



A Hybrid Ant Colony System for Vehicle Routing Problem with Time Windows

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Abstract: Vehicle Routing Problem with Time Windows (VRPTW) is a well known NP hard combinatorial scheduling optimization problem in which minimum number of routes have to be determined to serve all the customers within their specified time windows. Different analytic and heuristic approaches have been tried to solve such problems. The proposed hybrid algorithm uses ant colony system as well as random search incorporating simulated annealing concepts to solve VRPTW problem. From simulation results, it is shown that the proposed algorithm has satisfactory performance.

Keywords: Vehicle Routing Problem with Time Windows, Ant Colony System, Random Search, Simulated annealing.

I. Introduction

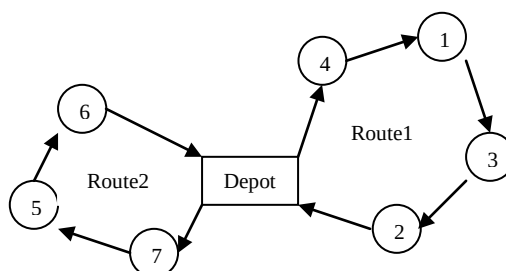
Vehicle routing and Scheduling has been of major interest to the scientific community for the last 50 years due to the inherent complexity of the associated problems and its wide applicability in transportation logistic. A large number of different approaches, in recent years mostly meta-heuristics have been proposed for different variants of vehicle routing and scheduling problems. VRPTW is an extension of the Vehicle Routing Problems (VRP) arising in transportation logistics that usually involve scheduling in constrained environment.

The Vehicle Routing Problem (VRP) is a well known NP-hard combinatorial optimization problem. VRP may be described as follows: Given a set of vehicles with fixed and identical capacity located at common depot and a set of geographically scattered locations (cities, stores, schools, customers, warehouses etc.) which have varying demands. It is desired to determine minimum cost routes of the vehicles in such a way that each location is visited once and only once by one of the vehicles under the restriction that all routes start and ultimately end at the depot. The routes are designed in such a way that the total demands at locations enroute do not exceed the carrying capacity of the vehicle.

VRPTW is an extension of the VRP with the additional restriction that at each location there is a time window. The vehicle can visit the location in this specified time window only. The windows are of two types: Soft time windows and hard time windows. A soft time window can be violated at some cost whereas a hard time window has to be strictly adhered to. It does not accept a vehicle after the latest time specified in the time window. However, if a vehicle arrives at such a destination prior to earliest specified time of the window, it will have to wait till start of the time window. However this waiting penalizes the management either in the direct waiting cost or the increased number of vehicles to be used as because of such a waiting it will be able to service fewer destinations later even when capacity is available in the vehicle.

The objective of VRPTW is to service all the customers as per their requirement while minimizing the number of vehicles required as well as the total travel distance by all the vehicles used without violating capacity constraints of the vehicles and the customers' time window requirements such that each customer is visited only once by one of the vehicles. All the routes are to start and ultimately end at the originating depot.

Figure 1 Typical output for VRPTW



VRPTW arise in a variety of real life situations. VRPTW arises in milk float, mail delivery, school bus routing, solid waste collection, heating oil distribution, newspaper distribution, transportation of persons, parcel pick-up and delivery, dial-a-ride systems, airline/railway fleet routing and several such other situations.

The early work on VRPTW can be broadly divided into two categories: exact optimization and heuristic algorithms. Using exact optimization techniques, [2], [3] obtained significant improvements in Solomon's benchmark problem. Survey of the VRPTW literature by heuristics has been given by [1], [6]. Tabu search is used by [5], [23] to solve these problems whereas [4] considered ant colony optimization approach. Large neighborhood search (LNS) is applied by [8]. This was extended by [7] as Adaptive-LNS approach to solve VRPTW problems and genetic algorithm is used by [9], [10] to solve these problems. A complete survey of the VRPTW literature, which includes methods of both the categories, is given by [11].

Ant Colony Optimization (ACO) was introduced by [16] as a novel nature-inspired metaheuristic for the solution of hard combinatorial optimization problems. The behavior that provided the inspiration for ACO is the ants' foraging behavior, and in particular, how ants can find shortest paths between food sources and their nest. When searching for food, ants initially explore the area surrounding their nest in a random manner. While moving, ants leave a chemical pheromone trail on the ground. Ants can smell pheromone. When choosing their way, they tend to choose, in probability, paths marked by strong pheromone concentrations. As soon as an ant finds a food source, it evaluates the quantity and the quality of the food and carries some of it back to the nest. During the return trip, the quantity of pheromone that an ant leaves on the ground may depend on the quantity and quality of the food. The pheromone trails will guide other ants to the food source. This characteristic of real ant colonies is exploited in artificial ant colonies in order to solve combinatorial optimization problems.

Ant algorithms have been applied to many combinatorial problems successfully, including traveling salesman problem [17], quadratic assignment problem [12], job-shop scheduling [15], sequential ordering problem [18] and graph coloring problem [14]. If taking the central depot as the nest and customers as the food, the VRP is very similar to food seeking behaviors of ant colonies in nature.

