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Thermal Buckling Analysis of a laminated composite plate resting on Elastic foundation using Stochastic Finite Element Method Based on Micromechanical model

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Abstract— The present work deals with the effect of temperature on the buckling response of a laminated composite plate subjected to thermo mechanical loadings. Mechanical loading consists of uniaxial, biaxial, and shear. The distribution of temperature on the surface is considered to be uniform. The properties such as material properties, thermal expansion coefficients and lamina thickness are modeled as basic random. The degradation in material properties due to temperature is taken into account using a micromechanical model. The mathematical formulation is based on higher order shear deformation theory and von Karman's nonlinear kinematics. A C^0 finite element method based on higher-order shear deformation plate theory is used for deriving the standard eigenvalue problem. A Taylor series based mean-centered first order perturbation technique is used to find out the second order statistics of the thermal buckling loads. The effects of temperature rise, fiber-volume fractions, and plate parameters on buckling response of the plate are presented.

Keywords- Thermal Buckling, Random System Properties, Stochastic Finite Element, Perturbation Technique, Micromechanical.

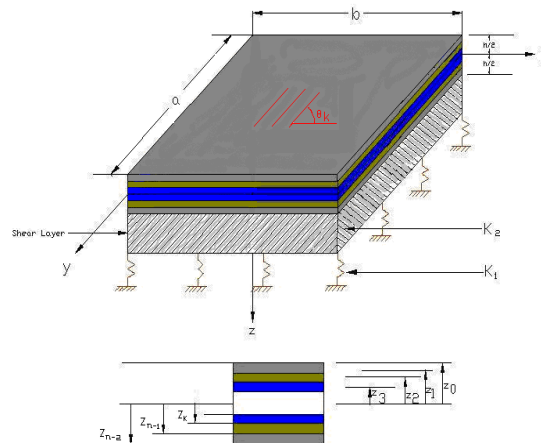
I. INTRODUCTION

Laminated composite plates when subjected to temperature environments the thermal stresses are developed at the edges of the plates due to constraint thermal expansion coefficients. These thermal stresses induce thermal buckling loads which may have affected the structural behavior of the plates consequently result buckling of the plate. Therefore, it necessitates to understand the linear buckling behavior of the composite plates induced by thermal loading. One of the aims of the designer is to control undesirable buckling that may eventually lead to the failure of the structures due to fatigue and creep. Considerable investigations have been made in the past by researchers on the prediction of the nonlinear buckling response of conventional structures and composite structures in thermal environments based on deterministic analysis which is insufficient to predict the system behavior due to various system randomness as they only give the mean response and ignore accountability for the deviation caused by inherent randomness in the system properties. Notable among them are Liu and Huang [1] and Lee and Lee [2]. Zhang and Ellingwood [3] have evaluated the effect of random material field characteristics on the instability of a simply supported beam on elastic foundation and a frame using perturbation technique. A comprehensive summary of extensive literature is available on the response analysis of the structures with deterministic material properties to random excitations by Nigam and Narayanan [4]. Zhang et al. [5] have investigated the stochastic perturbation method to vector-valued and matrix-valued function for response and reliability of uncertain structures. Some published literature is available on the analysis of the structures made of metallic materials with random material properties. It is evident from the literature that the studies on the nonlinear thermal response buckling of laminated composite plates with random material properties are not dealt by the researcher to the best of author's knowledge.

1 FORMULATION

Consider a rectangular laminated composite plate of length a , width b , and thickness h , which consists of N number of orthotropic layers. All orthotropic layers of the composite plate are of uniform thickness. The mid plane of the plate considered as the reference plane. The thickness coordinates z of the top and bottom surfaces of any (k_{th}) layer are denoted by $h_{(k+1)}$ and h_{k_0} respectively. The fibers of k_{th} layer are oriented at an angle θ_k to

the x-axis, supported at four edges defined in the (x, y, z) coordinate system with x- and y-axes located in the middle plane and origin



placed at the corner of the plate as shown in Fig. 1.

