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Theoretical Analysis of Two Dimensional Optical Lattice Trapping Potential

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Abstract: This paper will focus on theoretical treatment of two-dimensional optical lattice potential 2DOLP which are usually used in experimental that lead to produced Bose-Einstein condensation BEC in ultra cold gases. This analysis will focus mainly on the k and q factors and the role which these factors play in term of values and shapes of the trapping potential. This specific study will give us the overall view of the region of confinement that the 2DOLP have employed

Keywords: Laser cooled atom, BEC atom, Trapping, condensation, Atom Laser

I. Introduction

Artificial crystals of light, consisting of hundreds of thousands of optical micro-traps, are normally created by interfering optical laser beams. These so-called optical lattices precede as resourceful potential landscapes to trap ultra-cold quantum gases of bosons and fermions. They form influential model systems of quantum many-body systems in periodic potentials for probing nonlinear wave dynamics and strongly connected quantum phases, building fundamental quantum gates or observing Fermi surfaces in periodic potentials [1]. Optical lattices represent a fast-paced modern and interdisciplinary field of research. An optical lattice is simply a set of standing wave lasers. The electric field of these lasers can interact with atoms - the atoms observe a potential and therefore gather in the potential minima. In the case of a typical one-dimensional setup, the wavelength of the opposing lasers is chosen so that the light shift is negative. This means that the potential minima occur at the intensity maxima of the standing wave. Furthermore, the natural beam width can constrain the system to being one-dimensional.

To keep the atoms from distributing over too large a distance, the lattice is superimposed with an additional trap. This trap is generated by a dipole laser beam focused at the position of the atom cloud, perpendicular to the beam axis; this creates a Gaussian intensity profile. For small excursions from the trap centre this is a near harmonic trap. Along the beam axis, the trapping frequency is too low, though: atoms could spread out many 100 μm . To close the trap in this direction, a second (and later a third) perpendicular laser beam is focused onto the atom cloud.

If one of these laser beams is now collimated after passing through the atom cloud and retro-reflected on a mirror, the intensity and thus the trap-depth at the trap centre is doubled; but now a standing wave forms, with its first node at the surface of the retro-reflecting mirror. The interference pattern extends back to the atom cloud, producing an intensity modulation with a distance of half the laser wavelength between intensity maxima. A 2D or 3D lattice is formed by also retro-reflecting the other laser beams. The standing waves intersect and lattice sites are where all standing waves have an intensity maximum. Consider the oblate traps of one standing wave as parallel planes. Then two perpendicular groups of planes intersecting with each other form an array of cigar-shaped traps in a regular 2D lattice. A third group of parallel planes divide these 2D lattice sites into spherically symmetric traps arranged in a 3D optical lattice [2-10]. In some literatures, many authors investigated the effect of gravitation [11] by adding the gravitational potential as an external interaction. In this paper, we analyze in two dimension optical lattice external trapping potential which are typically used in experiments of BEC.

II. Theory

Bose-Einstein condensates of neutral atoms are quantum objects allowing the direct observation of fundamental effects of quantum mechanics on a macroscopic scale. Far-detuned laser light can be used to engineer different kinds of potentials with an unprecedented control of the forces exerted on a quantum system.

These two points justify our interest in the experimental study of Bose-Einstein condensates in optical potentials with the aim to investigate fundamental problems often born in the context of solid state physics. The close analogy between the periodic potential seen by an electron moving in a solid crystal and the optical lattice potential seen by a laser cooled atom allows us to apply condensed matter formalism to the atomic physics problem. A full quantum theory of laser cooling in optical lattices starts with the Hamiltonian [12]:

$$H = \frac{P^2}{2M} + U_0 (\bar{\epsilon}_L(\mathbf{r}) \cdot \hat{\mathbf{d}})^+ (\bar{\epsilon}_L(\mathbf{r}) \cdot \hat{\mathbf{d}}) \quad (1)$$

$$\text{Where, } \hat{\mathbf{d}} \equiv \sum_{q, m_g} C_{m_g}^{m_e+q} |e; j_e, m_e + q\rangle \langle g; j_g, m_g| e_q^* \quad (2)$$

