



International Journal of Emerging Technologies in Computational and Applied Sciences (IJETCAS)

www.iasir.net

Efficient lossy Image compression Using Adaptive Directional Lifting Based 9/7 wavelet Transform

Sanjay H. Dabhole¹,

Virajit A. Gundale²,

Johan Potgieter³

PhD Scholar, UNEM, Costa Rica

UNEM, Costa Rica

UNEM, Costa Rica

S.G.M.R. Polytechnic, Mahagaon, Kolhapur, INDIA-416503

Sanjay_dabhole@rediffmail.com

Abstract: Traditional 2-D Discrete wavelets transform (DWT) implemented by separable 1-D transform in horizontal and vertical directions does not take image orientation features in a local window into account. However, the lifting scheme provides a general and flexible tool for the construction of wavelet decompositions and perfect reconstruction filter banks. To improve it, we propose to introduce adaptive directional lifting structure based on DWT and thereby derive a new scheme for 9/7 DWT transform, whose transform defined structure is dependent on directional angle and interpolation used there and locally adapts the filtering directions to image content based on directional lifting. We propose an adaptive version of this scheme followed by SPIHT. The experimental results show that the performance of the proposed system with SPIHT can dramatically outperform the conventional DWT up to 6.4 dB even without overhead cost.

Keywords: Adaptive Directional Lifting, Discrete Wavelet Transform, Set Partition in Hierarchical Tree, Peak Signal to Noise ratio, Predict.

I. Introduction

Wavelet family is broad. Wavelet basis choice is conditioned by the application at hand or the given objective. In coding, some wavelets are more adequate for smooth regions and others behave better near discontinuities. Hence, many researchers have proposed adaptive schemes that modify the underlying wavelet basis according to local signal characteristics. Filter banks were the fundamental tool to create discrete wavelet transforms. They are formed by the analysis and synthesis low pass and high pass filters and the intermediate stage composed by a down- and up-sampling. Initially, the complexity and challenge of adaptively was to assure the filter bank reversibility in order to recover the original data. The wavelet transforms with spatially adaptive mother wavelets, chosen according to the underlying local signal characteristics are expected to be more effective in representing such signals. The lifting framework for wavelet transforms provides the flexibility of designing wavelet transforms with such nonlinear basis functions. Each one dimensional (1-D) Discrete Wavelet Transform (DWT) can be factored in to one or more lifting stages. A typical lifting stage is comprised of four steps: Split, Predict, Update and Normalize. The prediction and update steps in lifting can be made linear or nonlinear.

The lifting scheme proposed by Sweldens is an efficient tool for constructing second generation wavelets and has advantages such as faster implementation, fully in-place calculation, perfect reconstruction with low memory and computational complexity (Daubechies and Sweldens, 1997; Sweldens, 1996). A lifting stage is comprised of the four steps namely Split, Predict, Update and Normalize.

Split: The first step of lifting is to separate the original sequence (X) into two sub sequences containing the odd indexed samples (X_o) and the even indexed samples (X_e). This subsampling step is also called the lazy wavelet transform. $X_o : d_i \leftarrow X_{2i+1}$ and $X_e : s_i \leftarrow X_{2i}$ (1)

for $i = 0 \dots (L/2 - 1)$, where L is the signal length.

Predict: The two sets X_o and X_e are closely correlated. So a predictor $P(\cdot)$ can be used to predict one set from the other. In this prediction step, which is also called dual lifting, the odd indexed samples are predicted using the neighboring even indexed samples and the prediction error (detail) is recorded replacing the original sample value, thus providing in-place calculation

$d_i \leftarrow d_i - P_{(SA)}$ (2) where $A = (i - [N/2] + 1 \dots i + [N/2])$ is the number of dual vanishing moments in d, which set the smoothness of the P function.

