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An Algorithm for Improving the Dynamic Accuracy in Systems for Measuring the Parameters of Moving Objects

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Abstract: The present paper considers an algorithm for improving the dynamic accuracy of measuring systems which determine the parameters of moving objects. The algorithm is an additional tool that provides accuracy to systems that use a gyro-free principle for measuring the parameters determining the deviation of the moving object from the vertical in dynamic mode. It is based on the Kalman algorithm underlying the proposed dynamic mode of the sensitive element of the measuring system. As a result of the algorithm operation the influence of a number of inertial effects of random nature is eliminated. Thus the measurement accuracy in dynamic mode is improved without the use of expensive stabilizing elements and systems. The presented experimental results prove the effectiveness of the proposed algorithm in terms of the dynamic accuracy of similar measuring systems.

Keywords: metrology, dynamic accuracy, Kalman filter, measuring systems

I. Introduction

One of the most quickly advancing areas in the contemporary Stochastic Control Theory is the Filtration Theory or the Estimation Theory [1]. Two interrelated applied problems are known: that for optimal estimation and that for optimal control of physical processes in the presence of random disturbances and random errors in the measuring devices [2]. The estimation problem requires an approximation of the process behavior according to data from low-accuracy measurements. The control problem involves determination of the input control effects of the process under consideration to achieve the objective set. These two problems are combined on the basis of similar mathematical methods used in their analysis.

Furthermore, the stochasticity in the system behaviour under the conditions of the two problems is an important feature characterizing signal processing. This behavior is caused by the presence of disturbance sources of random characteristics and any type of secondary (in terms of the solution of the problem) processes of unpredictable behaviour.

The optimal estimation problem can also be viewed as a metrological problem that searches for an accurate measurement result and can be solved, to a great extent, by the methods of the Automatic Control Theory. In this context the problems related to the provision of dynamic accuracy in systems measuring the parameters of moving objects can be analyzed. The operating conditions of those measuring systems are mainly determined by the characteristics of the inertial forces and moments caused by the motion of the moving objects. The latter could be aircrafts, ships, road and off-road transport means. The motion characteristics of the moving objects listed above are different but a common structure of the proposed algorithm is used. In order to completely solve similar problems, it is necessary to specify the parameters of the quantities involved in the measuring procedure. Therefore the concept of the algorithm will be based on the existing conditions upon measuring the parameters defining the position of a ship against the sea surface, such as heel and trim.

The above mentioned inertial forces and moments create conditions under which the sensitive elements of the measuring instruments mounted on the ship hull operate in an inertial field of force. Under the influence of the inertial field of force the sensitive elements change their laws of motion [3]. Some energy is accumulated thus changing their forecast motion. This energy determines the natural fluctuations of the system. However, usually some external energy is continuously passed to the system as well. It defines the forced fluctuations. In this way the actual motion of the instrument sensitive elements may considerably differ from the nominal one thus leading to inaccuracy of the measurement result. This inaccuracy is quantitatively represented by the dynamic error. Actually, this is one of the components of the dynamic error. However, within the instruments used for measuring the parameters of moving objects it is the basic component of this error and often reaches such high values that it could be identified with the total dynamic error, i.e. the other components have inconsiderable influence on its formation. Due to its origin this component of the dynamic error is often referred to as an inertial component. Studies in this area show [4, 5] that if there are not satisfactory solutions in the metrological procedures of the

measuring instruments, the inertial components can reach high values which lead to uncertainty of the measurement result.

Within measurement engineering there are a number of methods and tools for eliminating the inertial component of the dynamic error from the measurement result. Some of them have a sophisticated design, which results in raising the share of the instrumental errors, reducing reliability, and increasing the costs of the respective measuring instrument [6]. Others are accuracy-efficient but only within a narrow range of change of the quantity under measurement.

An overall solution to these problems can be found by means of the approach proposed in [7]. Unlike the existing measurement methods and tools which use stabilization of the vertical in the inertial space, the proposed approach is based on methods where the dynamic error is eliminated in real time. However, it is necessary to introduce an additional procedure in order to protect the measuring instrument from the presence of disturbance sources of random characteristics and additional secondary processes of unpredictable behaviour. Otherwise additional errors will be superimposed on the output signal thus reducing the accuracy of the measurement result.

Therefore the aim of the present paper is to develop and present an algorithm for eliminating the influence of the above mentioned additional disturbance sources. This algorithm is designed to improve the dynamic accuracy in systems used for measuring the parameters of moving objects.