II. STRESS AND STRAIN RELATION.

The stress-strain relations of thermo-elasticity for the k th lamina oriented as an arbitrary angle with respect to the reference axis for an orthotropic layer under consideration of with stain in a plain stress state is given by Reddy (1996).

$$\{\sigma\}_k = [\bar{Q}]_k \{\varepsilon\}_k$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix}_k - \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_{12} \\ 0 \\ 0 \end{Bmatrix}_k \delta T$$

(1)

where, $\lambda_1 = \bar{Q}_{11}\alpha_1 + \bar{Q}_{12}\alpha_2 + \bar{Q}_{16}\alpha_{12}$; $\lambda_2 = \bar{Q}_{12}\alpha_1 + \bar{Q}_{22}\alpha_2 + \bar{Q}_{26}\alpha_{12}$; $\lambda_{12} = \bar{Q}_{16}\alpha_1 + \bar{Q}_{26}\alpha_2 + \bar{Q}_{66}\alpha_{12}$ where, $\{\bar{Q}\}_k$, $\{\sigma\}_k$ and $\{\varepsilon\}_k$ are transformed stiffness matrix, stress and strain vectors of the k th lamina, respectively and α_1 , α_2 , and α_{12} are the expansion coefficients along X, Y and Z directions which can be obtained from the thermal coefficients in the longitudinal (α_l) and transverse (α_t) directions of the fibers using transformed matrix.

A. Strain energy equations.

The strain energy of the plate is given by

$$U = \frac{1}{2} \int_V \{\varepsilon\}^T [\sigma] dV \quad (2)$$

Using Equations the strain energy (U) can be rewritten as

$$U = \frac{1}{2} \int_A \{\varepsilon_i\}^T [\bar{Q}] \{\varepsilon_i\} dA + \frac{1}{2} \int_A \{\varepsilon_i\}^T \{\bar{Q}\} \{\varepsilon_{nl}\} dA + \frac{1}{2} \int_A \{\varepsilon_{nl}\}^T \{\bar{Q}\} \{\varepsilon_i\} dA + \frac{1}{2} \int_A \{\varepsilon_{nl}\}^T \{\bar{Q}\} \{\varepsilon_{nl}\} dA, \quad (3)$$

Using linear strain displacement vector expressed in terms of thickness coordinate relations and mid-plane strain vector can be expressed as

$$U = \frac{1}{2} \int_A \{\bar{\varepsilon}_i\}^T [D] \{\bar{\varepsilon}_i\} dA + \frac{1}{2} \int_A [\bar{\varepsilon}_i]^T [D_3] \{A\} \{\phi\} dA + \frac{1}{2} \int_A \{A\}^T \{\phi\} [D_4] \{\bar{\varepsilon}_i\} dA + \frac{1}{2} \int_A \{A\}^T \{\phi\} [D_5] \{A\} \{\phi\} dA, \quad (4)$$

where $[D]$, $[D_3]$, $[D_4]$ and $[D_5]$ are the laminate stiffness matrices and mid plain strain vector $\{\bar{\varepsilon}_i\}$ is defined as

$$\{\bar{\varepsilon}_l\} = (\varepsilon_1^0 \ \varepsilon_2^0 \ \varepsilon_6^0 \ k_1^0 \ k_2^0 \ k_6^0 \ k_1^2 \ k_2^2 \ k_6^2 \ \varepsilon_4^0 \ \varepsilon_5^0 \ k_4^2 \ k_5^2)^T \quad (5)$$

III. THE EQUATION OF MOTION AND ITS SOLUTION TECHNIQUE

The governing equation for thermally induced nonlinear free vibration of the plate analysis can be derived using the Lagrange's equation of motion.

$$\delta \int_{t_1}^{t_2} (U - W - T) dT = 0 \quad (6)$$

Substituting Equations one obtains as in the form of non linear generalized Eigen value problem as

$$[K]\{q\} = \lambda[M]\{q\} \quad (7)$$

here, $[K] = [K_l] + [K_{nl}] - \lambda_T [K_g]$ with $[K_{nl}] = \left[\frac{1}{2} N_1(q) + \frac{1}{3} N_2(q) \right]$ where $\{q\}$, $[K_l]$, $[K_{nl}]$, $[M]$ and

$\lambda = \omega^2$ are defined as global displacement vector, global linear, nonlinear and geometric stiffness matrices, global mass matrix and critical buckling temperature respectively. $N_1(q)$ and $N_2(q)$ are global nonlinear stiffness matrices which are linearly and quadratic ally dependent on the displacement vector, respectively.

