



Stress Analysis of a Rectangular Plate with Circular Hole Using Three Dimensional Finite Element Model

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Abstract: Geometric irregularities such as holes, notches, keyways, shoulders provided on shafts etc. are common features provided in machine members. But such irregularities often lead to stress concentration near the irregularity due to which the stress near the irregularity is higher than the average stress in the whole member. In this study, a rectangular plate with circular hole at center is analyzed by using three dimensional finite element analysis. The plate is subjected to tensile loading and the effect of ratio of thickness of plate to hole diameter ie. T/D on the stresses and displacements is studied. Further, the stress concentration factor for different T/D ratios is calculated theoretically and is compared with stress concentration factor computed by using ANSYS.

Keywords: Finite Element Analysis, Aluminium 7075 alloy, T/D ratio, ANSYS

I. Introduction

Machine elements are generally provided with geometric irregularities such as holes, notches, keyways etc. Due to such irregularities in machine parts, the stress distribution around the irregularities is disturbed and the elementary equations of stress cannot define the state of stress in the stress concentrated regions in machine elements. Various methods of stress analysis include experimental methods such as photoelasticity, brittle coating, electrical strain gauges etc. and numerical or analytical methods like Finite Element Analysis, Boundary element analysis and Complex Variable Approach[1]. In this study, stress and displacement behaviour of a rectangular plate with circular hole at center under tensile loading is studied by developing a three dimensional Finite Element model of the problem.

Lightweight aluminium alloys like Al 7075 finds many structural applications in aerospace and automobile due to their excellent mechanical properties and light weight. Generally a rectangular plate with a central hole is a common appearance in structures. But stress concentration around the hole makes the member highly prone to failure across the hole region. Many solutions to decrease the stress concentration includes providing relief holes, notches etc.

Gunwant and Singh [2] analyzed a rectangular plate with elliptical hole using analytical and FEA and studies the effect of aspect ratio (ratio of minor to major diameter of the ellipse) on the stress concentration factor and conclude that as the aspect ratio increases, the stress concentration factor goes on decreasing. Also it was found out that irrespective of the aspect ratio, maximum stress occurs at the corners of the ellipse. Nagpal et al. [3] studies the effect of D/A ratio (ratio of diameter of hole to plate width) using FEA on a rectangular plate having circular hole and concluded that as the D/A increases, the stresses in X , XY and von mises stress also goes on increases but stress in Y direction does not follow the same trend. Studies shows a decrease in SCF on increasing D/A ratio. SCF was mitigated by providing relief holes at optimized position and size. Nagpal et al.[4] derived an equation to calculate the size, position of the auxiliary hole. Mitigation curves were also suggested to optimize the V notch position. A mathematical analysis of the isotropic plates subjected to in plane loading is performed by Peterson [5] to calculate stress concentration factors. Kawadkar et al.[6] studied the effect of different orientations of the geometric irregularities on stress concentration using experimental and Finite element method. Different hole profiles such as rectangular, circular and triangular were studied. They draw the conclusion that the stress concentration increases with increase in the orientations of the holes.

II. Problem Description

In this research work, the effect of T/D ie. ratio of thickness of the plate to diameter of the hole on stresses induced and the displacements produced due to the applied load. The plate material is aluminium alloy Al 7075. The material is assumed to be isotropic and the Young's modulus for the material is 72000Mpa and the Poisson's ratio is 0.33. The analysis is carried out on five T/d ratios which are 0.1, 0.2, 0.3, 0.4 and 0.5. Stress concentration factor for a rectangular plate of finite width and centred circular hole is given by the following equation ([5],[7]).

$$K_{th} = \sigma_{max} / \sigma_{nom} \quad (1)$$

where $\sigma_{\max}$ is the maximum stress which is likely to occur near the irregularity whereas σ_{nom} is the nominal stress in the member. σ_{nom} is calculated by using elementary stress equations[8] i.e. ($\sigma = F/A$) where F is the force which is calculated by multiplying the pressure load (in MPa) by the area on which it is applied, i.e. ($W \cdot T$). The area considered in calculating the nominal stress is net cross sectional area i.e. ($\{W-D\} \cdot T$) which is area considering the irregularities such as hole in this study. The stress concentration factor also depends on the ratio $2R/W$ (R =radius of hole) and can be calculated by the following relation.[9]

$$K = 3.00 - 3.13(2R/W) + 3.66(2R/W)^2 - 1.53(2R/W)^3 \quad (2)$$

III. Finite Element Analysis

Finite element analysis of the problem is performed in popular FEA package ANSYS 13. Model of the geometry is generated and analysed in ANSYS. The plate is fixed at one end and a tensile load of 8 Mpa is applied on the other end of the plate. The model is meshed with SOLID186 element. This element has 20 nodes per element and 20 degrees of freedom at each of the node. This element has the capabilities like plasticity, creep, stress stiffening, large deflections etc. It is also suitable for modeling irregular meshes.[10]. The mesh is refined near the hole. The plate geometry and the FE model of the plate are shown in the Fig.1 and 2 respectively.