Update: In the second lifting step, known as primal lifting (U), the even samples are replaced with smoothed values using the update operator $U(\cdot)$ on previously computed details. The $U(\cdot)$ operator is designed to maintain the correct running average of the original sequence, in order to avoid aliasing.

$$S_i \leftarrow S_i + U_{(dB)} \quad (3) \text{ Where, } B = \left(i + \frac{\tilde{N}}{2} \dots \dots \dots i + \frac{\tilde{N}}{2} - 1 \right)$$

is the number of preserving moments. The $U(\cdot)$ operator preserves the first moments in the s sequence. The lazy wavelet is lifted to a transform with required properties (number of vanishing moments in analysis and synthesis filters as in the filter bank approach) by applying the dual and primal lifting pair of operations one or more times.

Normalize: The outputs of the lifting are weighted by k_e and k_o . These values serve to normalize the energy of the underlying scaling and wavelet functions, respectively.

$$d_i \leftarrow d_i X k_o \text{ and } S_i \leftarrow S_i X k_e \quad (4)$$

The normalization factor k_e and k_o are $\sqrt{2}$ and $1/\sqrt{2}$ respectively. (Cohen, A. Daubechies I. and Feauveau I.C. 1992 Daubechies and Sweldens, 1997, Uytterhoeven, G. Roose and A. Bultheel 1997). For 2D signals, upon the completion of the 1-D lifting based horizontal transform, the 1-D lifting based vertical transform is performed in the same way.

In this paper, hence, we propose the 2D direct and adaptive directional Lifting based on DWT, called a 2D ADL structure of DWT. The proposed structure is realized by introducing an adaptive directional filtering framework into a 2D direct lifting structure based on 9/7 DWT. The lifting structure of DWT has been proposed for the integer-to-integer transform [10]. For 2D signals such as images, a 2D separable lifting structure of DWT is realized by applying the 1D lifting structures to images twice, vertically and horizontally. Iwahashiet O. have proposed the 2D direct lifting structure based on 9/7 DWT, by interchanging and merging some lifting in the 2D separable lifting structure of 9/7 DWT [11], [12]. In this paper, we generalize the polyphase representation of 2D directional lifting structure based on 2-channel 1D filter banks (FBs) such as DWT. It contains the class of the 2D directional lifting structure of 9/7 DWT [12]. Our proposed structure can design 2D adaptive lifting structures based on 9/7 DWT followed by it is compressed using Set Partitioning in Hierarchical Trees (SPIHT) algorithm, the most popularly used image compression algorithm (Said, William and Pearlman 1996).

Furthermore, to improve the efficiency of image coding, we develop the generalized polyphase representation for the adaptive directional filtering. Changing the sampling matrix of the proposed structure, we can change the direction of the transform. The proposed adaptive directional filtering is realized by changing the sampling matrix by sub-regions of images, according to feature directions of the sub-regions. With advantages of the 2D adaptive structure and the adaptive directional filtering, the proposed structure improves the performance of the lossy image coding application. The proposed method is simpler than the 1D one and can select the filtering directions which are different from 1ADL. Finally, lossy image coding results of the proposed structure are shown to validate the advantage of the proposed structure.

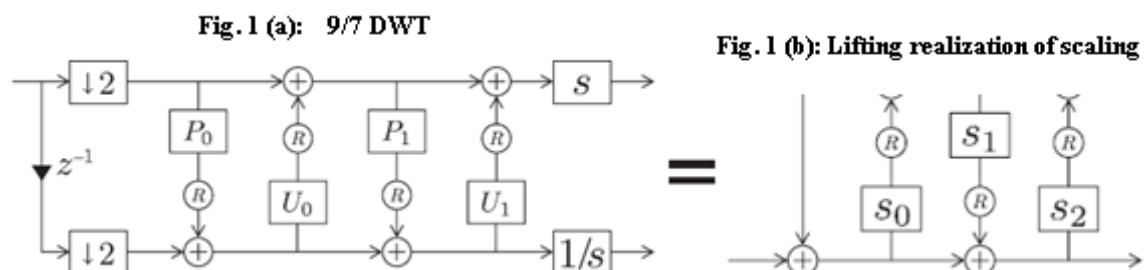
Section 2 summarizes the 2D lifting structure and sect. 3 discusses the 1D ADL structure of DWT. We propose the generalized polyphase representation of the 2D ADL structure of DWT and its adaptive directional filtering in Sect. 4. Section 5 shows some image coding results and conclusions are presented in Sect. 6.

