



International Journal of Engineering, Business and Enterprise Applications (IJEBA)

www.iasir.net

PRICE LIMITS AND INFORMATIONAL EFFICIENCY

Tamir Levy * and Joseph Yagil **

*Netanya Academic College

School of Business

1 University St.

Kiryat Yitzhak Rabin

Netanya 42365, ISRAEL

**Haifa University

Faculty of Management

Haifa, 31905, ISRAEL

Abstract: *Many of the empirical studies involving the estimation of the return-generating process (RGP) under conditions of a price-limit regime base their arguments on the equality of the aggregate market return during a k-day limit sequence and the aggregate theoretical return during those periods had the exchange not adopted a price-limit regime. Using an extension of Vives' (1995) model, we investigate whether this equality, which we call the Return Identity Proposition (RIP), actually holds. We demonstrate that the RIP may not hold when noise traders act in the markets. Simulation tests indicate that stocks for which noise trading is relatively high exhibit a higher number of limit hits, a larger number of limit days and a higher percentage of price reversals on the limit removal day. These results imply that while the RIP may be a practical tool for empirical studies of price limits, it may not hold when noise trading is present.*

Keywords: *Keywords: Price limits; Return-generating process; Noise traders.*

I. Introduction

Daily price limits are adopted by many securities exchanges in countries such as the USA, Canada, Japan and numerous countries in Europe and Asia, in order to increase the stability of the market. These limits confine the price of the financial asset during all trading stages of any trading day to a range, usually determined based on the previous day's closing price. By so doing, exchanges artificially change the return-generating process (RGP) of the asset. Such changes pose a problem involving the estimation of the RGP without knowing the theoretical prices.

Several suggestions have been made in the literature to solve the price-estimation problem. Roll (1984) discusses the relationship between the value and the price of frozen orange juice futures. He assumes the limits do not influence the RGP. Therefore, he proposes treating the aggregate rate of return during the limit period as if it were equal to the theoretical (equilibrium) aggregate rate of return.¹

We term this proposition the Return Identity Proposition (RIP). The RIP was adopted (directly or indirectly) in subsequent studies, such as those by Sutrick (1993), Yang and Brorsen (1995), Kim and Rhee (1997), Park (2000), Kim (2001) and Wei (2002).² Still, the basis of the proposition, beyond Roll's arguments, is lacking. Most researchers have not defined it, nor have they assigned it a specific name or explored its basis. As far as we know, the literature contains no theoretical discussion of the RIP.

While prior empirical studies employed the RIP, theoretical studies have not investigated it. Thus, an attempt is made here to fill the gap by examining the conditions necessary for the existence of the RIP. The purpose of this paper, therefore, is to test the RIP, with the specific goal of investigating whether the market RGP equals the equilibrium RGP, or, equivalently, whether the RIP holds. The importance of the investigation lies in finding a theoretical basis for empirical studies of the price-limit phenomenon. The issue of the existence of the RIP may also help answer the question about the effect of price limits on the statistical properties of market returns.

Theoretical studies of price limits include those by Brennan (1986), Subrahmanyam (1994), Kodres and O'Brien (1994), Chowdhry and Nanda (1998), Chou, Lin and Yu (2000, 2003), Harel, Harpaz and Yagil (2005 and 2010) and Levy and Yagil (2005).³

Empirical studies of price limits have investigated several issues and reached a number of significant findings. First, following an up-limit hit, the price usually continues to rise on the subsequent day (Park, 2000). In addition, stocks that have frequent limit hits have strong returns, high trading volumes, and receive more news coverage (Seasholes and Wu, 2007). While trading activity increases after either trading halts or price limits have been activated, volatility stays the same after trading halts but increases after price-limit hits are reached (Yong et al., 2008). Finally, Haun and Chou (2004) show that intraday price limits do not seem to have a strong influence on return dynamics.

Another question dealt with in the literature is whether price limits have a cooling off effect that stabilizes prices once they approach a limit, or a magnetic effect that accelerates prices toward the limits (Arak and Cook, 1997;

Fernandes and Rocha, 2007). Subsequent research has concluded that a strong cooling off effect prevails (Abad and Pascual, 2007). The empirical literature on price limits has also tested a variety of models to determine the most effective ones. Brennan's original model (1986) was broadened to two periods (Chou, Lin and Yu, 2000), and was also extended to investigate whether the imposition of spot price limits can further reduce the default risk (Chou, Lin and Yu, 2003). Other models include a censored stochastic volatility model for capturing important features of a return series censored by price limits (Hsieh and Yang, 2009), and a Censored-GARCH model with price limits used to estimate the return-generating process (Wei, 2002). Harel, Harpaz and Yagil (2005) developed a Bayesian forecasting model in the presence of return limits and provide some numerical predictions. An additional forecasting model is offered in Harel, Harpaz and Yagil (2010) who also applied it to a sample of futures contracts. Finally, research indicates that the near-limit model performs better than five other models proposed in the literature in terms of its ability to predict returns (Levy and Yagil, 2006). Related works include the literature on circuit breakers and trading halts.

Despite the broad range of both theoretical and empirical studies of price limits, none of them has examined the RIP either theoretically or empirically. This study attempts to fill this gap. By extending Vives' (1995) model to the issue of price limits, we examine potential sources for the price distortion and find that when noise traders act in the market, the RIP may not hold. We demonstrate explicitly how the limits distort the return-generating process. These results imply that while the RIP is a practical tool for empirical studies of price limits, it may not hold when noise trading is present. In addition, our findings indicate that both the probability of price limits and the length of the limit sequence depend on the level of trading information.

The organization of this study is as follows. Section 2 describes the basic model; Section 3 incorporates the price-limit features into the basic model; Section 4 introduces numerical simulations and discusses the results; Finally, Section 5 provides a brief summary and concluding remarks.

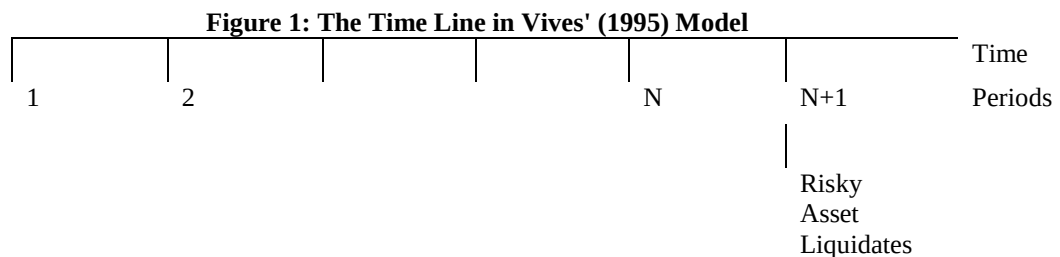
II. The Model

This section describes the model we use to investigate the conditions under which the RIP exists. Several features of our price-limit model are borrowed from Vives' (1995) model, which is not a price-limit model⁴. We will therefore begin our discussion with a general model for a no-price-limit market, and proceed later in Section 3 with a price-limit model.