A. Solutions- perturbation technique.

Due to the result of randomness in basic input variables, all the quantities in Equation (15), i.e. $[K]$, $\{q\}$ and λ are random as well. M. Based on this, the mean centered first order perturbation techniques in which all the system parameters are expanded in Taylor series is used to determine the stochastic characteristics of the nonlinear frequency of laminated composite plates.

In general, without any loss of generality any arbitrary random variable can be expressed as the sum of its mean and the zero mean random part, denoted by superscripts 'd' and 'r', respectively

$$K = K^d + K^r, \quad \lambda_i = \lambda_i^d + \lambda_i^r, \quad q_i = q_i^d + q_i^r, \quad (8)$$

where, $\lambda_i^d = \omega_i^{d^2}$, $i=1, 2, \dots, p$. The parameter p indicates the size of eigen problem.

Using Taylor series and neglecting the higher order terms since the first order approximation is sufficient to yield results with desired accuracy having low variability as is the case in most of the sensitive applications Kleiber and Hein(1992). one obtains as

$$\text{Zeroth order perturbation Equation:} \quad [K^d]\{q_i^d\} = \lambda_i^d [M]\{q_i^d\} \quad (9)$$

First order perturbation equation:

$$[K^d]\{q_i^r\} + [K^r]\{q_i^d\} = \lambda_i^r [M]\{q_i^d\} + \lambda_i^d [M]\{q_i^r\} \quad (10)$$

IV. Micromechanical approach

The material properties of the fiber composite at different temperature are evaluated using micromechanical model. Since the effect of temperature and moisture concentration is dominant in matrix material. The degradation of the fiber composite material properties is estimated by degrading the matrix property only.

The elastic constants are obtained from the following equations Upadhyay et al. (2010).

$$E_{11} = E_{f1}V_f + F_m E_m V_m \quad (11)$$

$$E_{22} = (1 - \sqrt{V_f}) F_m E_m + \frac{F_m E_m \sqrt{V_f}}{1 - \sqrt{V_f} \left(1 - \frac{F_m E_m}{E_{f2}} \right)} \quad (12)$$

$$G_{12} = (1 - \sqrt{V_f}) F_m G_m + \frac{F_m G_m \sqrt{V_f}}{1 - \sqrt{V_f} \left(1 - \frac{F_m G_m}{G_{f12}} \right)} \quad (13)$$

$$\nu_{12} = \nu_{f12}V_f + \nu_m V_m \quad (14)$$

where "V" is the volume fraction and subscripts "f" and "m" are used for fiber and matrix, respectively. The effect of increased temperature on the coefficients of thermal expansion (α) is opposite from the corresponding effect on strength and stiffness. The matrix thermal property retention ratio is approximated as

$$F_h = \frac{1}{F_m} \quad (15)$$

Coefficients of thermal expansion are expressed as Upadhyay et al.(2010).

$$\alpha_{11} = \frac{E_{f1} V_f \alpha_{f1} + F_m E_m V_m F_h \alpha_m}{E_{f1} V_f + F_m E_m V_m} \quad (16)$$

$$\alpha_{22} = (1 + \nu_{f12}) V_f \alpha_{f2} + (1 + \nu_m) V_m F_h \alpha_m - \nu_{12} \alpha_{11} \quad (17)$$

The elastic constants are obtained from the following equations Shen. Hui-Shen.(2001).

