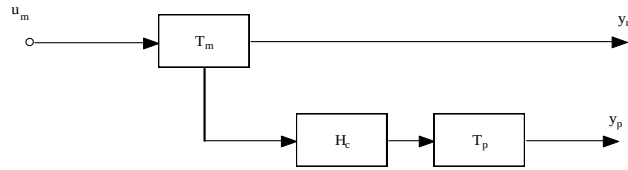


Figure 3. Basis for PTCS

Here

$$H_c(\gamma) = \sum_{i=0}^{\infty} h_i \gamma^i \quad (22)$$

$$\text{and } T_m(\gamma) = \sum_{i=0}^{\infty} k_{i,m} \gamma^i \quad (23)$$

Along the same lines as in PCMS, set

$$\sum_{i=0}^{\infty} k_{i,m} \gamma^i = \sum_{j=0}^{\infty} k_{j,p} \gamma^j \sum_{l=0}^{\infty} h_l \gamma^l \quad (24)$$

Truncating this identity up to $i = \eta$ on the l.h.s., replacing $\{k_{j,p}\}$ by their estimates $\{\hat{k}_{j,p}\}$ and following the steps similar to those in PCMS finally results in

$$\sum_{j=0}^i k_{i-j,p} h_j = k_{i,m}; \quad i = 1, 2, \dots, \eta \quad (25)$$

$$\hat{K}_p h = k_m \quad (26)$$

Where

$$\hat{K}_p = \begin{bmatrix} \hat{k}_{0,p} & 0 & 0 & \dots & 0 \\ \hat{k}_{1,p} & \hat{k}_{0,p} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{k}_{\eta,p} & \hat{k}_{\eta-1,p} & & & \hat{k}_{0,p} \end{bmatrix}$$

$$h = [h_0 \ h_1 \ \dots \ h_{\eta-1} \ 1]^T \quad (27)$$

$$\text{and } k_m = [k_{0,m} \ k_{1,m} \ \dots \ k_{\eta-1,m} \ k_{\eta,m}]^T \quad (28)$$

As seen earlier $\hat{k}_{0,p}$ and, hence, $k_{0,p}$ are finite and nonzero. The matrix K_p is non singular. We can solve for a unique h , which is necessary and sufficient to match the first $(\eta+1)$ time moments. The next step is to realize H_c by following the design procedure used in Section IV (b).

$$H_c(\gamma) = h_0 + h_1 \gamma + \dots + \gamma^\eta = \frac{B(s)}{A(s)} = \frac{b_0 + b_1 \gamma + \dots + b_\nu \gamma^\nu}{a_0 + a_1 \gamma + \dots + a_\mu \gamma^\mu}$$

through $\gamma^{\mu+\nu}$

Set $a_\mu = 1$. Cross multiplying and equating the coefficients of like powers of γ yields.

$$Ha = b \quad (30)$$

Where

$$H = \begin{bmatrix} h_0 & 0 & 0 & \dots & 0 \\ h_1 & h_0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_\eta & h_{\eta-1} & \dots & h_0 & \end{bmatrix}$$

$a = [a_0, a_1, \dots, a_{\eta-1}]^T$ and $b = [b_0, b_1, \dots, b_{\eta-1}, b_{\eta}]^T \dots$
Specify A as done in Section V (b). Then compute b.