A. General Model For a No-Price-Limit Market

A.1 The Assets

We consider an $N+1$ -period exchange economy with long-term agents. Trading occurs over the first N periods. Following Vives, we assume there are two types of assets in the market, a safe asset and a risky asset. The safe asset pays an unitary return. The risky asset liquidates at the end of Period $N+1$, and has a random fundamental value V , where V is normally distributed with a mean $\bar{V}$ and a variance of σ_V^2 . Figure 1 describes the time line of the model.



A.2 The Traders

We assume there are three types of traders in the market: informed traders, noise traders and competitive risk-neutral market makers.

• Informed Traders

There is a continuum of informed agents, indexed in the interval $[0,1]$. The informed traders are long-term informed agents, who maximize the expected utility of final wealth. Trader i has an exponential utility function, U , with a Constant Absolute Risk Aversion (CARA) coefficient, ρ :

$$U(W_{iN}) = -e^{-\rho W_{iN}} \quad \text{for} \quad \rho > 0. \quad (1)$$

The trader maximizes her utility from her wealth in Period N .

Each Period t , Trader i is endowed with a small piece of information (a private signal) about the liquidation value of the risky asset, V . The private signal (S_{it}) equals:

$$S_{it} = V + \varepsilon_{it} \quad , \quad (2)$$

where ε_{it} is the error term. It is normally distributed with a zero mean and a variance of σ_{ε}^2 . The terms V and ε_{it} are uncorrelated, and errors are also uncorrelated across agents and periods. S_{it} is therefore normally distributed with a mean $\bar{V}$ and a variance of $(\sigma_V^2 + \sigma_{\varepsilon}^2)$. The precision level of the signals $\tau_{\varepsilon_t} = (\sigma_{\varepsilon_t})^{-1}$ is the same across agents in the same period but may be different across periods. Vives mentions that a trade occurs in Period t only if $\tau_{\varepsilon_t} > 0$. He also considers two informational conditions: the case of the concentrated arrival of information and the case of a constant flow of information. In the case of the concentrated arrival of information, the precision level of signals is positive in the first period only (that is, $\tau_{\varepsilon_t} = 0$ for $t=2, \dots, N$). On the other hand, in the case of a constant flow of information the precision level of the signals is the same in all periods ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$ for all t).

In Period n , Trader i has a vector of private signals $S_i^n = (S_{i1}, \dots, S_{in})$ available. Informed agent i in Period n submits a demand schedule (a limit order) $X_{in}(\tilde{S}_{in}, P^{n-1}, \cdot)$ indicating the position desired at every price P_n^* , contingent on the information available (the sufficient statistic for the private information and the sequence of past prices $P^{n-1} = \{P_1^*, \dots, P_{n-1}^*\}$). In Period t , informed agent i buys or sells according to whether her/his private estimate of V (S_{it}) is larger or smaller than the market estimate, P_t^* .

Vives demonstrates that the demand function of the informed trader for the risky asset is:

$$X_t = \frac{E[(V - P) | S_n, P]}{\rho \text{Var}[(V - P) | S_n, P]} \quad . \quad (3)$$

As in other information models, the demand increases with the difference between the value and the price of the risky asset, and decreases with both the coefficient of the risk aversion and the variance of the difference between the value and the price of the risky asset.

Vives compares two trading models (“short-term” and “long-term”) in a dynamic model with asymmetric information. In the short-term model, informed traders have a short horizon and maximize the (expected) utility of the short-term return. In the long-term model informed speculators have long horizons and maximize the (expected) utility of consumption in the final period. In both models the quality of the information a speculator has at any point in time is the same.

- Noise Traders

Noise traders’ demand depends on the random variable, u_n , which is normally distributed with a zero mean and a variance of u , σ_u^2 . Expected trading volume increases with noise trading σ_u^2 . In fact, as is usual in this type of model, there is a trade because of the presence of noise traders and because informed agents have better information than risk-neutral market makers.

- Market Makers

Competitive risk-neutral market makers observe the noisy limit book schedule

$$B(P_{n+1}^*) = \int_0^1 X_{n+1}(\tilde{S}_{n+1}, P_{n+1}^*) di + u_{n+1} = z_{n+1} + \zeta(P_{n+1}^*), \quad (4)$$

and set the price efficiently: $P^* = E(V | L(\cdot))$, where ζ is a linear function of past and current prices, and the random variable z_{n+1} represents the net trading intensity of informed agents in period n . ζ can also be thought of as the new information in the current price filtered from the net aggregate action of informed agents.

A.3 Equilibrium

Proposition 1 characterizes the unique linear equilibrium, with long-term informed agents maximizing the expectation of the utility of final wealth⁵:

Proposition 1. At any linear equilibrium, $P_{N+1}^* = V$, and for $n = 1, \dots, N$:

$$P_n^* = \lambda_n Z_n + (1 - \lambda_n \Delta a_n) P_{n-1}^* \quad , \quad (5)$$

with:

$$\lambda_n = (\tau_u \Delta a_n) / \tau_n, Z_n = \Delta a_n V + u_n, \tau_n = \tau_v + \tau_u + a_n, \tau_v = \sigma^{-1} v, \tau_u = \sigma^{-1} u, a_n = \rho^{-1} \sum_{t=1}^n \tau_{\varepsilon_t},$$

$$\Delta a_n = \rho^{-1} \tau_{\varepsilon_n}, \text{ and } (1 - \lambda_n \Delta a_n) > 0$$

where a_n is the trading intensity of Period n informed agents.

Eq. (5) demonstrates two sources of price changes in the model: noise traders' demand, u_t , and informed traders' precision level, τ_{ε_t} . Recall that Vives considers two informational conditions: the case of the concentrated arrival of information and the case of a constant flow of information. In the case of the concentrated arrival of information, the precision level of signals is positive in the first period only (that is, $\tau_{\varepsilon_t} = 0$ for $t=2, \dots, N$). On the other hand, in the case of a constant flow of information the precision level of signals is the same in all periods ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$ for all t).

If private information is received only in the first period, $\Delta a_n = 0$, for $n \geq 2$, and there is no informed trading after the first period.

The equilibrium price-and-return expressions derived above allow us to investigate how a price-limit regime affects the equilibrium prices. Therefore, we will explore how each of the two sources of price changes in the model--noise traders' demand (u_t) and informed traders' precision level (τ_{ε_t})--affects the returns on the limit removal day (namely, the day the limit is removed, called here after the LRD) and on subsequent days after the limit has been removed.