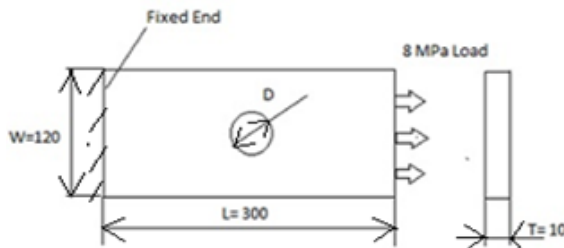


Fig. 1 Plate geometry(All dimensions are in mm)

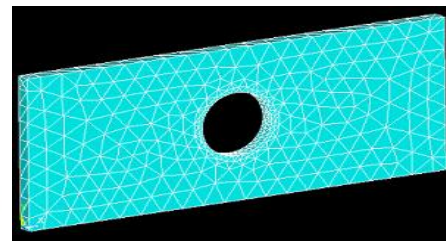


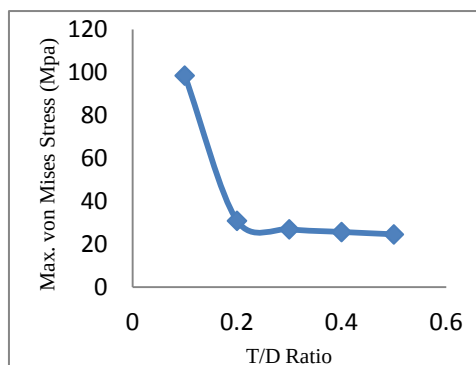
Fig. 2. 3-D FE model of the plate with hole

IV. Results and Discussion

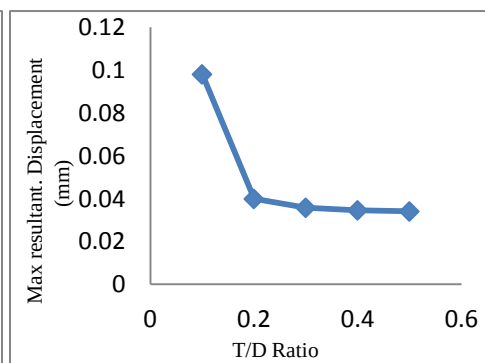
The various stress and displacement values are shown in the Table 1. By studying the table, it is clear that as the T/D ratio increases the maximum von Mises also increases. Same is the case with stress in X direction. But the stress in Y direction increases from T/D ratio 0.1 to 0.4 and decreases slightly further. The displacements (resultant, X and Y directions) decreases as the T/D ratio increases which is highly expected because as the T/D ratio increases, the hole diameter gets reduced, material in the plate increases which makes it more strong to restrict the deformation induced due to loading. The variation of von Mises stress and resultant displacement with T/D ratio are graphically represented in graphs 1-2 and stress concentration around the hole and displacement plots are shown in the Figures 3-7.

S.No	T/D Ratio	Max. von Mises Stress (Mpa)	X component of stress (Max. Value in Mpa)	Y Component of stress (Max. Value in Mpa)	Max Resultant Displacement (mm)	X Component of displacement (mm)	Y component of displacement (mm)
1	0.1	98.5025	99.976	6.74203	0.098023	0.098021	0.054961
2	0.2	30.8862	31.444	6.97365	0.039919	0.039918	0.006138
3	0.3	26.9271	27.3621	7.09829	0.035792	0.035769	0.003695
4	0.4	25.6535	26.4661	7.14378	0.034584	0.034535	0.003001
5	0.5	24.5474	25.9444	7.12384	0.03405	0.033987	0.002692

