



**International Journal of Engineering, Business and Enterprise
Applications (IJEBA)**
www.iasir.net

**Heat and Mass Transfer on Flow Past an Exponentially Accelerated Infinite
Vertical Plate With Variable Temperature and Mass Diffusion Through a
Porous Medium**

Amit¹

Department of Mathematics, GLA University, Mathura
India

R.K.Srivastava² Saraswat

Department of Mathematics, Agra College, Agra
Dr. B. R. Ambedkar University, Agra
India

Abstract: This paper analyzes heat and mass transfer on flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion through a porous medium. The dimensionless governing partial differential equations are solved by using Laplace transform method. Exact analytical solutions satisfy governing equations and the conditions are obtained. The velocity profiles are studied for different physical parameters. It is observed that the solutions are significantly affected by the radiation parameter Q , thermal Grashof number (Gr), mass Grashof number (Gc), permeability parameter (K), Schmidt number (Sc) and time t . The graphs are plotted for the variations in the velocity for different parameters.

Keywords: Porous Medium, exponentially accelerated infinite, vertical plate, Variable temperature, variable mass diffusion.

I. Introduction

Effect of heat and mass transfer plays vital role, in space craft design, in the cooling of liquid metal of nuclear reactors, pollution of environment etc. Flow through porous media have numerous engineering problems, for example, in the underground water resources, rain water harvesting, the movement of oil and natural gas through oil sandstone reservoirs, purification of crude oil, paper and pulp industry, membrane separation process, flow of blood. The purpose of present study is to study the Heat and Mass Transfer Effects on Flow Past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion through a porous medium.

Soundalgekar [1] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the Laplace-transform technique. Heat transfer effects on flow past an impulsively started infinite vertical plate in the presence of constant heat flux and variable temperature are studied by Soundalgekar and Patil [2,3]. Muthucumaraswamy et al. [4] studied the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Muthucumaraswamy and Valliamal [5] considered first order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion. Muthucumaraswamy et al. [6] studied heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. Basant kumar Jha [7] analyzed MHD free convection and mass transform flow through a porous medium. Recently R. Muthucumaraswamy, and A. Vijayalakshmi [8] studied the effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature. V. Rajesh [9] studied MHD effect of free convection and mass transform flow through a porous medium with variable temperature. Muthucumaraswamy Sathappan K.E and Nataragan R [10] studied the diffusion and heat transfer effects on exponentially accelerated vertical plate with variable temperature. Muthucumaraswamy R., Sundar Raj M. and Subramanian VSA [11] analyzed unsteady flow past an accelerated infinite vertical plate with variable temperature and mass diffusion. K. K. asogwa et al [12] studied the flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion.

Here the exponentially accelerated infinite vertical plate with variable temperature and mass diffusion through a porous medium has been considered. x^* axis is taken in vertically positive direction and flow is taken along x^* axis. y^* axis is taken in the direction normal to the plate. Initially in the stationary condition the temperature T_∞^* of the plate and the fluid is same at all points with concentration C_∞^* . The plate starts oscillatory motion in its own plane at time $t^* > 0$ with velocity $u^* = u_0 e^{at^*}$. The temperature and the concentration of the plate raised linearly with time. Then by usual Boussinesq's approximation the equation of unsteady flow governed is given by

$$\frac{\partial u^*}{\partial t^*} = g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \nu \left(\frac{u^*}{K} \right) \quad (1)$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q}{\partial y^*}$$

(2)

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - Q^* C^* \quad (3)$$

With initial and boundary conditions

$$t^* \leq 0, \quad u^* = 0, \quad T^* = T_\infty^*, \quad C^* = C_\infty^* \quad \text{for all } y^*$$

$$t^* > 0, \quad u^* = u_0 e^{at^*}, \quad T^* = T_\infty^* + (T_w^* - T_\infty^*)e^{at^*}, \quad C^* = C_\infty^* + (C_w^* - C_\infty^*) \left(\frac{u_0^2}{\nu} \right)^{\frac{1}{3}} t^* \quad \text{at } y^* = 0$$

$$u^* = 0, \quad T^* \rightarrow T_\infty^*, \quad C^* \rightarrow C_\infty^* \quad \text{as } y^* \rightarrow \infty \quad (4)$$

Normalizing the above equations by using following non-dimensionless parameters,

$$u = \left(\frac{u^*}{\nu u_0} \right)^{\frac{1}{3}}, \quad t = t^* \left(\frac{u_0^2}{\nu} \right)^{\frac{1}{3}}, \quad y = y^* \left(\frac{u_0}{\nu} \right)^{\frac{1}{3}}, \quad \theta = \left(\frac{T^* - T_\infty^*}{T_w^* - T_\infty^*} \right), \quad Sc = \frac{\nu}{D}, \quad Gr = \left(\frac{g\beta^*(T_w^* - T_\infty^*)}{u_0} \right),$$

$$Gc = \left(\frac{g\beta^*(C_w^* - C_\infty^*)}{u_0} \right), \quad C = \left(\frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} \right), \quad Pr = \frac{\mu C_p}{k}, \quad \frac{1}{K} = \frac{u_0^2 K^*}{\nu^2}, \quad R = \frac{4\sigma T_d^3}{ka_*} \quad (5)$$

Equation (1),(2),(3) and (4) become

$$\frac{\partial u}{\partial t} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} - Ku \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\alpha} \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - QC \quad (8)$$

With dimensionless initial and boundary conditions,

$$u = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } y \leq 0, \quad t \leq 0$$

$$u = e^{at}, \quad \theta = e^{at}, \quad C = t, \quad \text{at } y = 0, \quad t > 0$$

$$u = 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (9)$$

All physical variables are defined in nomenclature.