III. Price Limits in the General Model

What type of price relationship should exist in a market regulated by daily price limits? A daily price-limit rule restricts all prices during each trading day to the previous day's closing price plus (minus) an up (down) limit. More formally, let P_t^M be Period t's market price rather than the equilibrium price (P_t^*).

The market price at any Period t is subject to up or down limits that equal L dollars, and is related to the equilibrium price as follows:

$$P_t^M = \begin{cases} P_{t-1}^M + L & \text{if } P_t^M \geq P_{t-1}^M + L \\ P_t^* & \text{if } P_{t-1}^M - L < P_t^* < P_{t-1}^M + L \\ P_{t-1}^M - L & \text{if } P_t^M \geq P_{t-1}^M - L \end{cases}, \quad (6)$$

where P_t^M and P_t^* are the market and equilibrium prices in Trading Period t, respectively, and P_{t-1}^M is the price in the last period. In a given Period t, if the equilibrium price of the risky asset is higher than the down limit ($P_{t-1}^M - L$) or lower than the up limit ($P_{t-1}^M + L$), the price limits will not be effective. On the other hand, if the equilibrium price of the risky asset is lower than the down limit ($P_{t-1}^M - L$) or higher than the up limit ($P_{t-1}^M + L$), trading stops, and a limited (market) price is determined. Trading restarts at a subsequent period in which, once again, the market price is bounded by the down and up price limits. Theoretically, the price limit can exist only for J trading days. Actually, however, the risky asset's return is bounded by the price limits for k trading days, $k \leq J$, and Day $t+k+1$ is the day the limit removal day.

Given that the market and equilibrium prices are different, we distinguish between the expected *theoretical* (equilibrium) return (R^*) and the expected *market* return (R^M). When no price limits exist, the two returns are identical. However, when a limit mechanism is present, the two returns can be different.

The Return Identity Proposition (RIP) asserts that the accumulated market return during these $k+1$ days equals the equilibrium return that would have been accumulated under no-limit conditions. The RIP is described by Proposition 2 below.

Proposition 2. If the Return Identity Proposition (RIP) exists, the accumulated market return $\left(\sum_{i=0}^{k+1} R^M_{t+i}\right)$ and the

accumulated equilibrium return $\left(\sum_{i=0}^{k+1} R^*_{t+i}\right)$ in a k -day limit sequence become equal; i.e.,

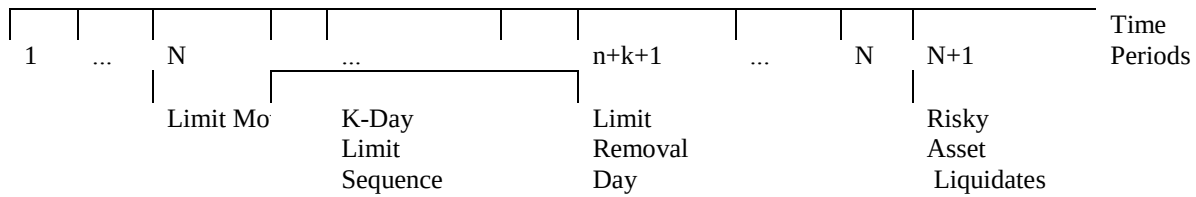
$$P^M_{t+k+1} = P^*_{t+k+1} \text{ and } \sum_{i=0}^{k+1} R^M_{t+i} = R^M_t + R^M_{t+1} + \dots + R^M_{t+k+1} = R^*_t + R^*_{t+1} + \dots + R^*_{t+k+1} = \sum_{i=0}^{k+1} R^*_{t+i}, \quad (7)$$

where R^M_i and R^*_i are the market and equilibrium returns in Period i , respectively.

The results for a k -day limit sequence, discussed in the following section, are general, and apply to any sequence.⁶

Figure 2 describes the time line of the model in the case of a limit regime.

Figure 2: The Time Line under a Price-Limit Regime



A. The Price-Limit Sources

In the following sections we will analyze the two cases of the information structure noted by Vives and cited in Section 2.1 above - the case of the concentrated arrival of information and the case of a constant flow of information. We will explore the causes for the limit move in each of the two cases. Eq. (5) identifies two sources of price changes in the model: noise traders' demand, u_t , and informed traders' precision level, τ_{ε_t} . Therefore, the limit move can stem from each of the two sources of price changes or from a combination of the two.

- When information arrives in concentrated form, informed traders receive private information only in the first period (that is, $\tau_{\varepsilon_t} = 0$ for $t = 2, \dots, N$). As a result, the limits can stem from both sources of price changes - noise traders' demand, u_t , and informed traders' precision level, τ_{ε_t} .
- When there is a constant flow of information, informed traders receive private information during all periods, with an equal precision level of signals during every period ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$ for all t). As a result, the limits arise due only to the noise traders' demand (u_t), but not due to the informed traders' precision level, τ_{ε_t} .

What effect do the two informational conditions have on the return-generating process? We will discuss that issue in the next sub-sections, using the assumption that informed traders receive positive signals in some Period t ($S^n_i > 0$), which cause an up-limit move.

B. The Case of a Concentrated Arrival of Information

The case of a concentrated arrival of information involves two sub-cases: (1) the case where the limits are due to both informed and noise trading; and (2) the case where the limits are due to informed trading only. We begin here with the first case.

B.1 Price Limits Due to Both Informed and Noise Trading

Suppose that in Period n the informed traders receive certain positive information (signal). In addition, the uninformed traders' demand is positive. As a result, both types of traders buy the asset. In a no-limit condition the price increases to its equilibrium level in Period n . Since the precision level of the signals in Period n and the subsequent periods (Period $n+1$ and further) is equal to zero, the informed trader does not trade during the periods that follow Period n . Assuming the noise traders' demand equals zero in any other period except Period n , there will be no trading following Period n .