Here P and $\mathbf{r}$ refer to the atom's center of mass, $\bar{\epsilon}_L(\mathbf{r})$ is the laser polarization at $\mathbf{r}$, $C_{m_g}^{m_e}$ is short-hand for the Clebsch-Gordan coefficient coupling the states $|j_e, m_e\rangle$ and $\langle j_g, m_g|$, and e_q are the spherical basis vectors ($q=0, \pm 1$). The above Hamiltonian, Eq. (1) governs only the coherent evolution of the atom. To treat laser cooling one must include dissipative processes, i.e. optical pumping between ground state sublevels. The starting point for most treatments of this problem is the master equation for the atomic density operator ρ following adiabatic elimination of the excited state [13],

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [H, \rho] - \frac{1}{2} \gamma_s \{ \Lambda, \rho \} + \gamma_s \sum_h \int d^2 k_s N_h(k_s) \{ W_h(k_s) \rho W_h^\dagger(k_s) \} \quad (3)$$

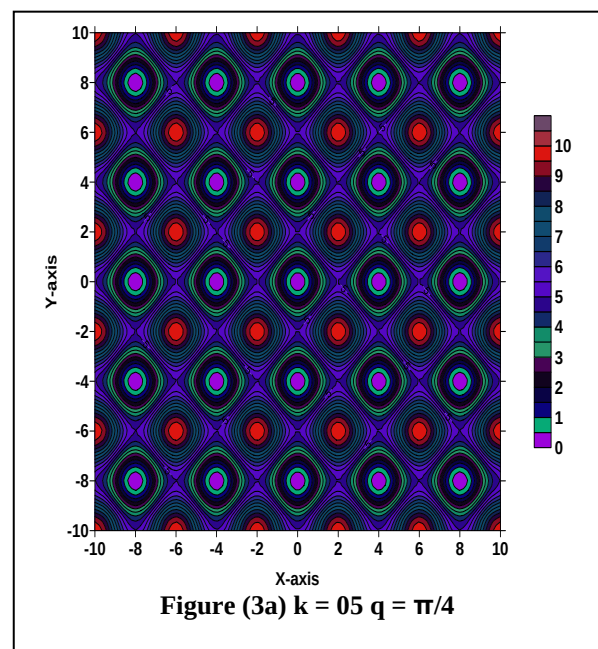
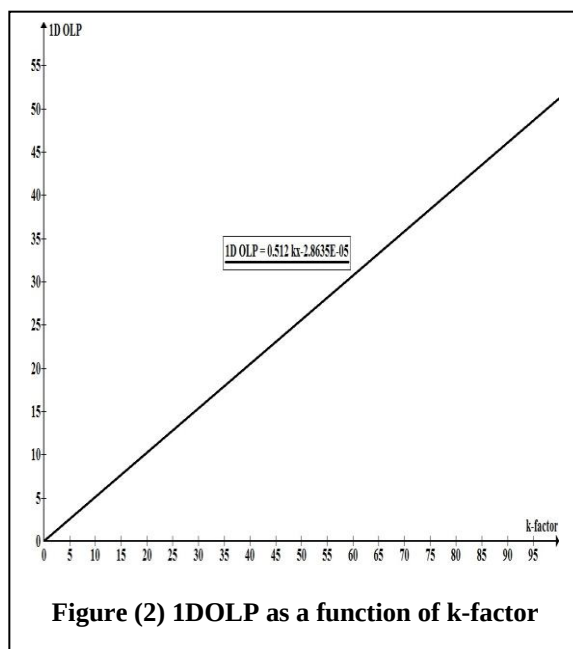
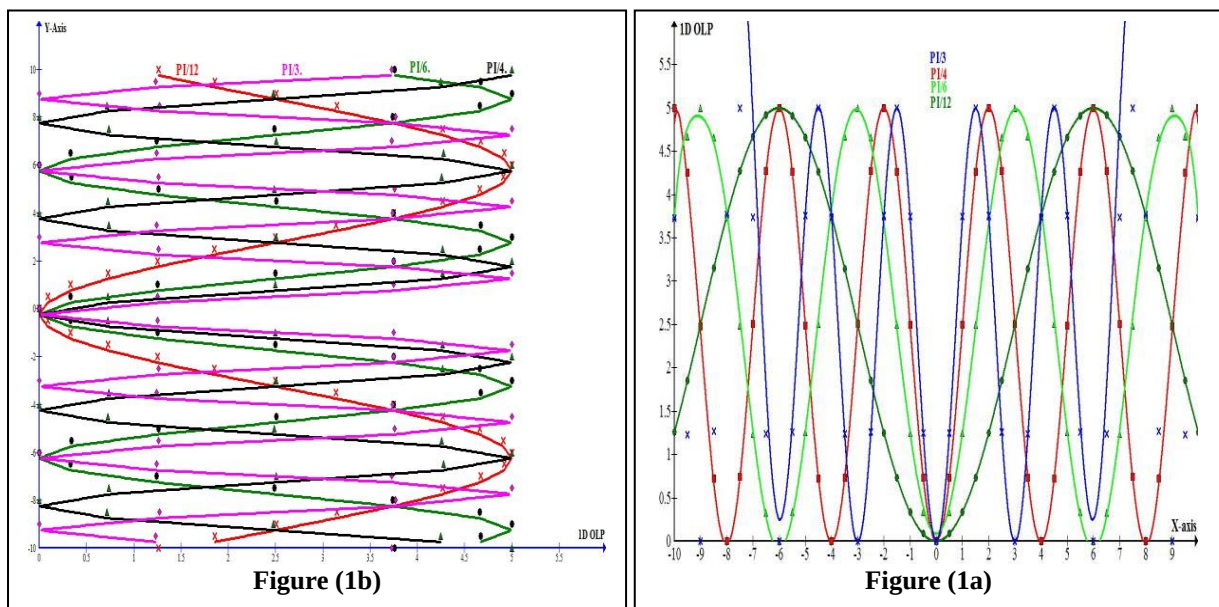
The first term is the coherent Schrödinger evolution governed by the Hamiltonian and the last two are the dissipative processes. The second term represents population decay, i. e. depletion of each ground state due to optical pumping into all others, with $\{, \}$ the anti-commutate and