II. A structural model of the algorithm

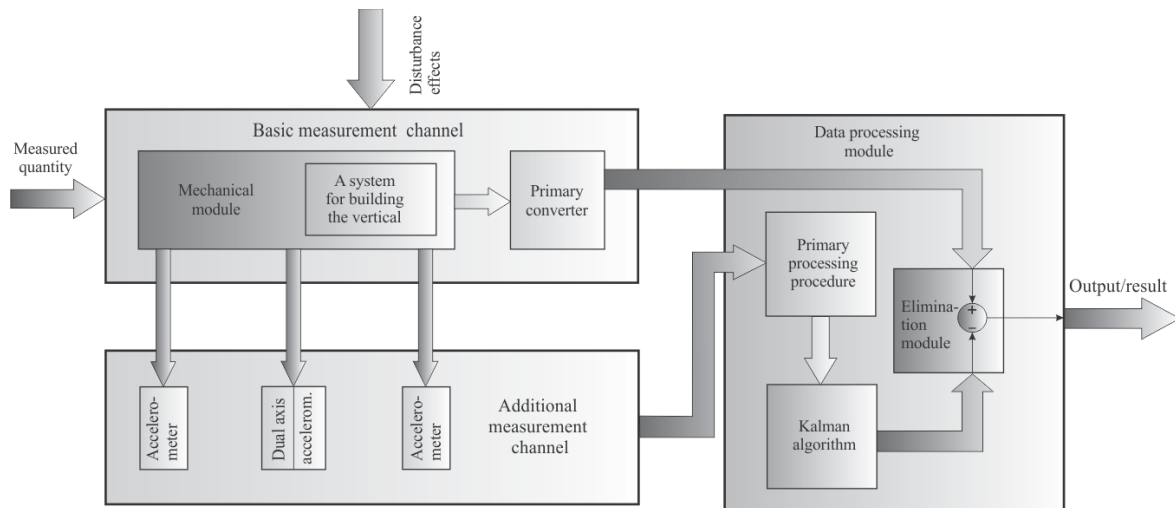
The most appropriate way to develop the algorithm is to apply the methods used for the solution of the optimal estimation problem [8]. In this case the measurement result is linked to a particular criterion that minimizes the error, which is actually the basic requirement, as a quality criterion, towards measuring systems. On the other hand, the development of the algorithm on this base is related to the application of the methods of the Automatic Control Theory.

The automatic control systems and the measuring systems differ in structure since the latter have a primary transformer at its input, whose input signal is inaccessible for both immediate measurement and correction. Measuring system, as a whole, do not make possible the provision of output-input feedbacks in their metrological circuits. Therefore it is not possible to immediately use the results from the automatic control methods in the measuring systems. However, additional measuring channels operating in parallel with the basic channels can be connected to the metrological circuits. The additional channels may possess the specific structure of correcting devices used for eliminating the dynamic error. Within them the concepts of a number of methods from the automatic control theory can be effectively used.

Taking into consideration the characteristics of the quantities that build the measuring environment where the measuring systems operate, it can be admitted that the most appropriate form of processing data obtained from such additional measuring channels is the Kalman algorithm. The characteristics of this algorithm fit very well into the solution of a number of problems arising in the process of accuracy optimization of the measuring instruments that define the parameters of all moving objects listed above [9]. In that case the algorithm will be developed in the context of the set objective, which is related to the improvement of the dynamic accuracy of the measuring system presented in [7]. This procedure is part of the additional channel and aims at eliminating the influence of a number of secondary disturbances on the measurement result. However, the algorithm can also be used in many other systems operating in dynamic mode in the area of metrology and automation since the mathematical model is based on the physical pendulum, which is one of the most widely spread elements in those systems.

The block diagram of the measuring system presented in [7] is shown in fig.1. The system is designed to measure the ship roll and pitch, as well as the quantities related to them, such as heel and trim. It is based on a gyro-free reproduction of the vertical. The latter is developed by using a simplified design and a physical pendulum. The measuring system consists of two measuring channels operating in parallel (fig.1). The data obtained from the basic measuring channel contains a dynamic error mainly caused by the deviations of the physical pendulum from the ideal astronomical vertical. These fluctuations are determined by the inertial effects caused by the ship fluctuations (e.g. heave, sway, surge, roll, pitch, yaw) and the vibrations in the place where the system has been mounted.

Unlike the existing measuring methods and tools which stabilize the vertical in the inertial space so as to improve the dynamic accuracy, the proposed approach involves a method for eliminating the dynamic error in real time. The measuring procedure related to the definition of the dynamic error is performed in the additional channel (fig.1). The latter consists of two pairs of identical MEMS accelerometers used for measuring linear acceleration. Within the mechanical module of the system the accelerometers are positioned in such a way that an algorithm for determining the dynamic error along the two measuring coordinates (heel and trim) can be developed from their output signals. This algorithm is described in details in [7]. The additional measuring channel operates in parallel with the basic one so as to provide a possibility for eliminating the dynamic error from the measurement result in real time.

Figure 1 Block diagram of the measuring system.

As it is mentioned in [7], if the dynamic error is determined only on the basis of the measuring procedure described above, the accuracy is not sufficient. The motion of the moving object initiates not only basic but also secondary disturbance processes of unpredictable behaviour for the sensitive elements of the measuring instruments in the additional channel. The mathematical model of the primary processing procedure (fig.1) for determining the dynamic error takes mainly into consideration the influence of the basic disturbance processes. The model remains insensitive to the secondary disturbance processes though. The magnitude and influence of the latter on the system's dynamic accuracy are different. They are determined by a number of parameters of random nature and are connected with the motion of the moving object. In some cases, however, under their influence the accumulated error can reach inadmissibly high values.