In the case of a limit regime, however, the situation is different. Suppose that both informed and noise trading ($\tau_{\varepsilon_n}, u_n > 0$) in Period n caused a limit move for k periods, beginning in Period n . Suppose also that in the periods subsequent to Period n , informed and noise trading equal zero ($u_t = \tau_{\varepsilon_t} = 0$, $t = n+1, n+2, \dots$); i.e., the precision level of the signals received by the informed traders is equal to zero. As a result, Period n 's equilibrium price is higher than the market price, $P^*_n > P^M_n > P^*_{n-1}$, which in turn is greater than the equilibrium price in Period $n-1$, due to the up-limit move. Since the informed traders know what effect the information they received in Period n

should have caused, they take this information into consideration. As a result, in Period $n+k+1$ (the limit removal day), there will be a left over (LO_{n+k+1}), equal to the difference between Period $n+k$'s equilibrium price (P^*_{n+k}) and Period $n+k$'s market price (P^M_{n+k}). Since in the periods subsequent to Period n , informed and noise trading equals zero, Period n 's equilibrium price (P^*_n) and Period $n+k$'s market price (P^M_{n+k}) are equal. The left over is given by:

$$LO_{n+k+1} = P^*_n - P^M_{n+k} = \lambda_n Z'_n + (1 - \lambda_n \Delta a_n) P^*_{n-1} - P^M_{n+k} \quad (8)$$

In Period $n+k+1$, the left over causes a further increase in the market price of the risky asset, toward the equilibrium price. Note that we are dealing here with a case of an up-limit move. For the alternative case of a down-limit move, the left over in Eq. 7 will be negative. Since noise trading is uncorrelated across periods, the market price will be lower than the equilibrium price. The difference between the theoretical and the market prices remains,

$P^*_{n+k+1} = P^*_{n+k} = P^*_n > P^M_{n+k+1} > P^M_{n+k} > P^M_n$. This outcome is described by Result 1 below.

Result 1: *If the limit move is due to both informed and noise trading as well as the concentrated arrival of information, there will be a gap between the equilibrium price and the market price during the limit removal period. Furthermore, the gap will remain constant over a period of time. In other words, the RIP will not hold in this case.*

B.2 Price Limits Due to Informed Trading Only

In the previous case (in Section 3.2.1) the limits stem from two sources of price changes – noise traders' demand and informed trading. What happens, however, when only the informed trading triggers the limit move? Suppose the signals received by the informed traders are positive and that the precision level of the signals is positive in the first period only (that is, $\tau_{\varepsilon_1} > 0$ and $\tau_{\varepsilon_t} = 0$ for $t=2, \dots, N$). Assume further that the noise traders' demand equals zero during all periods; $u_t = 0$ for $t=1, \dots, N$. As a result, the price of the risky asset in Period n increases.

In the case of a no-limit regime, Period $n+1$'s equilibrium price (P^*_{n+1}) does not change. In other words, it is equal to Period n 's equilibrium price (P^*_n). The reason for such a pattern is that in Period $n+1$ the trading intensity is the same as in Period n , $a_n = a_{n+1}$. Therefore, as Eq. (4) implies, no trading affects the equilibrium price.

On the other hand, in the case of a price-limit regime, the limit exists for k periods. Suppose the private information that the informed traders received in Period n caused a limit move in Period n . Given that the informed traders know what effect the concentrated arrival of information should have caused, there would be a left over (LO_{n+k+1}), equal to the difference between Period n 's equilibrium price (P^*_n) and Period $n+k$'s market price (P^M_{n+k}), that will affect Period $n+k+1$'s market price (P^M_{n+k+1}) and be given by Eq. (8) above. The effect of the left over causes Period $n+k+1$'s market price (P^M_{n+k+1}) to increase to its Period n 's equilibrium value (P^*_n). This outcome is represented by Result 2 below.

Result 2: *If the limit move is due to informed trading and a concentrated arrival of information, the equilibrium price and the market price will be equal during the limit removal period, implying the existence of the RIP.*

C. The Case of a Constant Flow of Information

In the preceding section (Section 3.2) we discussed the first type of informational situation – the case of a concentrated arrival of information. This section deals with the second kind of informational situation - the case of a constant flow of information. In such a case the informed traders receive positive signals during all periods, with an equal precision level of signals during every period ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$ for all t). We emphasize here that the flow of information is constant in every period, and thus its impact is identical. Assuming the information that the informed traders receive is positive, the risky asset's price will increase gradually from period to period. If in some Period n the noise traders' demand is positive, the price will increase further. The equilibrium price in Period n equals P^*_n .

The derivative of Eq. (5) with respect to P^*_n becomes:

$$\partial P^*_{n+1} / \partial P^*_n = 1 - \lambda_{n+1} \Delta a_{n+1} \quad (9)$$

It then follows that there are two sub-cases regarding Period $n+1$'s price. The first is the case where the term " $1 - \lambda_{n+1} \Delta a_{n+1}$ " in Eq. (5) is positive, while in the second case, this term is negative. In the first case, when a limit regime exists, the price after the limit has been removed continues to increase. The subsequent increase stems from the fact that the noise traders' demand during the limit-move period causes a price increase that is lower than the magnitude of the increase that would have been attained, given the informed traders' information.

In the second case where the term " $1 - \lambda_{n+1} \Delta a_{n+1}$ " in Eq. (5) is negative, the price on the limit removal day decreases. The decrease stems from the fact that the noise traders' demand during the limit-move period causes the price to increase by more than it should have increased given the informed traders' information.

We discuss here only the second case, where the term " $1 - \lambda_{n+1} \Delta a_{n+1}$ " is negative, because the conclusion as to whether the RIP exists is the same in both cases. In the case of a price-limit regime, the limits restrict the risky asset's price to a certain limit, and therefore restrains the move. Since a constant flow of information has an equal effect on the price in every period, a limit move becomes possible only in the presence of a noise trader's demand. As a result, Period n 's market price (P_n^M) becomes lower than the equilibrium price, and continues to increase toward the equilibrium price (P_n^*) during the k limit days.

The sort of trading strategy employed on the limit removal day will depend on whether the traders are informed traders or noise traders. The former know what effect the constant flow of information should have caused. They therefore know that there is a left over (LO_{n+k+1}), equal to the difference between Period n 's equilibrium price (P_n^*) and Period $n+k$'s market price (P_{n+k}^M) that will affect Period $n+k+1$'s market price (P_{n+k+1}^M). This left over is expressed by Eq. (8) above.

The left over causes Period $n+k+1$'s market price (P_{n+k+1}^M) to increase to its Period n 's equilibrium value (P_n^*). Since the price increases by more than what the informed traders expected, they will sell the risky asset. Unlike informed trading, noise trading is equal to a random variable u_n , and is not correlated across periods, nor with the limit move. In other words, the noise traders do not change their demands when the limit is removed. To illustrate, differentiating Period $n+1$'s price, given by Eq. (5), with respect to the uninformed traders' demand (u_n) produces zero ($\partial P_{n+1}^* / \partial u_n = 0$). Due to the constant flow of information, the informed trading is constant. As a result, the limit move can stem only from uninformed trading. Since the uninformed demand is a random variable, it is uncorrelated across periods. Consequently, when a limit regime is present, the market price will fall on the limit removal day, and the limit move will be described by Result 3 below:

Result 3: *If the limit move is due to noise trading, the limits will cause the RGP of the theoretical price and the RGP of the market price to deviate; consequently, the RIP will not hold.*

In the following sections, we will demonstrate Results 1, 2 and 3 described above employing numerical simulations that test the model.