II. 2D Directional Lifting Structure of DWT

1D DWT is classified into 2-channel FB. Its lifting structure can be expressed in the polyphase representation as follows [10], [13].

$$\begin{matrix} H_L(z) \\ H_H(z) \end{matrix} = \check{E}(z^2) d_2(z) \text{ and } \check{E}(z) = \begin{bmatrix} s & 0 \\ 0 & 1/s \end{bmatrix} \prod_{i=N-1}^0 \left\{ \begin{bmatrix} 1 & U_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ P_i(z) & 1 \end{bmatrix} \right\} \quad (5)$$

This structure with $N = 2$ is shown in Fig. 1(a), where R de-notes a rounding operator. This figure shows that 1D



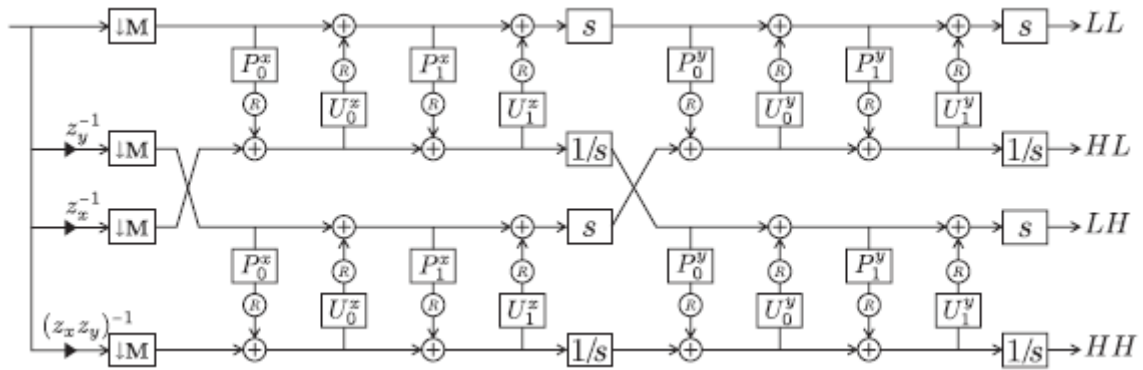
signals are divided into even samples and odd samples, sample is transformed with the other sample, and even and odd samples are conclusively output as low pass and high pass coefficients, respectively. In general, down

arrows from even signals to odd signals are called a prediction step, and up-arrows are called an update step. The pair of scaling factors can be realized as a lifting

$$\begin{bmatrix} s & 0 \\ 0 & 1/s \end{bmatrix} = \begin{bmatrix} 1 & s - s^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/s & 1 \end{bmatrix} \begin{bmatrix} 1 & s - 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad (6)$$

Figure 1(b) show the lifting realization of scaling factors, where $s_0 = s - 1$, $s_1 = -1/s$ and $s_2 = s - s^2$. The lifting structure with rounding operators can transform signals from integer to integer. Note that scaling parts can also be constructed by lifting steps. According to the implementation, the 2D adaptive lifting structure of DWT can be given by

Fig.2 2D adaptive Lifting Structure of 9/7 DWT



$$H(z) = \begin{bmatrix} H_{LL}(z) \\ H_{HL}(z) \\ H_{LH}(z) \\ H_{HH}(z) \end{bmatrix} = \begin{bmatrix} H_L(z_x)H_L(z_y) \\ H_L(z_x)H_H(z_y) \\ H_H(z_x)H_L(z_y) \\ H_H(z_x)H_H(z_y) \end{bmatrix} = \begin{bmatrix} H_L(z_y) & 0 \\ H_H(z_y) & 0 \\ 0 & H_L(z_y) \\ 0 & H_H(z_y) \end{bmatrix} \times \begin{bmatrix} H_L(z_x) \\ H_H(z_x) \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} \check{E}(z_y^2) & 0 \\ 0 & \check{E}(z_y^2) \end{bmatrix} \begin{bmatrix} d_2(z_y) & 0 \\ 0 & d_2(z_y) \end{bmatrix} \check{E} z_x^2 d_2(z_x)$$