the operator $\Lambda \equiv (\bar{\epsilon}_L(\mathbf{r}) \cdot \hat{\mathbf{d}})^+ (\bar{\epsilon}_L(\mathbf{r}) \cdot \hat{\mathbf{d}})$. The third term is a population feeding term due to optical pumping replenishing the populations of each of the ground states. Here the operator $W_h(k_s) = (e^{ik_s \cdot \mathbf{r}} e_h \cdot \hat{\mathbf{d}})^+ (\bar{\epsilon}_L(\mathbf{r}) \cdot \hat{\mathbf{d}})$ represents absorption of a lattice photon, followed by emission of a fluorescence photon with wave vector k_s and helicity h along the quantization axis. The function $N_h(k_s)$ is the probability distribution for a fluorescence photon with helicity h to propagate along the direction k_s . If one scales all energies by the recoil energy, we see that the master equation depends solely on two dimensionless parameters, $\bar{U}_0 \equiv U_0 / E_R$, and $\bar{\gamma}_s \equiv \hbar \gamma_s / E_R$. The first parameter determines the time scale for the coherent processes such as the oscillation of atoms deeply bound at the bottom of the wells, and the second determines the time scale for the dissipative processes such as optical pumping. In principle, all information is contained within Eq. (3), which is an exact description of the atom-laser dynamics in the low saturation limit.