IV. Numerical Simulations

The model's main result derived above (and discussed in the next section) is that when there are informed traders who expect the limit move, the RIP will hold. We will first explain the empirical difficulties associated with testing the theoretical model, and then present numerical simulations.

A. The Difficulties Associated With Testing the Model

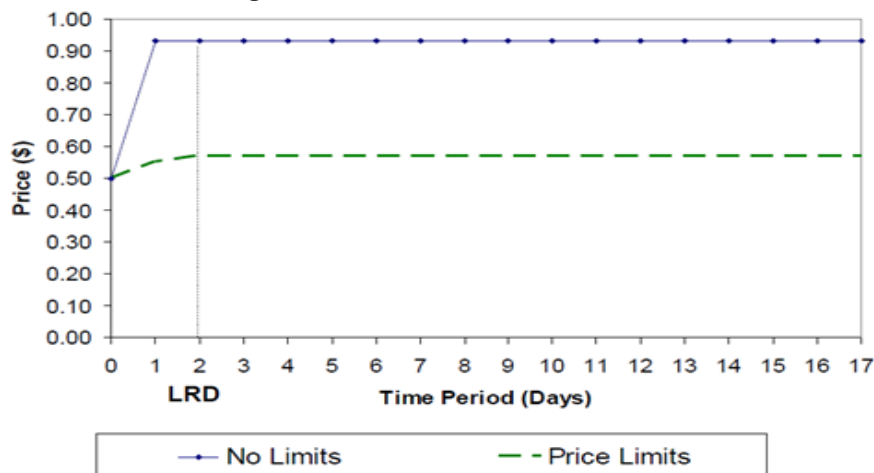
An empirical investigation of the RIP involves a comparison of the risky asset's return under conditions of price limits with the return of the same asset under conditions of no limits. This comparison, however, is empirically problematic. Since, in any time period, securities exchanges adopt only one of the two regimes (limit or no-limit), it is impossible to obtain the two returns simultaneously. The theoretical model presented above raises a second estimation problem. The model is based on the private signal of informed traders. The measurement procedure for this variable is not simple. There is a major problem in estimating the private information each trader has. Most prior studies, including Vives' (1995), that investigated the information issue did not conduct an empirical test. Instead, they used numerical simulations to test the models presented in those studies. This approach is adopted here as well.⁷

B. Price Limits Due to Both Informed and Noise Trading in the Case of a Concentrated Arrival of Information

When the effect of price limits is due to both informed and noise trading in the case of a concentrated arrival of information, Result 1 described above will follow implying that the RIP will not hold. To demonstrate the effect of price limits in this case, we have used the following simulation. To make Vives' no-limit model compatible with our limit model, most of the parameter values used in the simulation are identical to those of Vives (1995) and are as follows⁸: $\rho = 2.5$, $\sigma_\varepsilon^2 = 1.05$, $\sigma_u^2 = 0.05$, $\sigma_v^2 = 1$, and $V = 1$. Recall that the notation is as follows: ρ is the constant coefficient of absolute risk aversion; σ_ε^2 is the variance of the error term; σ_u^2 is the variance of the random variable u_n on which the demand of noise traders' depends and V is the random fundamental value whose variance is σ_v^2 . For the price-limit case, we adopted a limit of 0.05 dollars ($L = \$0.05$), and for the Period $n-1$ equilibrium price we used the value of $\$0.50$ ⁹. We then assumed that a concentrated arrival of

information had occurred in Period 1, with a trading intensity (a_n) of 0.38, and a trading intensity of zero in Period 3. Furthermore, we assumed that ($a_{n+1} = 0$ for $n > 2$). Substituting these parameter values stated above in Eq. (5) yields a 1-day limit sequence. In other words, only one period is bounded by a limit move. The results are essentially unchanged when the limit sequence exceeds one day; i.e., for an N-day limit sequence. In addition, we assumed that $u_2=1$, and the noise trading of all other periods equals zero. As a result, the equilibrium prices of any period following Period 1 were equal to \$0.93, whereas the market price was \$0.55 in Period 2 and \$0.57 in Period 3 and later. The simulation results are demonstrated in Figure 3, where the vertical axis represents the prices and the horizontal axis denotes the time period. Note that the main simulation results implied by Figure 3 remain unchanged even if we use other parameter values that are different than those that Vives (1995) employed here. As Result 1 implies, if the limit moves are due to both informed and noise trading and the concentrated arrival of information, the equilibrium and market prices will deviate. Furthermore, this deviation will remain over a period of time. Consequently, the RIP does not hold in this case.

Figure 3: The Observed and Theoretical Prices in the Case of Price Limits due to Both Informed and Noise Trading and a Concentrated Arrival of Information



Notes: Figure 3 demonstrates the case of a limit move due to both informed and noise trading and a concentrated arrival of information. The vertical axis and the horizontal axis represent the price and the time period, respectively. LRD denotes the limit removal day. The results here are based on the numerical simulation in Section 4.2. They indicate that the equilibrium and market prices will deviate when the limit is removed and that the deviation will continue over time. Consequently, as implied by Result 1, the RIP does not hold in this case.

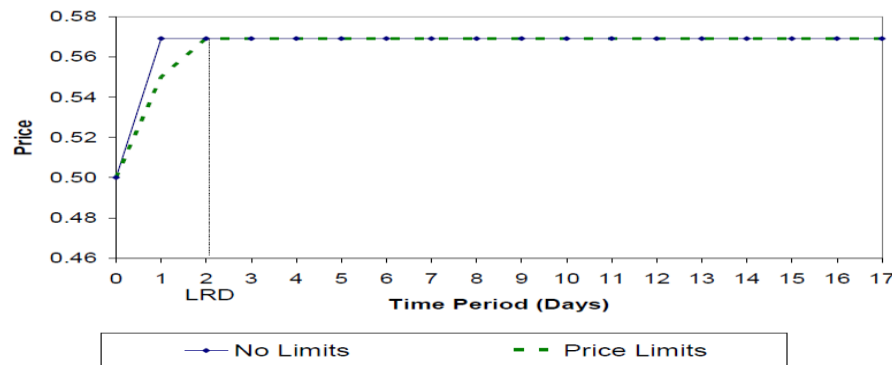
C. Price Limits Due to Informed Trading Only and a Concentrated Arrival of Information

In the case of a concentrated arrival of information, and price limits that are due to informed trading only, Result 2 will follow and the RIP will hold. To demonstrate the effect of price limits in this case, we have used a simulation similar to the one above. As in the preceding case (in Section 4.2), we used the following parameter values: $\rho = 2.5$, $\sigma_\varepsilon^2 = 1.05$, $\sigma_u^2 = 0.05$, $\sigma_v^2 = 1$, $V = 1$, $L = \$0.05$; and Period 1's equilibrium price equals 0.50 dollars. We then assumed that in Period 2 a concentrated arrival of information had occurred, with $a_2 = 0.38$ and $a_{n+1} = 0$ ($n > 2$). The value of zero for the trading intensity (i.e., the "a" coefficient) was used for any other periods. As a result, Period 2's equilibrium price equals \$0.57, whereas Period 2's market price equals \$0.55. Period $n+1$'s left over equals \$0.02, causing Period 3's market price to be \$0.57, which is identical to the equilibrium price of Period 3.