The most appropriate way of eliminating the influence of the above mentioned secondary disturbance processes on the procedure defining the dynamic error is to use the Kalman algorithm. In that case the optimal estimation problem is solved so as to specify the signal defining the dynamic error. Actually, the Kalman filter can be placed in three different positions within the common block diagram of the system. For example, in [7] the filter is placed at the end of the metrological circuit. Therefore the mathematical model defining the Kalman filter depends to a great extent on the parameters determining the dynamic characteristics of the specific ship, which impedes to a certain degree the design of these measuring systems. Greater universality is achieved in the block diagram proposed in the present paper (fig.1), where that disadvantage is avoided. The Kalman filter solves the accuracy optimization problem upon determining the dynamic error. Thus the matrices defining the algorithm are mainly formed by the parameters of the measuring system.

The block diagram of the Kalman filter developed for one of the two measuring channels of the system is shown in fig. 2. In this case the Kalman algorithm which measures the ship pitch and trim is viewed. However, an analogous filter with identical structure and mathematical model is connected to the metrological circuit of the second measuring channel. The block diagram can be provisionally divided into two loops operating in parallel. The first loop can be assumed as a basic one since the forecast estimate and the optimal value of the measured quantity are determined in it. The second loop operates in parallel with the first one. It aims at determining the current values of the elements from the correlation matrix of the forecast error.

To obtain the forecast estimate $\hat{\bar{x}}_k(-)$, it is necessary to integrate the differential equation [10, 11]

$$\frac{d\bar{x}(t)}{dt} = F(t)\bar{x}(t), \quad (1)$$

under an initial condition $\bar{x}(0) = \bar{x}(t_0)$.

In (1) $\bar{x}(t)$ is the n -th vector defining the state of the quantity whose optimal value is demanded by means of the Kalman filter developed. According to the block diagram in fig. 1 and the proposed processing procedure of the measuring system, vector $\bar{x}(t)$ is determined by the motion of the physical pendulum along the β coordinate, which actually defines the dynamic error in this measuring channel. By transforming equation (1) from a continuous time function into a discrete time function, the following expression is obtained for the forecast estimate [11]:

$$\hat{\bar{x}}_k(-) = \Phi_k \cdot \hat{\bar{x}}_{k-1}(+), \quad (2)$$

where Φ_k is a transition function of the $[n \times n]$ -dimensional state defining the dynamics of the system; $\hat{\bar{x}}_{k-1}(+)$ - the estimate of the vector of the state at the moment of time t_{k-1} .

Due to the stationary nature of the dynamic system, caused by the invariability of its basic parameters, the Φ_k

matrix does not change in time and its elements remain constant [12].

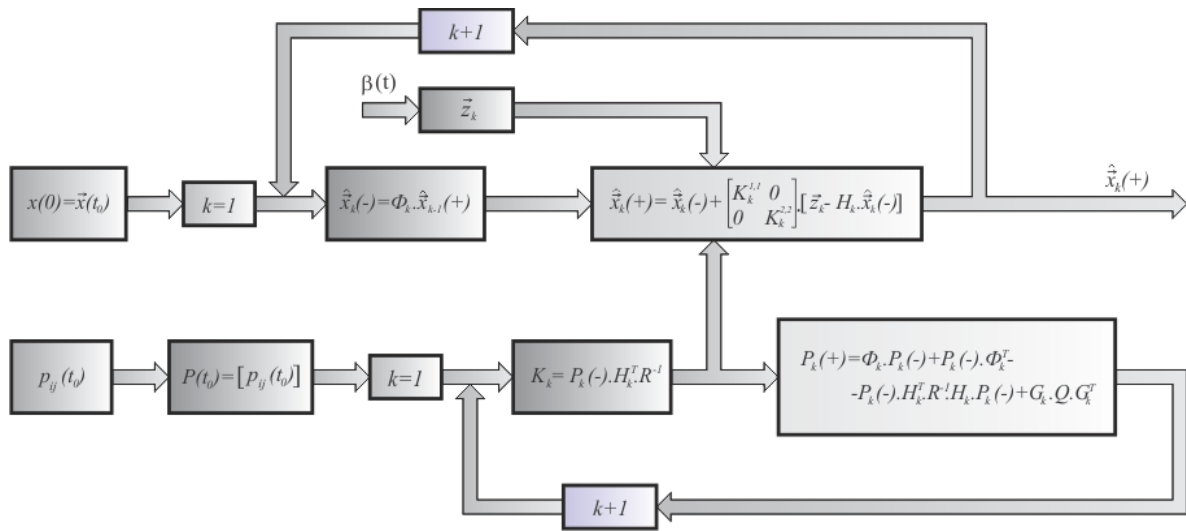
It is necessary to define the mathematical model of the coefficient correcting the forecast estimate since the Kalman filter operates according to the forecast-correction circuit (2). Therefore the continuous model of the linear dynamic system should be viewed. The latter could be illustrated by a vector-matrix differential equation defining the dynamics of the measuring system, and by a second equation defining the measuring process [10, 13].