Table 1 Stress and displacement results for various T/D ratios from FEA



Graph 1. Max.von Mises stress v/s T/D ratio



Graph 2 . Max.resultant displacement v/s T/D ratio

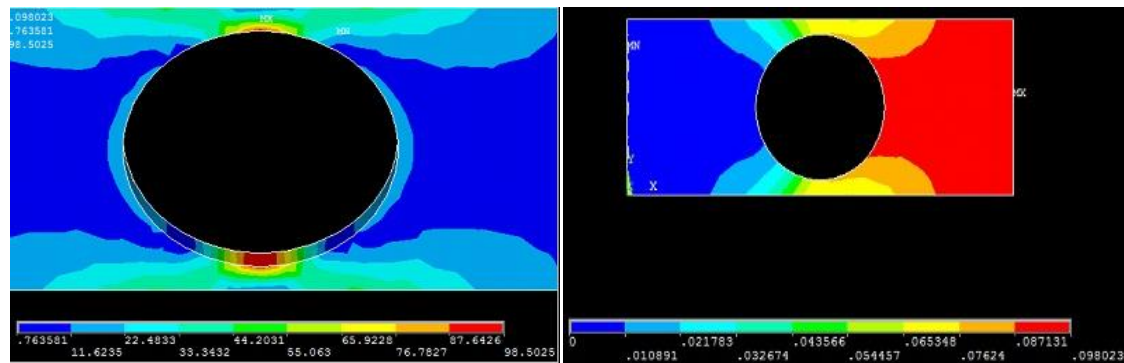


Fig. 3 Stress concentration (von Mises) around the hole(left) and displacement plot(right) for $T/D = 0.1$

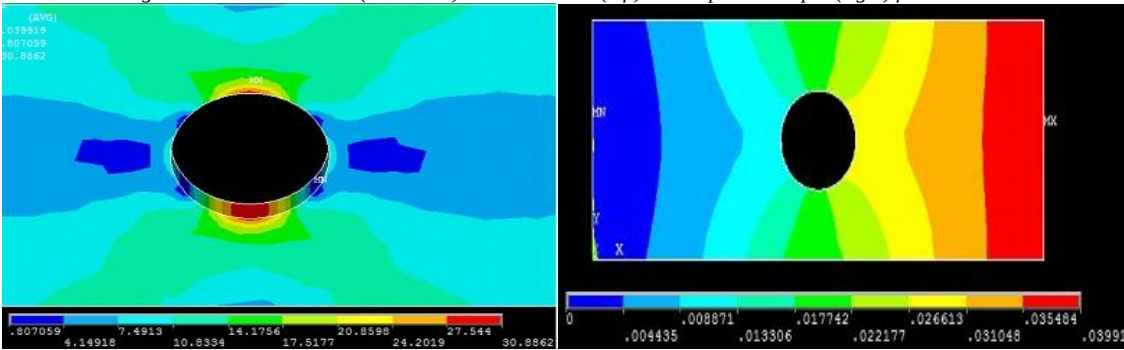


Fig. 4 Stress concentration (von Mises) around the hole(left) and displacement plot(right) for $T/D = 0.2$

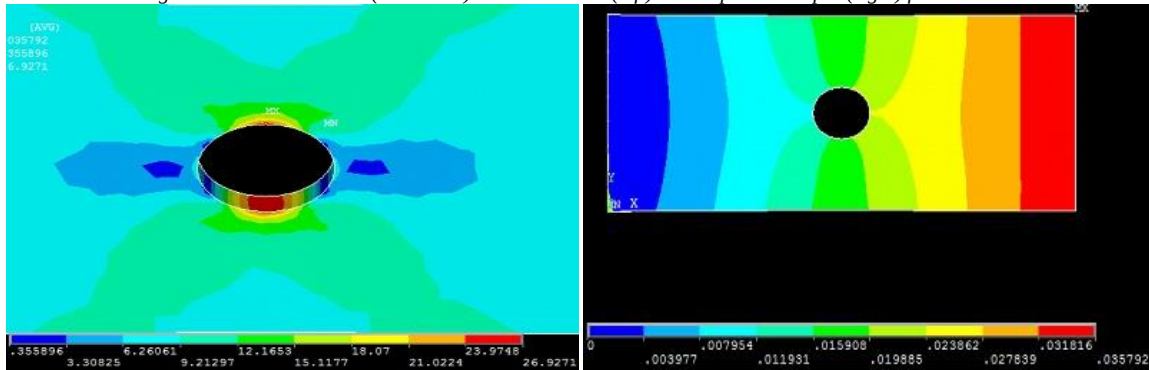


Fig. 5 Stress concentration (von Mises) around the hole(left) and displacement plot(right) for $T/D = 0.3$

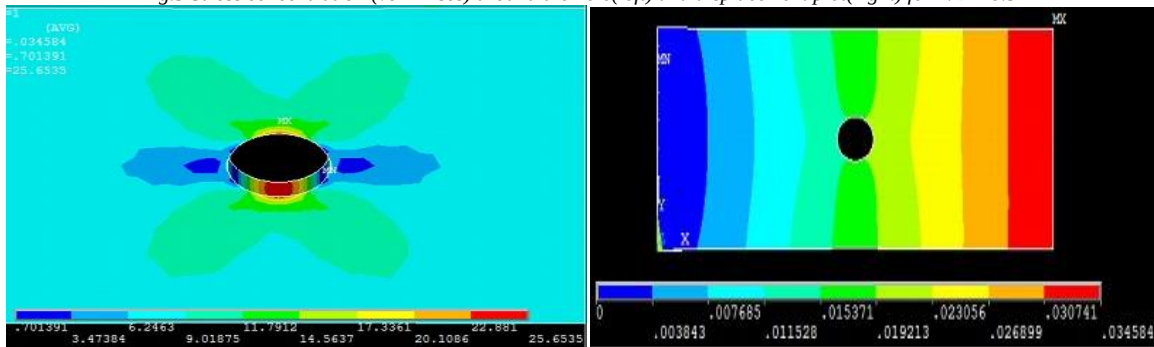


Fig. 6 Stress concentration (von Mises) around the hole(left) and displacement plot(right) for $T/D = 0.4$