The positive return on the limit removal day (LRD) stems from the informed traders' reaction. As demonstrated in Eq. (3), the value of the risky asset is greater than its price, $E[(V - P) | S_n, P] > 0$. Therefore, the informed traders will buy the asset ($X_t > 0$) on the LRD, and the price will continue to increase.

The simulation results are demonstrated in Figure 4, where the vertical axis represents the prices and the horizontal axis – the time period. As Result 2 implies, if the limit move is due to informed trading and the concentrated arrival of information, the equilibrium and market prices will be equal when the limit is removed, implying that the RIP will hold.

Figure 4: The Observed and Theoretical Prices in the Case of Price Limits due to Informed Trading and a Concentrated Arrival of Information



Notes: Figure 4 demonstrates the case of a limit move due to informed trading and a concentrated arrival of information. The vertical axis and the horizontal axis represent the price and the time period, respectively. LRD denotes the limit removal day. The results here are based on the numerical simulation in Section 4.3. The figure indicates that, as implied by Result 2, the equilibrium price will equal the market price on the LRD. Thus, the RIP holds.

D. Price Limits Due to Noise Trading in the Case of a Constant Flow of Information

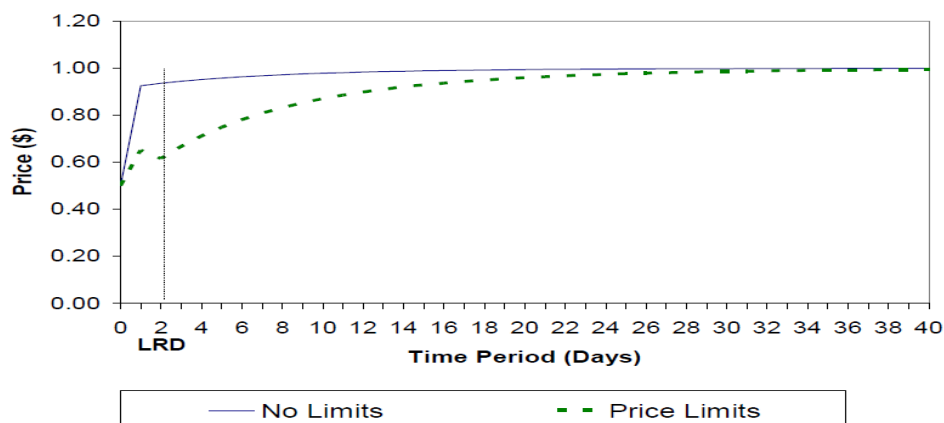
When there is a constant flow of information and price limits that are due to noise trading, the RIP, as shown in Result 3 described above, will not hold. Here, too, we employed essentially the same parameter values used in the preceding two cases. We have assumed that a constant flow of information occurs, with $a_n = a_{n+1} = 0.38$ (recall that the value of a_{n+1} in the preceding two cases was 0).

In addition, we assumed that $u_n = 1$, and the noise trading in all other periods equals zero. As a result, Period n's equilibrium price equals \$0.9242, whereas Period n's market price equals \$0.65. In the subsequent periods, both prices continue to increase.

As Result 3 predicts, the price limits reduce the overall price volatility in the entire sequence of N trading days, but increase the price volatility in the sequence that excludes the limit-move days. For the parameter values used in our simulation for the periods beginning in Period n, the variance of the equilibrium return is equal to 1.79%, whereas the variance of the market return is equal to 0.26%. In other words, the limits reduce the overall volatility. For the periods subsequent to the limit-move period, the variance of the equilibrium return is equal to 0.0008%, whereas the variance of the market return is equal to 0.0478%. In other words, the limits increase the price volatility in the periods subsequent to the limit-move day.

The simulation results are demonstrated in Figure 5, where the vertical axis represents the price and the horizontal axis – the time period. As Result 3 predicts, if the limit move is due to noise trading and a constant flow of information, the equilibrium and market prices will not be equal when the limit is removed, and the RIP will not hold.

Figure 5: The Observed and Theoretical Prices in the Case of Price Limits due to Noise Trading in the Case of a Constant Flow of Information



Notes: Figure 5 demonstrates the case of a limit move due to noise trading and a constant flow of information. The vertical axis and the horizontal axis represent the price and the time period, respectively. LRD denotes the limit removal day. The results here are based on the numerical simulation in Section 4.4. As implied by Result 3, the figure shows that when the limit is removed, the equilibrium price is not equal to the market price, implying that the RIP does not hold in this case.

The negative return on the LRD stems from the informed traders' reaction. As demonstrated by Eq. (3), the value of the risky asset is lower than its price, $E[(V - P) | S_n, P] < 0$. Therefore, the informed traders will sell the asset ($X_t < 0$) on the LRD, and the price will drop.

E. Discussion

Employing a simulation approach, we have explored above two information structures – the case of a concentrated arrival of information and the case of a constant flow of information – and two price-change sources – informed and uninformed trading. We have presented three results (summarized in Table 1) that follow from a positive signal that causes an up-limit move.

We have shown that the basic condition under which the RIP exists is dependent upon the cause for the limit move. If, on one hand, the limit move stems from noise trading, the RIP will not exist, as outlined in Results 1 and 3 above. If, on the other hand, the limit move is due to informed trading, the RIP will exist, as stated by Result 2 above. These general results also apply to the opposite case (not discussed here) of a negative signal and a down-limit move. The essence of the results remains unchanged if a negative signal, which causes a down-limit move, is assumed.

Table 1: The Three Main Results of the Study

		Limits Due to			RIP
		Result	Informed trade precision level	Noise trade demand	
Information	Concentrated arrival of information	1	√	√	Does Not Exist
		2	√	N.A.	Exists
Structure	Constant flow information	3	N.A.	√	Does Not Exist

Notes: Table 1 summarizes the existence of RIP in the circumstances described in Results 1, 2 and 3 discussed in Section 4.5. The sign "√" describes the causes for the limit. For example, the sign "√" under the column "informed traders' precision level" means that the informed traders' precision level causes the limit move when there is a concentrated arrival of information. N.A. (not applicable) denotes that there is no limit cause. For example, N.A. under the column "informed traders' precision level" means that the informed traders' precision level does not cause the limit move when there is a constant flow of information.