For the four subband filters, $H(z)$, $H_{HL}(z)$, $H(z)$ and $HHH(z)$. To define the polyphase matrix of 2D separable structure, we introduce the relationships as

$$\begin{cases} \check{d}_2 z_y \begin{bmatrix} s & 0 \\ 0 & 1/s \end{bmatrix} = \begin{bmatrix} sI & 0 \\ 0 & 1/sI \end{bmatrix} \check{d}_2 z_y, \\ \check{d}_2 z_y \begin{bmatrix} 1 & U_i(z_x^2) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & U_i(z_x^2)I \\ 0 & I \end{bmatrix} \check{d}_2 z_y, \\ \check{d}_2 z_y \begin{bmatrix} 1 & 0 \\ P_i(z_x^2) & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ P_i(z_x^2)I & I \end{bmatrix} \check{d}_2 z_y \end{cases} \quad (8)$$

Where, $\check{d}_2(z_y) = \text{diag}(d_2(z_y), d_2(z_y))$ from above relationships (6), (5) is rewritten as

$$H(z) = \begin{bmatrix} \check{E}(z_y^2) & 0 \\ 0 & \check{E}(z_y^2) \end{bmatrix} \times \begin{bmatrix} sI & 0 \\ 0 & 1/sI \end{bmatrix} \prod_{i=N-1}^0 \left\{ \begin{bmatrix} I & U_i(z_x^2)I \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ P_i(z_x^2) & I \end{bmatrix} \right\} \times \check{d}_2 z_y \check{d}_2 z_x \quad (9)$$

Consequently, the polyphase representation of the above 2D separable structure is described as

$$H(z) = E(z^M) d_M(z),$$

$$E(z) = \begin{bmatrix} \check{E}^y & 0 \\ 0 & \check{E}^y \end{bmatrix} \begin{bmatrix} sI & 0 \\ 0 & 1/sI \end{bmatrix} \prod_{i=N-1}^0 \left\{ \begin{bmatrix} I & U_i^x I \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ P_i^x I & I \end{bmatrix} \right\},$$

$$d_M(z) = [1, z_y^{-1}, z_y^{-1}, z_y^{-1}, z_y^{-1}, z_y^{-1}]^T \quad (10)$$

where, $\check{E}^j$, P_i^j and U_i^j ($j: x$ or y) denote $\check{E}(z_j)$, $P_i(z_j)$ and $U_i(z_j)$. Since the 2D FB is realized by the separable implementation of 1D FB, the decimation matrix M should be $\text{diag}(2, 2)$. For example, Fig. 2 shows the 2D adaptive lifting structure of 9/7 DWT. This figure shows that images are divided into four sets and transformed while maintaining integers.

III. 1D ADL Structure of DWT

Based on the lifting structure of 1D DWT, 1D ADL is realized by permuting the other directions of the transform except horizontal and vertical [5].

Fig. 3: Lifting Direction of 1D ADL

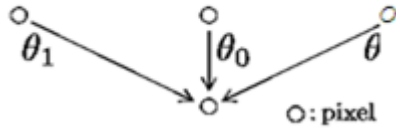


Figure 3 illustrates a 2D signal $x(n)$, where, $[n_x, n_y]^T$ and n_x and n_y are defined as vertical and horizon- indices in 2D signals. In the lifting structure of 1D DWT, the input signals are divided into even samples x_e and odd samples x_o as follows.

$$\begin{cases} x_e(n) = x([n_x, 2n_y]^T) \\ x_o(n) = x([n_x, 2n_y + 1]^T) \end{cases} \quad (11)$$