$$E_{11} = V_f E_f + V_m E_m \quad (18)$$

$$\frac{1}{E_{22}} = \frac{V_f}{E_f} + \frac{V_m}{E_m} - V_f V_m \frac{V_f^2 \frac{E_m}{E_f} + \nu_m^2 \frac{E_f}{E_m} - 2\nu_f \nu_m}{V_f E_f + V_m E_m} \quad (19)$$

$$\frac{1}{G_{12}} = \frac{V_f}{E_f} + \frac{V_m}{G_m} \quad (20)$$

$$\nu_{12} = V_f \nu_f + V_m \nu_m \quad (21)$$

where “V” is the volume fraction and subscripts “f” and “m” are used for fiber and matrix, respectively. The effect of increased temperature on the coefficients of thermal expansion (α) is opposite from the corresponding effect on strength and stiffness. Coefficients of thermal expansion are expressed as Shen.(2001).

$$\alpha_{11} = \frac{V_f E_f \alpha_f + V_m E_m \alpha_m}{V_f E_f + V_m E_m} \quad (22)$$

$$\alpha_{22} = (1 + \nu_f) V_f \alpha_f + (1 + \nu_m) V_m \alpha_m - \nu_{12} \alpha_{11} \quad (23)$$

other constants are related as Shen.(2001).

$$\rho = V_f \rho_f + V_m \rho_m \quad (24)$$

$$V_m + V_f = 1 \quad (25)$$

V. Monte- Carlo Simulation

In MCS approach, the samples for the random parameters are obtained by generating a set of random numbers of given sample size to fit the desired mean and Standard deviation (SD). For present work, MATLAB inbuilt command is used for generating random numbers corresponding to mean values of the material property to be varied. The formula for Mean and SD of property x to be varied are as:

$$\text{Mean} = \mu = \frac{\sum_{i=1}^n x_i}{n} \quad (26)$$

$$\text{Standard Deviation} = \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2} \quad (27)$$

where $i=1,2,3,\dots,n$.

The governing equation for thermal buckling of laminated composite plate can be derived using Variational principle Reddy (1981) which is generalization of the principle of virtual displacement. For the prebuckling analysis, the first variation of total potential energy $\Pi = (\Pi_1 + \Pi_2)$ must be zero.

$$[K]\{q\} = [F^T] \quad (28)$$

For the critical buckling state corresponding to the neutral equilibrium condition, the second variation of total potential energy (Π) must be zero. Following this conditions, ones obtains as standard eigenvalue problem

$$[K]\{q\} = \lambda [K_g]\{q\} \quad (29)$$

The Eq. 28 & 29 can be rewritten as

$$[[K] + \lambda [K_g]] = 0 \quad (30)$$

Where $[K]$, and $[K_g]$ are the stiffness matrix and geometric stiffness matrix respectively

VI. RESULTS AND DISCUSSION

In the present study, a procedure and a MATLAB code for performing stochastic analysis of the thermal buckling of laminated composite plates has been developed. The direct iterative based stochastic finite element method (DISFEM) approach is outlined for the linear response of the laminated composite plates subjected to temperature changes with random input variables through numerical examples. The approach has been validated by comparing the results with those available in literature and independent Monte Carlo Simulation with important sampling. A nine noded Lagrangian isoparametric element with 63 degrees of freedom (DOFs) for the present HSDT model has been used for discretizing the laminate. Based on convergence study, a (4×4) mesh has been used. Unless otherwise mentioned all the results reported in this paper have been obtained employing the full integration (3×3) rule The influence of scattering in the system properties on the thermal linear

buckling referred as a linear in the following text has been examined for the laminated composite plate with various temperature increments. The mean and standard deviation of the linear buckling load are obtained considering the all random input variables. However, the results are only presented taking SD/mean of the system property equal to 0.10 Liu et al.(1996) as the nature of the SD (Standard deviation) variation is linear and passing through the origin. Hence, the presented results would be sufficient to extrapolate the results for other SD/mean value keeping in mind the limitation of FOPT Liu et al.(1996). The basic random variables such as E_1 , E_2 , G_{12} , G_{13} , G_{23} , ν_{12} , α_1 , α_2 are sequenced and defined as appendix.