Our numerical simulations have demonstrated that the characteristics of the limit sequence are different for limits that are due to noise trading in the case of a constant flow of information (Section 4.5) compared with the other two cases - the case of limits due to both informed and noise trading and a concentrated arrival of information (Section 4.3), and the case of limits due to informed trading and a concentrated arrival of information (Section 4.4). The length of the sequence and the probability of a price reversal increase in the case of price limits due to noise trading accompanied by a constant flow of information. In other words, the asset's price depends on the level of the information the traders have, leading to the following result:

Result 4: *As the level of noise trading increases, the length of the sequence and the number of reversals increase.*

V. Conclusions

This study has investigated the impact of price limits on the equilibrium return of a risky asset. According to the Return Identity Proposition (RIP), there is an equality between the aggregate market returns during a k-day limit sequence and the aggregate theoretical returns during those periods. Therefore, price limits should not have an effect on prices. Several researchers adopt this approach and assume in their empirical works that the RIP holds.

In light of the existence of only a few theoretical works on price limits, we have developed a model of asymmetric information that we use to examine the conditions under which the RIP may or may not exist. Based on Vives (1995), we have demonstrated three results that may occur. Result 1 indicates that if the limit move is due to both informed and noise trading as well as a concentrated arrival of information, there will be a gap between the equilibrium price and market price in the limit removal period. Furthermore, the gap will remain constant over a period of time, meaning that the RIP will not hold. According to Result 2, if the limit move is due to informed trading and a concentrated arrival of information, the equilibrium price and market price will be equal in the limit removal period, and the RIP will hold. According to Result 3, if the limit move is due to noise trading, the limits will cause the return generating process (RGP) of the theoretical price and the RGP of the market price to deviate, and the RIP will not hold in this case.

To test these hypothesized results, we have conducted simulations based on Vives' (1995) parameter values. The results confirm our three results, even if different parameter values are employed.

The findings indicate that stocks involving a higher level of noise trading also involve a higher frequency of the following characteristics: (1) limit days, (2) long limit sequences and (3) price reversals on the limit removal day (LRD). The findings also imply that the volatility of returns for both the LRD and the limit sequence is statistically higher for stocks involving a higher level of noise trading. For these stocks we also found that the return on the LRD is negative for an up-limit sequence involving a lower level of noise trading. These findings appear consistent with the simulation's results.

One implication of the model developed here is that by adopting a price-limit regime, securities exchanges can cause a deviation of the return-generating process of the market return from the original process. Another implication of our results is that while the RIP can be a practical tool for use in empirical studies of price limits, it is also important to include cases where the RIP may not hold in these studies.

Given the deviation of the RGP of the market return from the theoretical return, we suggest that future research attempt to determine which (price-limit) empirical studies are more appropriate—those attempting to estimate the *market* return or those attempting to estimate the *theoretical* return. Another potentially interesting future study might test the existence of the RIP when uninformed traders use a model of time variance (similar to a GARCH model) in formulating their expectations. Finally, we suggest that future research should focus on finding an empirical estimator for returns on the LRD, both in the presence and absence of the RIP.