$$\frac{d\vec{x}}{dt} = F(t)\vec{x}(t) + G(t)\vec{w}(t), \quad (3)$$

$$\vec{z}(t) = \vec{y}(t) + \vec{v}(t) = H(t)\vec{x}(t) + \vec{v}(t), \quad (4)$$

where $\vec{w}(t)$ is r -th vector representing the signal at the system input; $\vec{y}(t)$ - m -th vector determining the error-free signal at the output of the measuring channel before the Kalman filter; $\vec{z}(t)$ - m -th vector defining the measurement result before the Kalman filter; $\vec{v}(t)$ - m -th vector determining the measurement error; $F(t)$, $G(t)$, $H(t)$ - respectively $[n \times n]$, $[n \times r]$, $[m \times n]$ matrices.

Figure 2 Block diagram of the Kalman filter



It should be noted that equations (3) and (4) are written for a non-stationary dynamic system. Taking into consideration the above mentioned stationariness of the system in terms of its dynamics, the given equations are reduced and the matrices involved in them do not depend on time. Then matrix F , under the conditions of the present problem, will be equal to the above considered matrix Φ_k , which appears its analogue in the discrete model of the algorithm. This circumstance is valid for the rest matrices as well.

The Kalman method presumes that the components of the vector $\vec{w}(t)$ are linearly related to random functions of a *white noise* type. The means of the former are zero and their correlation functions are determined by the equation [15]

$$M[\vec{w}(t) \vec{w}^T(\tau)] = Q(t) \delta(t - \tau), \quad (5)$$

where $Q(t)$ is a $[r \times r]$ -dimensional symmetric matrix defining the intensity of the white noise; $\delta(t - \tau)$ - delta-function.

The vector determining the measurement errors $\vec{v}(t)$ has analogous qualities:

$$M[\vec{v}(t) \vec{v}^T(\tau)] = R(t) \delta(t - \tau), \quad (6)$$

where $R(t)$ is a $[m \times m]$ -dimensional symmetric matrix.

If at the moment of time t_0 , the initial state of the measured parameter is defined by the vector $\vec{x}(t_0)$ which is a vector random quantity of normal distribution and mean

$$M[\vec{x}(t_0)] = \vec{x}_0, \quad (7)$$

then the correlation matrix will be [16]

$$P(t_0) = M\{[\vec{x}(t_0) - \vec{x}_0] \cdot [\vec{x}(t_0) - \vec{x}_0]^T\}. \quad (8)$$

In this case $P(t_0)$ is a diagonal matrix whose elements are equal to the vector variances of the measured parameter, i.e. $P(t_0) = [p_{ij}(t_0)]$, $i, j = 1, 2, \dots, n$.

Actually the optimal filtration problem according to the Kalman method involves the definition of the optimal estimate $\hat{x}(t)$ of the vector of the measured parameter state on the basis of the differential equation (3) and the

signal $\bar{z}(t)$ obtained as a result of measurement (4). Indeed the estimate $\hat{\bar{x}}(t)$ is the output signal of the Kalman filter. The difference $\bar{e}(t)$ between the vector $\bar{x}(t)$, defining the actual state of the measured parameter, and its estimate (the output signal of the filter) $\hat{\bar{x}}(t)$ determines the estimation error or the filter error, i.e.

$$\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t). \quad (9)$$

Within the Kalman algorithm the following condition is accepted as an estimate optimality criterion: vector $\hat{\bar{x}}(t)$ is an unshifted estimate of $\bar{x}(t)$ and minimizes the variance of the error $\bar{e}(t)$. In order for $\hat{\bar{x}}(t)$ to be an unshifted estimate of $\bar{x}(t)$, the following condition should be satisfied [17]

$$M[\hat{\bar{x}}] = M[\bar{x}(t)], \quad (t \geq t_0), \quad (10)$$

where, on the basis of (9), the following is obtained: $M[\bar{e}(t)] = 0$.

Then the correlation matrix determining the estimation error can be defined by (8) and (9) [16]

$$P(t) = M[\bar{e}(t) \cdot \bar{e}^T(t)] = M\{[\bar{x}(t) - \hat{\bar{x}}(t)] \cdot [\bar{x}(t) - \hat{\bar{x}}(t)]^T\}. \quad (11)$$

The correlation matrix $P(t)$ is a $[n \times n]$ -dimensional symmetric matrix. It can be defined as a solution of the matrix differential equation [10]

$$\begin{aligned} \frac{dP(t)}{dt} = & F(t) \cdot P(t) + P(t) \cdot F^T(t) - P(t) \cdot H^T(t) \cdot R^{-1}(t) \cdot H(t) \cdot P(t) + \\ & + G(t) \cdot Q(t) \cdot G^T(t). \end{aligned} \quad (12)$$