$$b_1 = E_{11}, b_2 = E_{22}, b_3 = G_{12}, b_4 = G_{13}, b_5 = G_{23}, b_6 = \nu_{12}, b_7 = \alpha_1, b_8 = \alpha_2, b_9 = h$$

The following material properties are used for computation:

$$E_{11}^d = 40E_{22}^d, G_{12}^d = G_{13}^d = 0.6E_{22}^d, G_{23}^d = 0.5E_{22}^d, \nu_{12}^d = 0.25, \rho = 1, \alpha_1^d = 1.14 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, \alpha_2^d = 11.4 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, \alpha_0^d = 1 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, E_{22}^d = 6.92 \times 10^9 \text{ Pa.}$$

In the present study various combination of edge support conditions namely clamped (C) and simply supported (S) have been used for the investigation. The boundary conditions for the plate are as :

All edges simply supported (SSSS):

$$v = w = \theta_y = \psi_y = 0, \text{ at } x = 0, a; \quad u = w = \theta_x = \psi_x = 0 \text{ at } y = 0, b \text{ All edges clamped (CCCC):}$$

$$u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0, \text{ at } x = 0, a \quad \text{and } y = 0, b;$$

Two opposite edges clamped and other two simply supported(CSCS):
 $u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0, \text{ at } x = 0 \quad \text{and } y = 0; \quad v = w = \theta_y = \psi_y = 0, \text{ at } x = a \quad u = w = \theta_x = \psi_x = 0, \text{ at } y = b$

A. Comparison results: mean and second-order statistics

In order to verify the accuracy of the present finite element formulation in dimensionless linear thermal buckling analysis of laminated composite angle-ply $[0^\circ/45^\circ/45^\circ/0^\circ]$ square plate with various volume fractions ratio and with uniform thermal loading condition having all edges simply supported is shown in Table 1 and compared with the results obtained by Liu and Huang (1996). Clearly, it is seen that the present finite element results obtained by HSDT are in good agreement with that obtained by Liu and Huang analysis using the first-order shear deformation plate theory. The maximum difference is about 2%.

Due to unavailability of results concerning the buckling of composite plates subjected to thermal loading with randomness in system properties, the existing result concerning the random linear buckling of laminated composite plates in thermal environment has been validated by comparing the results obtained from present DISFEM approach with an independent MCS approach with various buckling load. Fig.2 plots the normalized standard deviation, COV (i.e. the ratio of the standard deviation (SD) to the mean value), of the nonlinear standard deviation versus the COV of the random material constant for an all simply supported square $[0^\circ/45^\circ/45^\circ/0^\circ]$ plate subjected to in-plane thermal loading $\delta T = 100$ changing from 0 to 20%. It is assumed that one of the material property (i.e., E_{11}) change at a time keeping other as a deterministic, with their mean values of the material properties. The dashed line is the present [DISFEM] result that is obtained by using first-order perturbation approach based on HSDT in the von-Karman sense and the solid line is independent MCS approach. For the MCS approach, the samples are generated using Mat Lab to fit the desired mean and SD. It is assumed that one of the material property (i.e., E_{11}) change at a time keeping other as a deterministic, with their mean values of the material properties. For the MCS approach, the samples are generated using Mat Lab to fit the desired mean and SD. The effect of volume fractions ratio with support edge conditions SSSS(S2) with amplitude ratio on the mean and standard deviation of the dimensionless linear buckling load for lay-up angle-ply $[45^\circ/-45^\circ/-45^\circ/45^\circ]$ square laminate with all random input such as combination of material properties and thermal expansion coefficients as well as the stiffness separately, changing for $V_f = 0.55, 0.60, 0.65, 0.70$ is shown in Table 2(a) and Table 2(b) The mean buckling load increases as volume fractions ratio increases and the RVs (b_i , $i = 1, 2, \dots, 8$) and thermal expansion coefficient inputs RVs (b_i , $i = 7, 8$) are also changes From the table it can be seen that the dispersion in the buckling load is greatest for $V_f = 0.65$ while it is lowest for $V_f = 0.50$. In general, the presented results reveal that the effect of randomness in thermal expansion coefficients on the dispersion of the linear mean buckling load is significant and hence it is not desirable to ignore in the design.