Bullnheimer et al. [13] were the first to use ant system to solve the VRP. They presented a hybrid Ant System algorithm (HAS) that added the 2-opt heuristic and then based on Saving Algorithm to construct routes of ants. Gambardella [4] proposed a multiple Ant Colony System to vehicle routing problem with time windows (MACS-VRPTW) and improved some of the best-known solutions in the literature. In MACS-VRPTW, one colony minimizes the number of vehicles while the other colony minimizes the traveled distances.

The proposed algorithm for solving vehicle routing problem with time windows works in two phases. In the first phase it uses ant colony system to generate an optimal solution. In the next phase it perturbs iteratively the optimal solution obtained in phase I in a random search manner incorporating annealing concept to see if a still better solution is possible or not. Section II describes the ant colony system algorithm adopted by us in detail followed by random search method incorporating simulated annealing in section III. Application of the technique on selected 20 problems taken from [19] benchmark set of problems is considered in section IV. Conclusions based on present study are finally drawn in section V.

II. Ant Colony System

The proposed Ant Colony System is based on [17]. In every generation each of m ants constructs a tour covering all the nodes. Each ant is randomly assigned to a starting node and has to build a solution, that is, a complete tour. A tour is built node by node: each ant iteratively adds new nodes until all nodes have been visited. For ant k positioned at node v_i , it chooses the next node v_j probabilistically in the set of feasible nodes

N_i^k (i.e., the set of nodes that still have to be visited). The probabilistic rule used to construct a tour is given by Equation 1.

$$P_j^k(v_i) = \begin{cases} \arg \max_{j \in N_i^k} [\tau_{ij}^\alpha \eta_{ij}^\beta] & \text{if } q \leq q_0 \\ \text{Equation 2,} & q > q_0 \end{cases} \quad (1)$$

$$P_j^k(v_i) = \begin{cases} \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{m \in N_i^k} \tau_{im}^\alpha \eta_{im}^\beta}, & \text{if } j \in N_i^k \\ 0, & \text{if } j \notin N_i^k \end{cases} \quad (2)$$

where τ_{ij} is pheromone level on edge (i, j) , The heuristics value is $\eta_{ij} = 1/d_{ij}$ where d_{ij} is the distance between node i and node j , α and β are constants that determine the relative influence on τ and η . Moreover, q is a random number uniformly distributed in $[0, 1]$, and q_0 is a pre-defined parameter ($0 \leq q_0 \leq 1$). This state transition rule favors transitions toward nodes connected by short edges and with a large amount of pheromone.

In ant system, the global updating rule is implemented as follows. Once all ants have built their tours, pheromone is updated on all edges according to

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (3)$$

$$\Delta \tau_{ij}^k = \begin{cases} 1/L_k & \text{if } (i, j) \in T_k \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where

$0 < \rho < 1$ is pheromone evaporation parameter, $\Delta \tau_{ij}^k$ is the amount of pheromones deposited by ant k on edge (i, j) , T_k is the best tour found by ant k , L_k is the length of the tour T_k and m is the number of ants.

While building a solution, ants visit edges and change their pheromone level by applying the local updating rule. In this strategy all ants deposit pheromones, but the pheromone levels on the edges from the best tour found so far are enforced by elitist ants.

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k + \sigma \Delta \tau_{ij}^* \quad (5)$$

Besides the standard pheromone, σ ants deposit pheromones on the edges that belong to T^* the best solution found so far. The length of T^* is denoted with L^* and $\Delta \tau_{ij}^*$ is $1/L^*$, if $(i, j) \in T^*$.

III. Random Search Method

The application of random search concept incorporating simulated annealing in phase II of the algorithm to solve the VRPTW is as follows: Initially the best solution, say S of the phase I of the ant colony system algorithm is taken as the starting current solution. From this a new solution S' is obtained by randomly moving a customer of the current solution from one route to another route where it is feasible. The new solution if it decreases the objective function value (or leaves it unchanged) is accepted to replace the current solution; else it is accepted to replace the current solution in annealing type manner. More precisely, the new solution S' is accepted as the new current solution if $\Delta E \leq 0$, (where $\Delta E = \text{Fitness}(S') - \text{Fitness}(S)$) else to ensure the search to escape a local optimum, solutions that increases the objective function value are accepted if

$$\exp(-\Delta E/T_k) > \theta \quad (6)$$

where θ is a randomly selected number between $[0, 1]$, if $\Delta E > 0$, where T_k is a parameter called the "temperature". (The value of T_k is gradually decreased from a relatively large value to a small value close to zero. These values are controlled by a cooling schedule which specifies the initial and temperature values at each stage of the algorithm. Eqn. 6 implies that in a minimization problem large increases in objective function, so called uphill moves, are more likely to be accepted when T_k is high. As T_k approaches zero most uphill moves are rejected.