Equation (12) is a nonlinear differential equation of a Riccati type, which allows a certain variant. The initial value $P(t_0)$ is determined by the expression (8) and is assumed to be known. Taking into account the stationary nature of the dynamic system and the discrete form of the implementation of the algorithm, (12) is written in the following form

$$\begin{aligned} P_k(+) = & \Phi_k \cdot P_k(-) + P_k(-) \cdot \Phi_k^T - \\ & - P_k(-) \cdot H_k^T \cdot R^{-1} \cdot H_k \cdot P_k(-) + G_k \cdot Q \cdot G_k^T. \end{aligned} \quad (13)$$

In this way the error correlation matrix for each step is a function of the a priori value of this matrix. Within the next step of the error correlation matrix its a posteriori value is used for correction purposes. The algorithmic sense of that operation is illustrated in fig. 2. The error correlation matrix, whose value is restored at each time step, makes possible the correction of the forecast estimate according to an optimality criterion in relation to accuracy. Thus the metrological criteria referring to the quality of the measuring process are met. In the algorithm (fig.2) the estimate is corrected by means of the matrix amplification coefficient, whose discrete form is of the following type:

$$K_k = P_k(-) \cdot H_k^T \cdot R^{-1}. \quad (14)$$

Then the differential equation defining the optimal estimate vector is [10]

$$\frac{d\bar{x}(t)}{dt} = F(t) \cdot \bar{x}(t) + K_k(t) \cdot [\bar{z}(t) - H(t) \cdot \bar{x}(t)], \quad (15)$$

In terms of the algorithm, the discrete form of (15) is important. On the basis of the clarifications done about the specific dynamic system, it will be:

$$\hat{\bar{x}}_k(+) = \hat{\bar{x}}_k(-) + K_k \cdot [\bar{z}_k - H_k \cdot \hat{\bar{x}}_k(-)]. \quad (16)$$

From equation (16) and fig.2 it follows that the filter is built on the model forming the actual measuring signal, where, by means of the amplification coefficient K_k , the difference between the k -th value of the measuring signal before the filter $\bar{z}_k$ and the respective estimate $\hat{y}_k = H_k \cdot \hat{\bar{x}}_k(-)$ of its actual component enters.

III. Design of the elements of the Kalman filter

The concept of the Kalman filter assumes that the stochastic system can be described by the models of the dynamics and measurement. In that case the model of the dynamics is described by the second differential equation in the system (9) from [7]

$$(J_{x_1} + m_1 \cdot l^2) \cdot \ddot{\beta}(t) + k_\beta \cdot \dot{\beta}(t) + m_1 \cdot l \cdot g \cdot \beta(t) = -m_1 \cdot l \cdot \ddot{\xi}_0(t) - (J_{x_1} + m_1 \cdot l \cdot z) \cdot \ddot{\psi}(t), \quad (17)$$

where m_1 , l , J_{x_1} are mass, design and inertia parameters of the system sensitive element; k_β - damping coefficient; $\psi(t)$ - pitch; $\xi_0(t)$ - surge; z - the coordinate in the vertical direction of the suspension point of the system sensitive element with regard to the centre of gravity of the ship.

It is necessary to note that the measured quantity from the system measuring channel under consideration is $\psi(t)$. According to the metrological procedure it is measured with accuracy depending to a great extent on the accuracy related to the definition of the dynamic error $\beta(t)$. Namely the last quantity is subject to filtration in the present problem.

Equation (17) can be easily transformed in the type

$$\ddot{\beta}(t) + 2h\dot{\beta}(t) + w_0^2 \beta(t) = -\frac{w_0^2}{g} \ddot{\xi}_0(t) - \frac{w_0^2}{g} \left(\frac{J_{x_1}}{m_1 l} + z \right) \ddot{\psi}(t), \quad (18)$$

where $w_0 = \sqrt{\frac{m_1 l g}{(J_{x_1} + m_1 l^2)}}$ - the own frequency of the sensitive element (the physical pendulum);

$$2h = \frac{k_\beta}{(J_{x_1} + m_1 l^2)}; h - \text{damping factor.}$$

Quantities $\ddot{\xi}_0(t)$, $\ddot{\psi}(t)$ in the right part of equations (17) and (18) are random time functions. The Kalman method introduces a simplifying hypothesis according to which these quantities are assumed as random functions of a *white noise* type [9, 10, 11]. Then equation (18) will be transformed into

$$\ddot{\beta}(t) + 2h\dot{\beta}(t) + w_0^2 \beta(t) = k_1 w(t), \quad (19)$$

where $w(t)$ - scalar white noise of intensity $Q = 1$; $k_1 = -\frac{w_0^2}{g} \left(\frac{J_{x_1}}{m_1 l} + z + 1 \right)$.