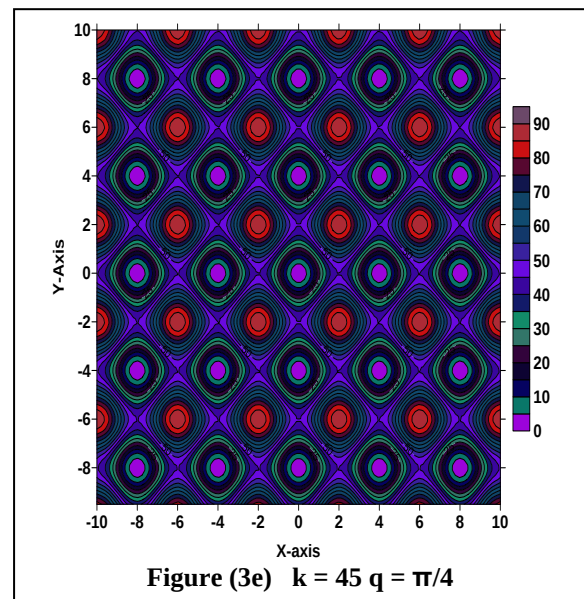
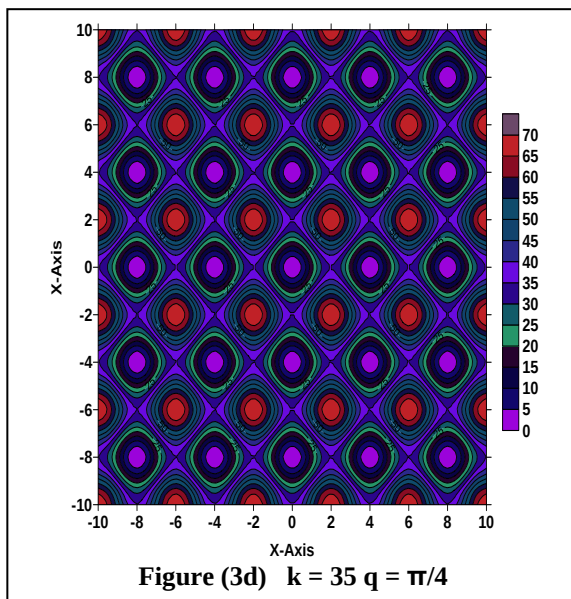
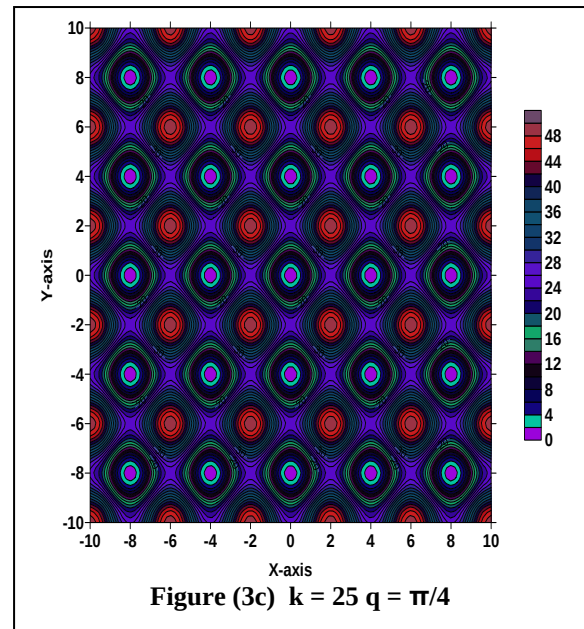
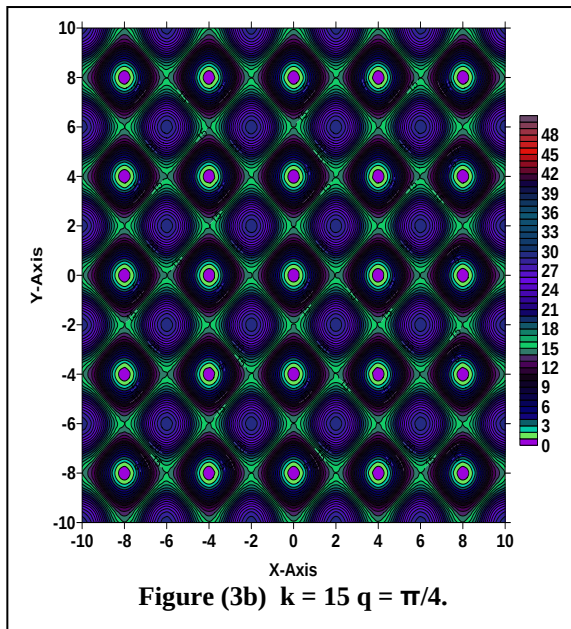
The master equation (3) is a full quantum picture of the atomic dynamics, together with external degrees of freedom. In certain circumstances, one can shorten the problem by treating the center of mass coordinates as classical variables. This *semi-classical* approximation is valid when the broaden in Doppler shifts due to the quantum uncertainty in momentum is small compared with the natural line-width, and when the spatial coherence length of the atomic wave function is small compared with the optical wavelength. In addition the internal degrees of freedom must slow down much faster than the external degrees of freedom so that one can treat the atom as a classical particle experiencing an instantaneous force. This approximation seem to be comparable with the early BEC experiment results, in that experiments, a quadratic harmonic oscillator well was used to trap the atoms. Recently more advanced and complicated traps have been applied for studying BECs in laboratories [16, 17, 18, 19]. In this paper, we will analysis a typical optical lattice potential trapping potentials which are widely used in current experiments $V_{opt}(\tau) = S_\tau E_\tau \sin^2(\hat{q}_\tau \tau)$, where $\hat{q}_\tau = 2\pi / \lambda_\tau$ is the angular frequency of the laser beam, with wavelength λ_τ , that creates the stationary 2D periodic lattice, $E_\tau = (\hbar^2 \hat{q}_\tau^2) / 2m$ is the recoil energy, and S_τ is a dimensionless parameter characterizing the intensity of the laser beam. The optical lattice potential has periodicity $T_\tau = \pi / \hat{q}_\tau = \lambda_\tau / 2$ along the τ -axis ($\tau = x, y, z$). The choices for the scaling parameters t_0 and r_0 , the dimensionless potential $V(r)$ with $\gamma_y = t_0 \omega_y$ and $\gamma_z = t_0 \omega_z$, the energy unit $E_0 = \hbar / t_0 \rightarrow E_0 = \hbar^2 / m r_0^2$, and the interaction parameter $\beta = 4\pi a_s N / r_0$. For external optical lattice trapping potentials t_0, r_0, k_r , and q_τ are reads as follow: $t_0 / \omega_r, r_0 = \sqrt{\hbar / m \omega_r}$, $k_r = 2\pi^2 r_0^2 S_\tau / \lambda_\tau^2$, $q_\tau = 2\pi r_0 / \lambda_\tau$, $\tau = x, y, z$.