In order to achieve good optimization results the simulated annealing metaheuristic has to be adjusted specific to the problem. With respect to the VRPTW following parameters have been taken into account:

- Initial temperature of annealing T_0 :** If T_0 is too high then almost all new solutions are accepted and the search produces a series of random solutions. When T_0 is too low, very few movements are allowed which reduce the scope of the search. After a series of experiments we decided to set constant initial temperature $T_0 = 10$.
- Cooling schedule:** One of the most popular temperature reduction functions is by [20]. It is based on geometric reduction $T_{k+1} = T_k \cdot \gamma$. In the present work we have set parameter γ as 0.96.
- Number of annealing steps executed in each temperature:** It is usually related to the size of a solution neighbourhood. In our computational experiments the numbers of annealing steps are set as n^2 , where n is the number of customers in the problem considered.
- Termination condition:** We set the termination criteria as $T_0 = 0$.

Contrary to the classical approach in which a solution to the problem is taken as the last solution obtained in the annealing process, we memorize the best solution found during the whole annealing process and record it as the best solution found. The algorithm has been coded in C++ and run on an Intel(R) Core(TM) 2 Duo 2.0 GHz for solving chosen test problems.

IV. Experimental Results

In this section we summarize the results of the computational experiments performed by us using the proposed algorithms on a set of benchmark test problems selected from Solomon's set of problems.

Solomon [19] generated a set of 56 problems which have been frequently used in the literature to assess and compare the performance of various algorithms developed for solving VRPTW problems. The problems vary in available fleet size, vehicle capacity, traveling time of vehicles, spatial and temporal distribution of customers to

be served. The classes R1 and R2 have customers randomly distributed, while the classes C1 and C2 have customers clustered. The classes RC1 and RC2 contain a subset of customers randomly disposed and the other part clustered. Problem sets R1, C1 and RC1 have a short scheduling horizon and allow only a few customers per route (approximately 5 to 10). In contrast, the sets R2, C2 and RC2 have a long scheduling horizon permitting many customers (more than 30) to be serviced by the same vehicle. Larger problems have one hundred customers to be served. Experiments have been done with the following parameter settings: $m=10$ ants, $q_0=0.9$, $\alpha=1$, $\beta=1$ and $\rho=0.1$, $\sigma=2$ on 20 problems from [19] set of problems. Each problem was solved ten times using developed algorithm.

We summarize the computational results that include the best known solutions, literatures, solutions obtained by proposed algorithm and the deviations of traveled distance from the best know solutions (RPD-relative percentage deviation) in Table 1. Bold numbers in tables indicate that the obtained solution are same as the best-known or has yielded an improvement on the currently best known solution in the literature. The proposed algorithm has yielded either improved results or identical solutions as compared to best-known solutions in ten problems out of selected twenty problems. In the present case the improvement is in terms of less total distance traveled than the best-known solution. However, it needs one more vehicle than the number of vehicles used in the best-known. Moreover average RPD of all instances is 0.99 %

Table 1: Results generated by Proposed Algorithm for Solomon's 100 customers set Problems

Problem	Proposed Algorithm (No. of Vehicles/ distance traveled)	Best Known [Ref.] (No. of Vehicles/ distance traveled)	Relative Percentage Deviation
C101	10 / 828.94	10 / 828.94 [22]	0.00
C102	10 / 828.94	10 / 828.94 [22]	0.00
C104	10 / 828.20	10 / 824.78 [22]	0.41
C108	10 / 828.94	10 / 828.94 [22]	0.00
R101	19 / 1670.6	19 / 1650.8 [22]	1.19
R102	18 / 1523.10	17 / 1486.12 [22]	2.49
R105	15 / 1370.02	14 / 1377.11 [8]	-0.51
RC101	15 / 1630.09	14 / 1696.94 [23]	-3.93
RC104	10 / 1145.94	10 / 1135.83 [22]	0.89
RC105	14 / 1540.18	13 / 1633.72 [21]	-5.72
C201	3 / 591.6	3 / 591.6 [9]	0.00
C203	3 / 591.17	3 / 591.17[22]	0.00
C207	3 / 590.6	3 / 590.6 [22]	0.00
C208	3 / 588.32	3 / 588.32 [22]	0.00
R201	4 / 1259.76	4 / 1252.37 [21]	0.59
R203	3 / 943.49	3 / 942.64 [21]	0.09
R204	3 / 803.92	2 / 849.62 [5]	-5.38
RC203	3 / 1131.78	3 / 1060.45 [21]	6.72
RC204	4 / 806.44	3 / 798.46 [4]	0.99
RC205	4 / 1352.39	4 / 1302.42 [21]	3.84

V. Conclusion

In the present study a hybrid approach based on ant colony system and random search incorporating simulated annealing concept has been presented to solve vehicle routing problem with time windows. Performance of proposed algorithms is comparable to those available in literature and in some cases even better. As for future work, it may be interesting to test proposed algorithm on some application of VRPTW.

VI. References

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