According to the type of the model (3) of the equation, defining the system dynamics, equation (19) is transformed from a second-order equation into a system of two first-order equations.

$$\begin{aligned} \dot{\beta}(t) &= \chi(t); \\ \dot{\chi}(t) &= -2h\chi(t) - w_0^2 \beta(t) + k_1 w(t). \end{aligned} \quad (20)$$

Vector $\bar{x}(t)$ determining the state of the quantity $\beta(t)$, in compliance with (20), will be

$$\bar{x}^T(t) = \begin{bmatrix} \beta & \chi \end{bmatrix}. \quad (21)$$

Then matrices F and G will be of the type

$$F = \Phi_k = \begin{bmatrix} 0 & 1 \\ -w_0^2 & -2h \end{bmatrix}; \quad G = G_k = \begin{bmatrix} 0 \\ k_1 \end{bmatrix}. \quad (22)$$

Within the specific problem, the equation of the measured signal $z(t)$ before the filter can be written in the following way:

$$z(t) = \beta(t) + v(t), \quad (23)$$

where $v(t)$ is the measurement error which can be considered white noise of intensity $R = 1$.

Then matrix H from equation (4), determined by the type of the input signal entering the Kalman filter (23), will be

$$H = H_k = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (24)$$

The diagonal matrix $P(t_0)$, whose elements are equal to the variances D_β and $D_\chi = D_{\dot{\beta}}$ of the components of the vector $\bar{x}^T(t) = \begin{bmatrix} \beta & \chi \end{bmatrix}$, is written on the basis of the correlation functions of quantities $\beta(t)$ and $\dot{\beta}(t) = \chi(t)$. This is due to equations $D_\beta = K_\beta(0)$ and $D_{\dot{\beta}} = K_{\dot{\beta}}(0)$, where $K_\beta(0)$ and $K_{\dot{\beta}}(0)$ are the values of the correlation functions of $\beta(t)$ and $\dot{\beta}(t) = \chi(t)$ when $\tau = 0$ [16, 18].

The spectral density of the stationary solution of the linear differential equation with constant coefficients (19) can be presented in the type [15, 16]

$$S_\beta(\omega) = |W(j\omega)|^2 S_w(\omega), \quad (25)$$

where $W(j\omega)$ is the transfer function $W(p)$ written with the argument $p = j\omega$.

From (19) it follows that the transfer function of the system sensitive element will be

$$W(p) = \frac{k_1}{p^2 + 2hp + w_0^2}. \quad (26)$$

By replacing the arguments in (26), the following is obtained

$$W(j\omega) = \frac{k_1}{-\omega^2 + 2hj\omega + w_0^2}. \quad (27)$$

Then on the basis of (25),

$$S_\beta(\omega) = \frac{k_1^2}{\omega^4 - w_0^2 \omega^2 + 4h^2 \omega^2} S_w(\omega). \quad (28)$$

Since the process $w(t)$ is white noise, then on the basis of the qualities of the latter it follows that the spectral density $S_w(\omega) \approx c = \text{const}$. Then equation (28) can be transformed into the type

$$S_\beta(\omega) = \frac{2D_\beta \mu}{\pi} \cdot \frac{\mu^2 + \lambda^2}{\omega^4 - \lambda^2 \omega^2 + 4\mu^2 \omega^2}, \quad (29)$$

$$\text{where } \mu = h; \quad \lambda = \sqrt{w_0^2 - h^2}; \quad D_\beta = \sigma_\beta^2 = \frac{\pi k_1}{2h.w_0^2} \cdot c. \quad (30)$$

From equation (29) the sought mathematical model of the correlation function $K_\beta(\tau)$ of the process $\beta(t)$ can be found on the basis of the common relationship $K(\tau) = 2 \cdot \int_0^\infty S(\omega) \cdot \cos \omega \tau \cdot d\omega$, i.e. [10, 15]

$$K_\beta(\tau) = D_\beta \cdot e^{-\mu|\tau|} \cdot \left(\cos \lambda \tau + \frac{\mu}{\lambda} \cdot \sin \lambda |\tau| \right). \quad (31)$$

Since function $\beta(t)$ is related to $\dot{\beta}(t)$ by a differential operator, their respective correlation functions will be in the following relationship [15]:

$$K_{\dot{\beta}}(t_1, t_2) = \frac{\partial^2 K_\beta(t_1, t_2)}{\partial t_1 \cdot \partial t_2} = -\frac{d^2 K_\beta(\tau)}{d\tau^2} = K_{\dot{\beta}}(\tau), \quad (32)$$

where $\tau = t_2 - t_1$.

It follows from (32) and (31) that:

$$K_{\dot{\beta}}(\tau) = D_\beta \cdot (\lambda^2 + \mu^2) \cdot e^{-\mu|\tau|} \cdot \left(\cos \lambda \tau - \frac{\mu}{\lambda} \cdot \sin \lambda |\tau| \right). \quad (33)$$

Then on the basis of (31) and (33) we can write the initial state of the correlation matrix $P(t_0)$ of the error, i.e.

$$P(t_0) = \begin{bmatrix} D_\beta & 0 \\ 0 & D_\beta \cdot (\lambda^2 + \mu^2) \end{bmatrix}. \quad (34)$$

The expressions obtained for matrices F , G , H , $P(t_0)$ and the specified intensities of the white noise $Q = 1$, R at the input of the dynamic system make possible to develop the Kalman algorithm on the basis of the block diagram in fig. 2. The algorithm improves the accuracy upon determining the function $\beta(t)$, which appears an important condition for increasing the dynamic accuracy of the measuring system. Moreover, the numerical values of the components of the given matrices are defined by the respective design parameters of the sensitive element of the system.