In the prediction step, odd samples $x_o(n)$ are predicted from the neighboring even samples $x_e(n)$ as

$$\tilde{x}_o(n) = x_o(n) + P_e(n) \quad (12)$$

Where $P_e(n)$ is expressed as

$$P_e(n) = P_i \left\{ x_e \left(\begin{bmatrix} n_x - \tan \theta_d \\ n_y \end{bmatrix} \right) + x_e \left(\begin{bmatrix} n_x + \tan \theta_d \\ n_y + 1 \end{bmatrix} \right) \right\}$$

P_i is a coefficient of filter and θ_d is arbitrary direction of filtering. Figure 3 shows $\theta_d = \{\pi/2, \pi/4, -\pi/4\}$. Next, in the update step, even samples $x_e(n)$ are updated from the new neighboring odd samples $\tilde{x}_o(n)$ as

Where $u_o(n)$ is expressed as

$$u_o(n) = u_i \left\{ \tilde{x}_o \left(\begin{bmatrix} n_x - \tan \theta_d \\ n_y - 1 \end{bmatrix} \right) + \tilde{x}_o \left(\begin{bmatrix} n_x + \tan \theta_d \\ n_y \end{bmatrix} \right) \right\},$$

u_i is a coefficient of filters. This set of lifting operations is repeated and conclusive samples are produced as lowpass and highpass coefficients.

The generalized representation of 2D directional lifting structure of DWT is defined as

$$E_{(z)} = \tilde{S} \prod_{i=N-1}^0 G_{i,j}(z) \quad \text{Where } \tilde{S} = S\tilde{S} = \text{diag}(s^2, 1, 1, 1/s^2) \quad (13)$$

IV. 2D ADL structure of 9/7 DWT

In this section, we present its examples based on 5/3 and 9/7 DWT. These DWTs are well known to be applied to the JPEG 2000. From equation (13), the polyphase structure of 2D ADL structure of 9/7 DWT is described as,

$$E_{(z)} = \tilde{S} G_{1,1}(z) G_{0,0}(z) \quad (14)$$

The ADL structure can transform a sub-region of images along its direction. Therefore, the structure efficiently reduces the entropy of transformed coefficients and achieves better image coding performance.

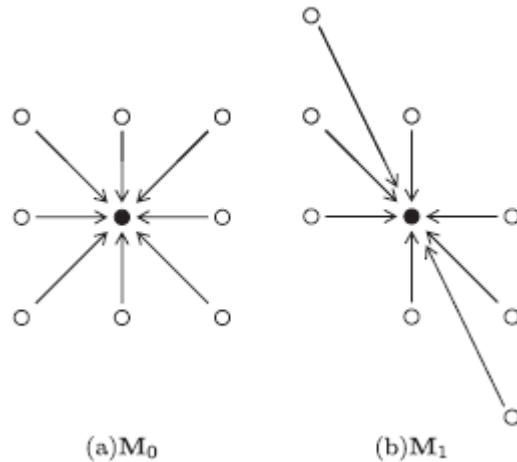
By changing the sampling matrix M depending on sub-regions of images, the ADL structure is realized. Matrices are defined as $M_0 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $M_1 = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ (17)

These dots are pixels, and black dots are transformed with neighbor white dots along arrows. This figure shows that neighbor pixels used for the lifting operation are changed by changing the sampling matrix in the sub-region. In other words, the direction of the transform can be changed by changing the sampling matrix.

V. Simulation

We apply the proposed ADL structure of 9/7 DWTs into the lossy image coding application. We have applied the adaptive algorithms to 256 x 256 8 bit images. MATLAB tool is used for computing the algorithms. The original image is transformed with CDF 9/7 wavelet through 1D and 2D adaptive directional lifting and it is compressed using Set Partitioning in Hierarchical Trees (SPIHT) algorithm. The performance of the adaptive algorithm is

Fig. 4: Lifting direction of 2D ADL



compared with the popular wavelet transforms as shown in Table 1 for the test images like Lena and peppers at different bit rates . The performance metric PSNR is computed and as shown in table 1.