III. Results and Discussion

In this section we will present results for optical lattice trapping potential usually used in experimental which lead to develop the atom Laser. A computer programs have written to calculate first the distribution of the optical-Lattice trapping potential along the axis of interest. It is assume that the working region of this potential is the XY-plane. Optical lattice potentials are applied along X-axis and Y-axis the shape of these potentials are

presented in figure (1). Figure (1a) shows the OLP along the X-axis for different values of q-factor (15° , 30° , 45° , and 60°). It is clear from this figure that as the value of q-factor increase the distribution of the OLP are broadening which means that the shape of potentials are change but still resolves its maximum values as well as the sinusoidal oscillations. The distribution of the OLP along Y-axis is presented in figure (1b) for the same values of q-factor. One can conclude from Figure (1a) and Figure (1b) that the distributions of OLP along X-axis and Y-Axis are matching in its principles and wavering. The relation connecting the average values of OLP and k-factor along x and y axis seem to be linear Follows the equation $OLP = 0.512 k$, with inaccuracy of order of 2.8635×10^{-5} , this presented in figure (2). To develop a comprehensive pictures about these potential more calculation are carried out in two dimensions. The contour level of the OLP is shown in figure (3) for k-factor acquire the values (5, 15, 25, 35, and 45), where q-factor set at value of $\pi/4$. In all these figures the distributions of the optical lattice trapping potential is a circularly symmetric and at the centre is near harmonic. The OLP trap has great flexibility and possible for a range of applications for example ideal circumstances are provided for accuracy measurements because the atoms are restrained in a region of low optical field strength in deficiency of any external field. Moreover, the 2D trap can be straight forward applied as a guide for atoms. We expect the OLP trap to amplify the atomic phase space density and expectantly to realize Bose Einstein condensation BEC.





VI. References

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