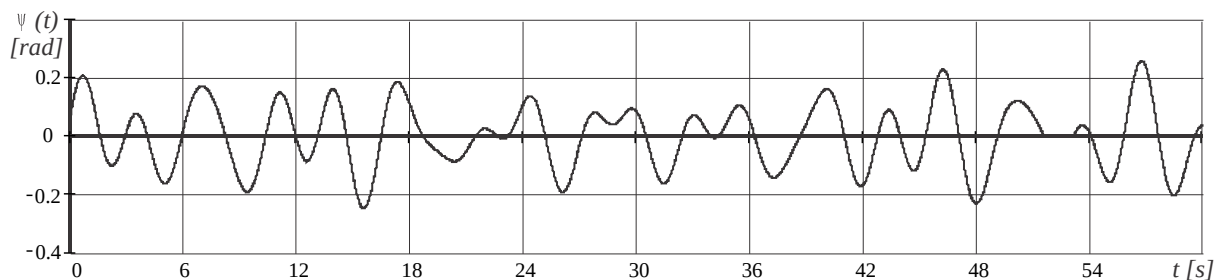
IV. Experiments and results

Some experiments have been performed to test the measuring approach proposed. The test involves an experimental definition of the dynamic error of the measuring system. In this respect some difficulties are encountered since there is not any well developed reference base in the area of dynamic measurements. This mainly refers to those types of metrological tests, where the reference quantities are random time functions.

Therefore it is necessary to develop equipment that possesses reference metrological qualities and makes possible to conduct the present experiments. On the basis of the characteristics of the ship motion in its six degrees of freedom, which have to be reproduced by the equipment, the most appropriate variant of building the equipment is to use parallel-kinematic mechanisms (a hexapod with six degrees of freedom). Those systems provide high sensitivity, manoeuvrability, high positioning accuracy, high allowable loads when the platform moves in its six degrees of freedom. This makes possible to reproduce an accurate reference shift in the form of both harmonic motions and random processes. It should be mentioned that the detailed description of the experimental equipment is not a subject of the present paper. Only the results from the experiments related to the dynamic error of the described measuring system, and obtained by means of that equipment, are presented. Figure 3 shows the graphic representation of the movement $\psi(t)$ of the hexapod operating platform along the trim coordinate.

Since the measuring system is mounted on the operating platform, the function $\psi(t)$ represents the reference signal against which the reading obtained at the system output is compared in order to determine the error sought.

Figure 3 Motion of the reference equipment along the angular coordinate ψ



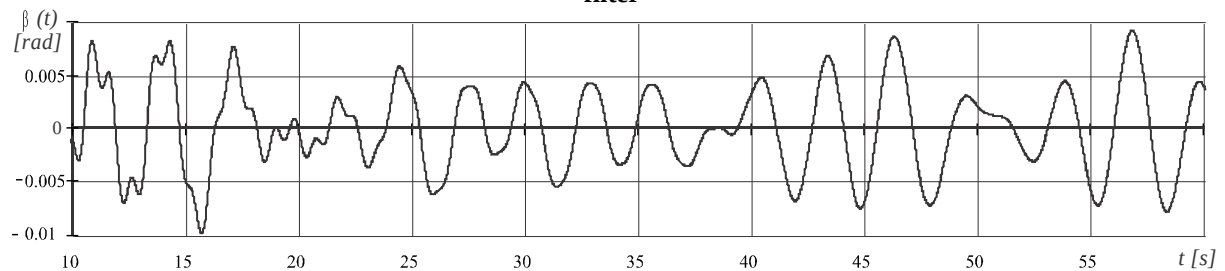
To illustrate the effectiveness of the proposed measuring algorithm, the results determining the dynamic error for the three possible variants of the system operation are presented. In its first variant, the system operates without an additional channel and a Kalman filter. The results of the operation of the system in terms of accuracy are shown in fig. 4. The dynamic error $\Delta\psi(t)$ is determined by the function $\beta(t)$ defining the deviation of the physical pendulum from the vertical, i.e. $\Delta\psi(t) = \beta(t)$. Figure 4 illustrates that in this variant the measuring system is very sensitive to the inertial effects caused by the motion of the operating platform. The maximal values of the dynamic error vary within the range of $0,5^\circ$ - $0,6^\circ$, and depending on the position of the system against the centre of gravity of the ship, they can reach values up to 2° , which is unacceptable in terms of accuracy.