References

- Abad, D and Roberto, P., 2007, "On the Magnet Effect of Price Limits," *European Financial Management*, 13, 833-852.
- Admati, A. R., 1985, "A Noisy Rational Expectations Equilibrium For Multi-Asset Securities Markets," *Econometrica*, 53, 629-657.
- Arak, M. and Cook, R.E., 1997, "Do Daily Price Limits Act as Magnets? The Case of Treasury Bond Futures," *Journal of Financial Services Research*, 12, 5-20.
- Blume, L., and Easley, D., 1992, "Evolution and Market Behavior," *Journal of Economic Theory*, 58, 9-40.
- Brennan, J. M., 1986, "A Theory Of Price Limits In The Futures Markets," *Journal of Financial Economics*, 16, 213-233.
- Brennan, J. M. and Cao, H. H., 1997, "International Portfolio Investment Flows," *Journal of Finance*, 52, 1851-1880.
- Chance, M.D., 1994, "Futures Pricing And The Cost Of Carry Under Price Limits," *The Journal of Futures Markets*, 14, 813-836.
- Chou, P.H., 1999, "Modeling Daily Price Limits," *International Review of Financial Analysis*, 8, 283-301.
- Chou, P.H., Lin, M.C. and Yu, M.T., 2000, "Price Limits, Margin Requirements, And Default Risk," *The Journal of Futures Markets*, 20, 573-602.
- Chou, P.H., Lin, M.C. and Yu, M.T., 2003, "The Effectiveness Of Coordinating Price Limits Across Futures And Spot Markets," *The Journal of Futures Markets*, 23, 577-602.
- Chowdhry, B. and Nanda, V., 1998, "Leverage And Market Stability: The Role Of Margin Rules and Price Limits," *Journal of Business*, 71, 179-210.
- Fernandes, M. and Marco A. R., 2007, "Are Price Limits on Futures Markets that Cool? Evidence from the Brazilian Mercantile and Futures Exchange," *Journal of Financial Econometrics*, 5, 219-242.
- Grossman, S. and Stiglitz, J.E., 1980, "On The Impossibility Of Informationally Efficient Markets," *American Economic Review*, 70, 393-408.
- Harel, A., Harpaz, G. and Yagil, J., 2005, "Forecasting Futures Returns In The Presence Of Price Limits," *The Journal Of Futures Markets*, 25, 199-210.
- Harel, A., Harpaz, G. and Yagil, J., 2010, "A New Paradigm For Forecasting Security Returns in a Market Regulated by Price Limits", *Review of Quantitative Finance and Accounting*, 35, 113-121.
- Hsieh, P. H and Yang, J. J., 2009, "A Censored Stochastic Volatility Approach to the Estimation of Price Limit Moves," *Journal of Empirical Finance*, 16, 337-351.
- Huang, Y.S., Fu, T.W., and Ke M.C., 2001, "Daily Price Limits and Stock Price Behavior: Evidence from the Taiwan Stock Exchange," *International Review of Economics and Finance*, 10, 263-288.
- Kim, A. K. and Rhee, G.S., 1997, "Price Limit Performance: Evidence From the Tokyo Stock Exchange," *Journal of Finance*, 52, 885-899.
- Kim, A. K., 2001, "Price Limits and Stock Market Volatility," *Economics Letters*, 71, 131-136.
- Kim, Y.H., and Yang, J.J., 2008, "The Effect of Price Limits on Intraday Volatility and Information Asymmetry," *Pacific-Basin Finance Journal*, 16, 522-538.
- Kodres, L.E. and O'Brien, P.D., 1994, "The Existence Of Pareto-Superior Price Limits," *American Economics Review*, 84, 919-932.
- Kodres, L.E. and Pritsker, M., 2002, "A Rational Expectations Model Of Financial Contagion," *Journal of Finance*, 57, 769-799.
- Lee, J.H. and Chou, R.K., 2004, "The Intraday Stock Return Characteristics Surrounding Price Limit Hits," *Journal of Multinational Financial Management*, 14, 485-501.
- Levy, T. and Yagil, J., 2005, "Observed Vs. Theoretical Prices Under Price Limit Regimes," *Journal Of Economics And Business*, 57, 208-237.
- Levy, T. and Yagil, J., 2006, "An Empirical Comparison of Price-Limit Models," *International Review of Finance*, 6, 157-176.
- Morgan, I.G. and Trevor, R.G., 1999, "Limit Moves As Censored Observations Of Equilibrium Futures Price In GARCH Processes," *Journal Of Business And Economic Statistics*, 17, 397-407.
- Naughton, T., 2004, "Are Price Limits Priced? Evidence from the Taiwan Stock Exchange," *Journal of Emerging Market Finance*, 3, 249-267.
- Park, H.W., 2000, "Examining Futures Price Changes And Volatility On Trading Day After A Limit-Lock Day," *The Journal of Futures Markets*, 20, 445-466.
- Roll, R., 1984, "Orange Juice And Weather," *American Economic Review*, 74, 861-880.
- Routledge, B.R., 1999, "Adaptive Learning In Financial Markets," *Review of Financial Studies*, 12, 1165-1202.
- Seaholes, M. S. and Wu, G., 2007, "Predictable Behavior, Profits, and Attention," *Journal of Empirical Finance*, 14, 590-610.
- Subrahmanyam, A., 1994, "Circuit Breakers And Market Volatility: A Theoretical Perspective," *The Journal Of Finance*, 49, 237-254.
- Sutrick, K.H., 1993, "Reducing The Bias In Empirical Studies Due To Limit Moves," *The Journal of Futures Markets*, 13, 527-543.
- Vives, X., 1995, "Short-Term Investment And The Informational Efficiency Of The Market," *Review of Financial Studies*, 8, 125-160.
- Wang, J., 1993, "A Model Of Intertemporal Asset Prices Under Asymmetric Information," *Review of Economic Studies*, 60, 249-282.
- Wang, J., 1994, "A Model Of Competitive Stock Trading Volume," *Journal of Political Economy*, 102, 127-168.
- Yang, S. R. and Brorsen, B. W., 1995, "Price Limit As An Explanation Of Thin-Tailedness In Pork Bellies Futures Prices," *The Journal of Futures Markets*, 15, 45-59.
- Yong H. K., Yagüe J. and Yang, J. J., 2008, "Relative Performance of Trading Halts and Price Limits: Evidence from the Spanish Stock Exchange," *International Review of Economics and Finance*, 17, 197-215.
- Wei, S.X., 2002, "A Censored-GARCH Model Of Asset Returns With Price Limits," *Journal of Empirical Finance*, 9, 197-223.

Endnotes

1. The terms “theoretical” and “equilibrium” will be used here interchangeably.
2. Sutrick (1993) investigates ways to reduce the estimation bias by estimating the equilibrium prices of assets traded under a price-limit regime. Yang and Brorsen (1995) support the hypothesis that price limits cause the thin-tailedness observed in the distributions of pork bellies futures. Kim and Rhee (1997) formulate and verify the delayed price discovery hypothesis using daily data from the Tokyo Stock Exchange (according to this hypothesis the limits slow down the price discovery process by preventing prices from effectively reaching their theoretical values). Park (2000) also documents the existence of the delayed price discovery hypothesis in futures markets by testing the effect of price limits on futures prices, using futures contracts traded on the Chicago Board of Trade (CBOT). Kim (2001) finds that price limits usually do not change volatility. Wei (2002) estimates the return-generating process using a censored-GARCH model.
3. Brennan (1986) was the first to infer that price limits can serve as a partial substitute for margin requirements. Kodres and O’Brien (1994) claim that price limits partially insure implementation risk. Chowdhry and Nanda (1998) provide a result similar to that of Brennan (1986). Chou, Lin and Yu (2000) broaden Brennan’s (1986) model to two periods, and Chou, Lin and Yu (2003) extend Brennan’s (1986) model to investigate whether the imposition of spot price limits can further reduce the default risk. Harel, Harpaz and Yagil (2005 and 2010) develop forecasting models in the presence of return limits and provide some numerical predictions. Finally, Levy and Yagil (2005) discuss the S-shape relationship between observed and theoretical asset prices in markets governed by a price-limit mechanism.
4. Grossman and Stiglitz’s (1980) model, which analyzes information acquisition under conditions of asymmetric information, was extended by Vives (1995). Grossman and Stiglitz’s model was first extended to a multiple-asset setting by Admati (1985) and has since been extended in a number of directions by others. Admati (1985) considers a continuum of investors who have diverse private information. Blume and Easley (1992) consider an infinitely repeated version of the model. Others have extended the model to a dynamic setting with either a single risky asset (Wang, 1993, 1994), or with multiple risky assets (Brennan and Cao, 1997). Routledge (1999) considers the adaptive learning of the traders. Kodres and Pritsker (2002) extend the model to explain financial market contagion.
5. Proposition 1 corresponds to Vives’ Propositions 2.1 and 4.1.
6. A sequence higher than a two-day limit is quite rare. For example, both Morgan and Trevor (1999) and Wei (2002) examined T-bills futures traded on the CME for the period from 1979 to 1982. Using the same sample, they observed 57 limit days, only 20 of which were two-day consecutive limit moves and none was beyond a two-day limit sequence.
7. Numerical simulations have been used extensively in studies of asymmetric-information models (e.g. Grossman and Stiglitz, 1980) and in the price-limit literature (e.g. Chance, 1994).
8. We also used an alternative set of parameter values and the main results remain unchanged.
9. Changing Period $n-1$ ’s equilibrium price (P^*_{n-1}) does not change the results.
10. Though the effects of price limits have been tested empirically before (e.g., Hsieh and Yang, 2009) the RIP, as stated above, has not been investigated yet.

Acknowledgments

We would like to thank the seminar participants at the University of Haifa and Ben-Gurion University for their comments and suggestions on an earlier draft of the paper. Remaining errors are our responsibility.