Table 1: PSNR in db for different level at different bitrates for the test images

Level	Rate (in bpp)	0.2	0.4	0.6	0.8	1.0
2	Lena	15.75	15.75	21.29	21.29	25.90
	peppers	15.32	15.32	22.12	22.11	27.38
4	Lena	30.47	35.25	38.14	40.73	42.61
	peppers	34.69	40.33	43.71	46.28	48.16
6	Lena	33.42	36.99	39.53	41.75	43.53
	peppers	38.55	42.54	45.11	47.43	49.11
8	Lena	33.52	37.06	39.60	41.79	43.58
	peppers	38.68	42.62	45.16	47.48	49.16

As a comparison benchmark the PSNR values, based on mean square error (MSE) are calculated for the proposed ADL based on 9/7 DWT with SPIHT algorithm. Table 1 indicates results of the lossy image coding application. In the lossy, these numbers are PSNR of reconstructed images in dB, and rates are described in bpp with respect to 1 to 8 levels. The proposed structure improves image coding application in the lossy for the same reason of 9/7 DWT. The 2D ADL is efficient for images having rich directional components because of the small number of rounding operators and the adaptive directional filtering. On the other hand, the proposed structure is slightly effective for images whose energy is concentrated on the low frequency region such as Lena or peppers.

Table 1 and fig. 5 (a) and 5 (b) seems that proposed structure work efficiently from level 3 to level 8 and maximum PSNR for peppers becomes 49.16 (peppers) or 43.58 (lena) and average PSNR becomes 45.20 and it is observed that about 5.1851 to 6.4 dB gains in PSNR is obtained with the proposed algorithm compared to previous work in [2] for same bit rate.

The visual comparison of 1D ADL and 2D ADL and lifting stages of the proposed algorithm is as shown in the figures 6 for the lena image with 1 bpp rate. Compared with the 1D and 2D ADL with 9/7 DWTs, the 2D ADL shows more efficient image coding results from level 4 to level 6 because of fewer rounding operations and adaptive directional filtering.

VI. Conclusion:

In this paper, the proposed directional adaptive transform has been applied to 256x256 8 bit images. We proposed 1D and 2D ADL structure based on 9/7 DWT followed by SPIHT and maintaining the advantage of the 2D the adaptive directional lifting, the proposed structure achieves efficient image coding results. This adaptive lifting transform appears promising for image compression. It reduces edge artifacts, ringing and gives improved PSNR up to 6.4 dB for smooth images like Lena or peppers.

Finally, 9/7 transform with spiht gives much better performance for lossy compression as compare conventional DWT method.