In its second variant the system operates with an additional channel and without a Kalman filter. When the additional channel operates, the dynamic accuracy of the system considerably increases. As it can be seen in fig.5, the maximal values of the dynamic error are within the range of $0,15^\circ$ - $0,16^\circ$. The latter is determined by the difference

$$\Delta\psi_{ch}(t) = \psi_{ch}(t) - \psi(t), \quad (35)$$

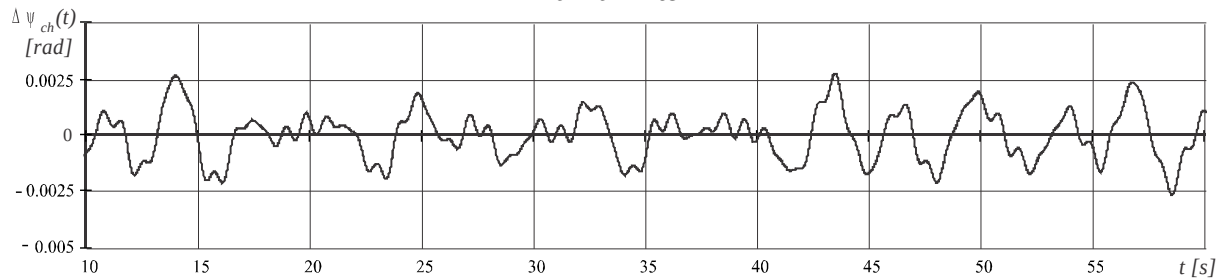
where $\psi_{ch}(t)$ is the signal obtained from the system when the additional channel is connected to it.

Figure 4 Function of the dynamic error of the quantity $\psi(t)$ without an additional channel and a Kalman filter



The experiments show that even under the most severe conditions caused by the inertial effects the system remains stable in terms of its dynamic accuracy and the error does not exceed $0,2^\circ$ - $0,3^\circ$.

Figure 5 Function of the dynamic error of the quantity $\psi(t)$ with an additional channel and without a Kalman filter

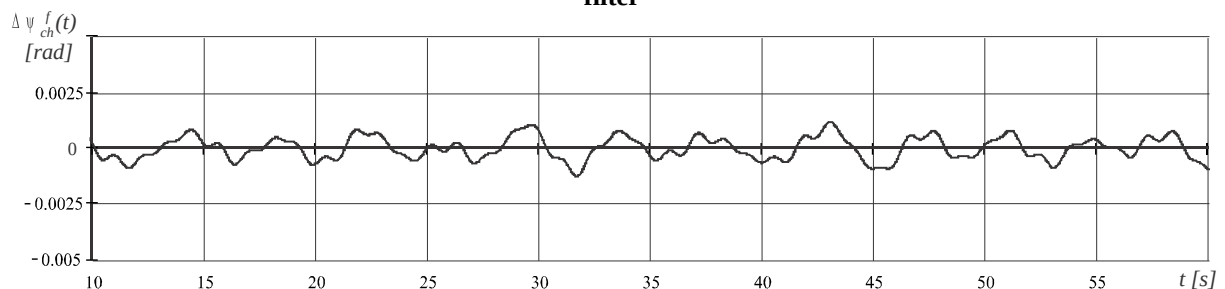


When the proposed algorithm is involved in the additional channel of the system, the maximal values of the dynamic error decrease. In this case the dynamic error is determined by an expression which is analogous to (35), i.e.

$$\Delta\psi_{ch}^f(t) = \psi_{ch}^f(t) - \psi(t), \quad (36)$$

where $\psi_{ch}^f(t)$ is the signal obtained from the system when the Kalman filter is connected to the additional channel.

Figure 6 Function of the dynamic error of the quantity $\psi(t)$ with an additional channel and a Kalman filter



The result from the operation of the Kalman filter is illustrated in fig. 6. The maximal values of the dynamic error vary within the range of $0,05^{\circ}$ - $0,07^{\circ}$.

V. Conclusions

The experimental results confirm the effectiveness of the proposed algorithm in terms of the dynamic accuracy of systems used for measuring moving objects. As a result of the operation of the additional channel and the Kalman filter the accuracy characteristics of the measuring system increase under the conditions of dynamic influences. This is implemented without the introduction of expensive stabilization elements and systems.

The present paper underlies a measuring system developed on the basis of a metrological procedure which differs from the methods applied into the existing systems and measuring tools. Unlike the latter which use stabilization of the vertical in the inertial space, the proposed approach is based on the elimination of the dynamic error in real time. The elimination of the dynamic error is based on the problem related to its accurate determination in a channel operating in parallel with the basic measuring channel. The proposed algorithm aims at presenting an effective solution of this problem. The algorithm is based on the Kalman method and seeks to eliminate the influence of a number of disturbance sources, each of which is of secondary significance. However, their cumulative effect may cause a considerable distortion of the signal defining the dynamic error. The algorithm is developed on the basis of the proposed dynamic model of the sensitive element of the measuring system.

The results from the operation of the filter are illustrated by the experiments done. The latter show that when a filter is involved in the measuring procedure, the maximal value of the dynamic error does not exceed $0,1^{\circ}$. The maximal dynamic error can reach values of $0,15^{\circ}$ - $0,2^{\circ}$ in rare cases when the measuring system is shifted at a considerable distance from the centre of gravity of the ship. The above mentioned proves that the proposed algorithm has the potential to be applied. Its use in similar measuring systems can considerably increase their dynamic accuracy and reduce their costs.

VI. References

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