(d). Plant Time Moment Controller with Feedback Scheme in Delta Domain

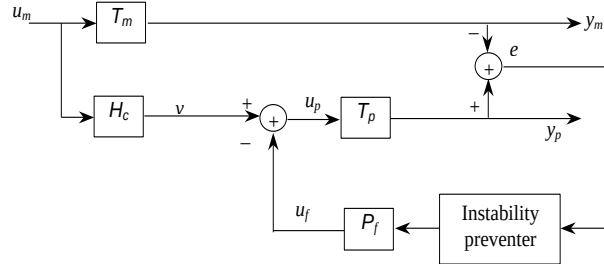


Figure 4. Implementation of PTCFS

In this PTCF scheme, v of Fig.4 is generated as

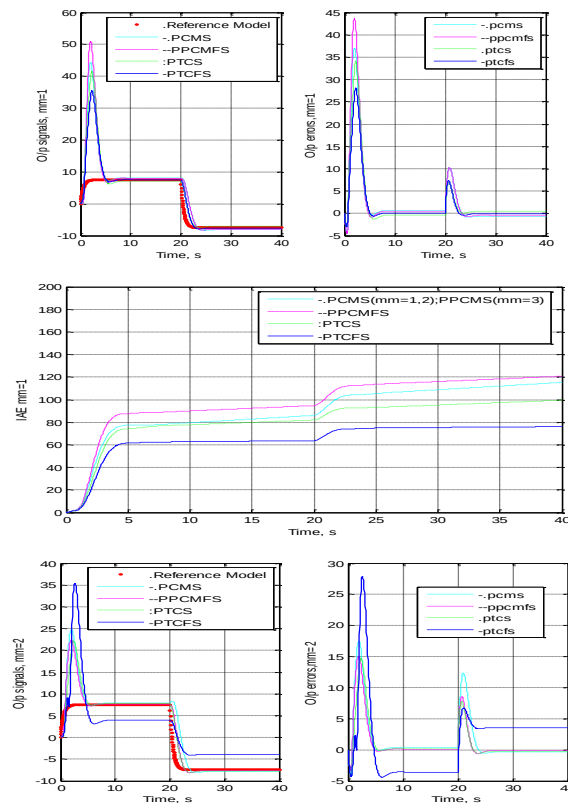
$$v = H_c u_m \quad (31)$$

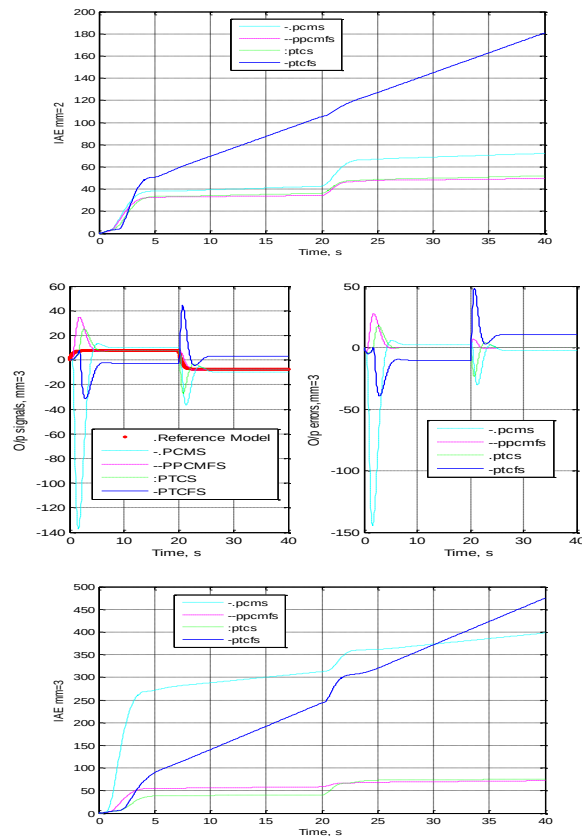
V. SIMULATIONS

1. Reference model expressed in s-domain

$$T_m(s) = \frac{15}{s + 2}$$

2. Sampling period $\Delta = 0.05$ sec
3. Period of revision of estimates and adaptation of controller $T_1 = 0.5$ sec
4. In the simulations, mm is the number of moments matched i.e. $\eta = mm - 1$
5. The reference model is excited by a unit step.
6. Only minimum phase results are shown.





VI. CONCLUSIONS

The outcome of our study with minimum phase plants shows that among the various control schemes proposed, Plant Time Moment Controller with Feedback Scheme in Delta Domain appears to be the best because it has the least IAE, which is evident from the results obtained from the figures. Moreover in this scheme, there is a steady improvement with every additional time moment matched, which is evident from the figures. As a future scope of work we can extend this discussion for multi-variable systems as well.

REFERENCES

- [1] K.J. Astrom & B. Wittenmark, "Adaptive Control" Addison-Wesley, 1995.
- [2] K.J. Astrom, "Adaptive Feedback Control" *proc. of IEEE*, pp. 185-217, 1987.
- [3] I.D. Landau, "A Survey of Model reference Adaptive Techniques (Theory & Applications)", *IEEE* pp. 353-379, 1974.
- [4] H.N. Shankar Model Reference Adaptive Control of Non-minimum Phase Plants. MSc (Engg.) thesis, Department of Electrical Engg., IISC, Bangalore, India, August 1993.
- [5] K. Rajgopal and H.N. Shankar, Adaptive Cascade and Feedback Compensators for Stable Minimum and Non-minimum Phase Plants Through Refined Time Moments Estimators, International Conference on Communications, Control & Signal Processing (CCSP-2000), IEEE Bangalore Section, Bangalore, July 2000.
- [6] H.N. Shankar Adaptive Control of a General Class of Finite Dimensional Stable LTI Systems Phd(Engg.) thesis, Department of Electrical Engg., IISC, Bangalore, India, March 2000.
- [7] Dr. Prasanta Sarkar Reduced Order Modeling and Controller Design in Delta Domain Phd(Engg) Thesis, Dept of Electrical Engg, IIT Kharagpur, August, 2001.
- [8] Richard H. Middleton, Graham C. Goodwin, David J. Hill and David Q. Mayne, Design issues in Adaptive Control, *IEEE Transactions on Automatic Control*, Vol. 33, No. 1, January, 1988, pp. 50-58.
- [9] Darijusz Janecki, Model Reference Adaptive Control Using Delta Operator, *IEEE Transactions on Automatic Control*, VOL. 33, NO. 8, August 1988, pp. 771-775.
- [10] G. R. Sivaramakumar. Model Matching and Adaptive Control of Finite and Infinite Dimensional Systems, PhD thesis, Department of Electrical Engg., IISC, Bangalore, India, August 1991.