Figure 5 (a): PSNR (dB) vs Rate(bpp) for Lena Image with 1 to 8 levels

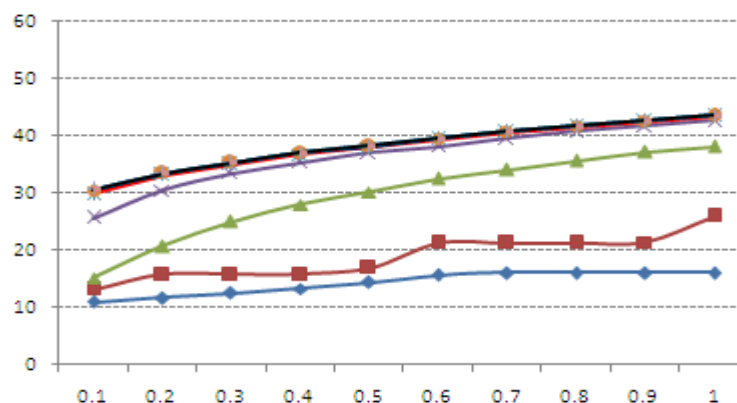


Figure 5 (b): Psnr (dB) vs Rate (bpp) of peppers image for 1 to 8 levels

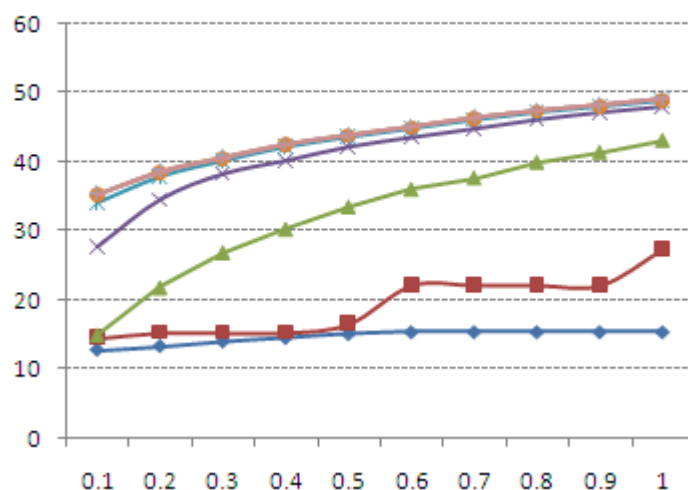


Fig. 5 shows the effect of 1D and 2 D ADL at 4 to 8 lifting levels with rate=1 for Lena image.

nlevel	1 D ADL	2 D ADL	Reconstructed
4			
5			
6			
7			

8



VII. References

- [1] ISO/IEC 10918-1, Information technology-JPEG 2000 image coding system: Core coding system.
- [2] R. Satyabama and S. Annadurai, "Image compression using Adaptive primal and dual Lifting scheme based on Mean square Error Minimization" European Journal of Scientific Research., ISSN 1450-216X Vol.63 No.1 (2011), pp. 99-10, Inc. 2011.
- [3] Omer N. Gerek and A.E. Qetin, "A 2-D orientation-adaptive prediction filter in lifting structures for image coding," IEEE Trans. Image Process., vol.15, no.1, pp.106-111, Jan. 2006.
- [4] C.L. Chang and B. Girod, "Direction-adaptive discrete wavelet transform for image compression," IEEE Trans. Image Process., vol.16, no.5, pp.1289-1302, May 2007.
- [5] W. Ding, E. Wu, X. Wu, S. Li, and H. Li, Adaptive directional lifting-based wavelet transform for image coding," IEEE Trans. Image Process., vol.16, no.2, pp.416-427, Feb. 2007.
- [6] W. Dong, G. Shi, and J. Xu, Adaptive nonseparable interpolation for image compression with directional wavelet transform," IEEE Signal Process. Lett., vol.15, no.1, pp.233-236, 2008.
- [7] Y. Liu and K.N. Ngan, "Weighted adaptive lifting-based wavelet transform for image coding," IEEE Trans. Image Process., vol.17, no.4, pp.500-511, April 2008.
- [8] G. Quellec, M. Lamard, G. Cazuguel, B. Cochener, and C. Roux, Adaptive nonseparable wavelet transform via lifting and its application to content-based image retrieval," IEEE Trans. Image Process., vol.19, no.1, pp.25-35, Jan. 2010.
- [9] Y. Tanaka, M. Hasegawa, S. Kato, M. Ikehara, and T.Q. Nguyen, Adaptive directional wavelet transform based on directional pre-filtering," IEEE Trans. Image Process., vol.19, no.4, pp.934-945, April 2010.
- [10] I. Daubechies and W. Sweldens, "Factoring wavelet transforms into lifting steps," J. Fourier M. Appl., vol.4, no.3, pp.247-269, 1998.
- [11] M. Iwahashi, "Four-band decomposition module with minimum rounding operations," IET Electronics Lett., vol.43, no.6, pp.27-28, March 2007.
- [12] M. Iwahashi and H. Kiya, "Reversible 9-7 DWT compatible to irreversible 9-7 DWT of JPEG 2000," IEICE Technical Report, IE2008-257, March 2009.
- [13] M. Iwahashi and H. Kiya, "Non separable 2D factorization of separable 2D DWT for lossless image coding," IEEE ICIP'09, pp.17-20, Nov. 2009.

VIII. Acknowledgments

This paper is partly supported by Research and Development Cell of Xtreme Solutions Ltd. Pune-India.