VII.CONCLUSION

Table 1 Comparisons of buckling loads P_x (KN) for perfect $(\pm 45^\circ)_{2T}$ laminated square plates ($a/h=10$) under sets of environmental conditions

$\Delta T=0$	$V_f=0.5$	$V_f=0.6$	$V_f=0.7$
	142.740	169.708	203.014
Present	141.1574	171.226	214.1426

Comparison of dimensionless mean buckling load with amplitude ratios of a $(0/45^\circ/45^\circ/0^\circ)$ laminated composite plates subjected to two sets of techniques to study the thermally induced geometrically linear and response of

laminated composite plate in the framework of higher order deformation theory with the von Karman sense. Thermal Increase the temperature results in decrease in nonlinear mean buckling load of laminated plate for the same amplitude ratio however, dispersion of linear mean buckling load of the plate increases with increase the temperature. The SSSS plate is most sensitive, while CCCC plate is least sensitive against simultaneous change in all random input variables and thermal expansion coefficients. In contrast the mean values of SSSS and CCCC plate show the opposite effect

Table 2(a)- Effect of volume fraction and input random variables $b_i \{i=1 \dots 9, 7-8, 9-10, 1 \dots 10=0.10\}$ on the buckling loads N_x (KN) for angle ply $(\pm 45^\circ)_T$ square laminated composite plate resting on Winkler ($k_1=100$, $k_2=0$) elastic foundation, plate thickness ratio ($a/h=50$), $T_0=25^\circ\text{C}$, Simple Support(S2) under Environmental Conditions

(Vf)	Winkler foundations ($k_1=100$, $k_2=00$)					
	$\Delta T=$	$\Delta T=100^\circ\text{C}$				
	$W_{m_{ax/h}}$ KN	$W_{m_{ax/h}}$ KN	Coefficient of variation (SD/ Mean)			
			b_i			
			(i=1-8)	(i=7.-8)	(i=9-10)	(i=1-10)
$V_f=0.5$	4.50	4.04	0.1205	0.0201	0.0128	0.1211
$V_f=0.55$	4.53	4.43	0.1202	0.0182	0.0128	0.1209
$V_f=0.60$	4.98	4.85	0.1200	0.0165	0.0129	0.1207
$V_f=0.65$	5.34	5.28	0.1197	0.0150	0.0130	0.1204

Table 2(b)- Effect of volume fraction and input random variables $b_i \{i=1 \dots 9, 7-8, 9-10, 1 \dots 10=0.10\}$ on the buckling loads N_x (KN) for angle ply $(\pm 45^\circ)_T$ square laminated composite plate resting on Winkler ($k_1=100$, $k_2=10$) elastic foundation, plate thickness ratio ($a/h=50$), $T_0=25^\circ\text{C}$, Simple Support(S2) under Environmental Conditions.

(Vf)	Pasternak foundations ($k_1=100$, $k_2=10$)					
	$\Delta T=0^\circ$	$\Delta T=100^\circ\text{C}$				
	$W_{m_{ax/h}}$ (KN)	$W_{m_{ax/h}}$ (KN)	Coefficient of variation (SD/ Mean)			
			b_i			
			(i=1-8)	(i=7.-8)	(i=9-10)	(i=1-10)
$V_f=0.5$	5.04	4.95	0.1290	0.0164	0.0151	0.1299
$V_f=0.55$	5.53	5.44	0.1283	0.0148	0.0150	0.1292
$V_f=0.6$	6.05	5.95	0.1279	0.0134	0.0150	0.1288
$V_f=0.65$	6.60	6.49	0.1276	0.0122	0.0150	0.1285

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