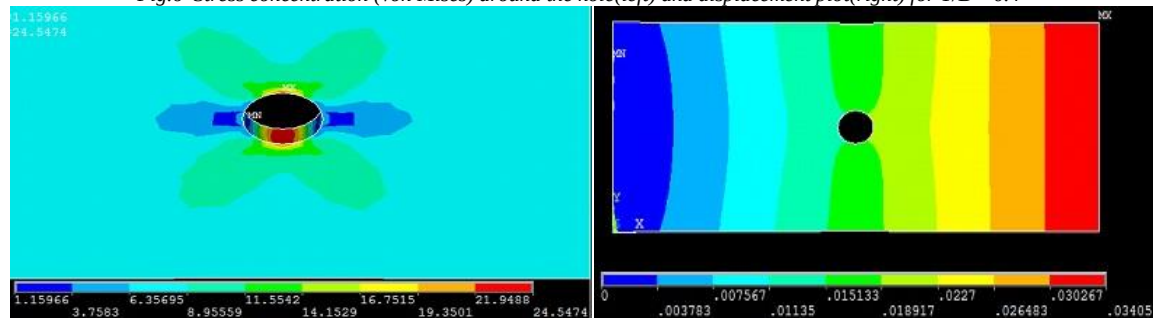
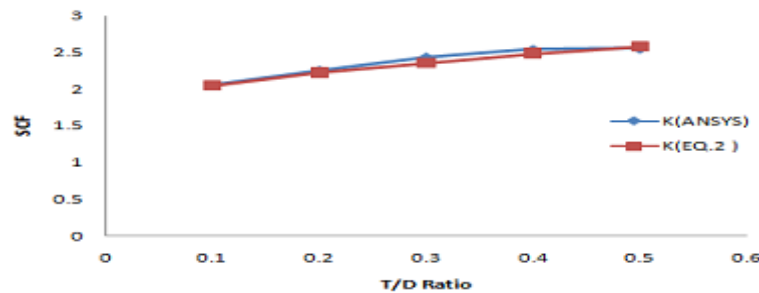


Fig. 7 Stress concentration (von Mises) around the hole(left) and displacement plot(right) for $T/D = 0.5$

For various T/D ratios, the stress concentration factors are calculated based on results of ANSYS (eq. 1) and theoretically (eq.2). The SCF values increases as the T/D ratio increases and this indicates an increase in SCF for reduction in diameter. The values of SCF based on eq 1 and 2 are compared and % error is calculated and the data is plotted in the Table. 2. Graph 3 shows the graphical representation of the experimental (from ANSYS) and theoretical values (eq.2) of SCF.

S No.	T/D Ratio	Max. von Mises stress, $\sigma_{\max}$	Nominal stress $\sigma_{\text{nom}} = F/A$	S.C.F, K_{ANSYS}	S.C.F, $K_{\text{eq. 2}}$	% Error
1	0.1	98.5025	48	2.052	2.048	0.175
2	0.2	30.8862	13.714	2.252	2.221	1.376
3	0.3	26.9271	11.07	2.432	2.354	3.207
4	0.4	25.6535	10.11	2.537	2.483	2.128
5	0.5	24.5474	9.6	2.557	2.573	0.622

Table 2. Comparison of SCF values obtained from FEA(eq. 1) and equation 2.



Graph 3. Comparison of different SCF values obtained

V. Conclusion

In this study, a rectangular plate with circular hole in center is analyzed with FEA and effect of T/D ratio on stress and displacement is studied. Also, effect of T/D ratio on the SCF is studied and the SCF values obtained with FEA and relation (eq. 2) were compared. Following conclusions can be drawn from the study.

1. Maximum stress occurs at the corners of the hole irrespective of the T/D ratio. Also, von Mises stress decreases with increase in T/D ratio.
2. The resultant displacement in the plate decreases with increase in the T/D ratio.
3. The SCF increases with increase in the T/D ratio. The SCF values obtained from FEA results are in a good agreement with the values obtained with the relation (eq. 2) with a maximum error of 3.207%. From these results, it can be concluded that Finite Element analysis is a valuable tool to study stress concentrations in structural and machine elements.

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