II. Solution

The governing equations (6),(7) and (8) with boundary condition are solved by laplace transform method. On taking laplace transform of the equation (6),(7),(8) and (9) we get,

$$\frac{d^2 \bar{u}}{dy^2} - (s + K)\bar{u} = -Gr\bar{\theta} - Gc\bar{C} \quad (10)$$

$$\frac{d^2 \bar{\theta}}{dy^2} - s\alpha\bar{\theta} = 0 \quad (11)$$

$$\frac{d^2 \bar{C}}{dy^2} - (s + K)Sc\bar{C} = 0 \quad (12)$$

$$\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{C} = 0 \quad \text{for all } y, t \leq 0$$

$$\bar{u} = \frac{1}{s-a}, \quad \bar{\theta} = \frac{1}{s-a}, \quad \bar{C} = \frac{1}{s^2} \quad \text{at } y=0, t > 0$$

$$\bar{u} = 0, \quad \bar{\theta} \rightarrow 0, \quad \bar{C} \rightarrow 0 \quad \text{as } y \rightarrow \infty, t > 0 \quad (13)$$

Where s is the parameter of Laplace transform

On solving above equations, we get,

$$\bar{u} = \left[\frac{1}{s-a} + \left\{ \frac{Gr}{(s-a)\{s(\alpha-1)-K\}} + \frac{Gc}{s^2\{s(Sc-1)+(QSc-K)\}} \right\} \right] e^{-y\sqrt{s+K}} - \left\{ \frac{Gre^{-y\sqrt{s\alpha}}}{(s-a)\{s(\alpha-1)-K\}} + \frac{Gce^{-y\sqrt{(s+Q)Sc}}}{s^2\{s(Sc-1)+(QSc-K)\}} \right\} \quad (14)$$

$$\bar{\theta} = \frac{e^{-y\sqrt{s\alpha}}}{s-a} \quad (15)$$

$$\bar{C} = \frac{e^{-y\sqrt{(Q+s)Sc}}}{s^2} \quad (16)$$

Taking inverse laplace transform of equation (14),(15)and(16),we get

$$u = \left(1 + \frac{Gr}{(\alpha-1)(a-b)} \right) \frac{e^{at}}{2} \left[e^{y\sqrt{K+a}} \operatorname{erfc}\{\eta + \sqrt{(K+a)t}\} + e^{-y\sqrt{K+a}} \operatorname{erfc}\{\eta - \sqrt{(K+a)t}\} \right] + \left(\frac{Gre^{bt}}{2(\alpha-1)(a-b)} \right) \left[e^{y\sqrt{K+b}} \operatorname{erfc}\{\eta + \sqrt{(K+b)t}\} + e^{-y\sqrt{K+b}} \operatorname{erfc}\{\eta - \sqrt{(K+b)t}\} \right] + \frac{Gc}{Sc-1} \left[\left(\frac{-1}{2d^2} - \frac{t}{2d} - \frac{y}{4d\sqrt{K}} \right) e^{y\sqrt{K}} \operatorname{erfc}(\eta + \sqrt{Kt}) + \left(\frac{-1}{2d^2} - \frac{t}{2d} + \frac{y}{4d\sqrt{K}} \right) e^{-y\sqrt{K}} \operatorname{erfc}(\eta - \sqrt{Kt}) \right] + \frac{Gc}{Sc-1} \left(\frac{e^{dt}}{2d^2} \right) \left[e^{y\sqrt{K+d}} \operatorname{erfc}(\eta + \sqrt{(K+d)t}) + e^{-y\sqrt{K+d}} \operatorname{erfc}(\eta - \sqrt{(K+d)t}) \right] + \left(\frac{Gre^{at}}{2(\alpha-1)(a-b)} \right) \left[e^{y\sqrt{\alpha a}} \operatorname{erfc}\{\eta\sqrt{\alpha} + \sqrt{at}\} + e^{-y\sqrt{\alpha a}} \operatorname{erfc}\{\eta\sqrt{\alpha} - \sqrt{at}\} \right]$$

$$\begin{aligned}
& - \left(\frac{Gr e^{bt}}{2(\alpha-1)(a-b)} \right) \left[e^{y\sqrt{\alpha b}} \operatorname{erfc} \left\{ \eta \sqrt{\alpha} + \sqrt{bt} \right\} + e^{-y\sqrt{\alpha b}} \operatorname{erfc} \left\{ \eta \sqrt{\alpha} - \sqrt{bt} \right\} \right] \\
& + \frac{Gc}{Sc-1} \left[\left(\frac{-1}{2d^2} - \frac{t}{2d} - \frac{y\sqrt{Sc}}{4d\sqrt{Q}} \right) e^{y\sqrt{ScQ}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Qt}) + \left(\frac{-1}{2d^2} - \frac{t}{2d} + \frac{y\sqrt{Sc}}{4d\sqrt{Q}} \right) e^{-y\sqrt{ScQ}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Qt}) \right] \\
& + \frac{Gc}{Sc-1} \left(\frac{e^{dt}}{2d^2} \right) \left[e^{y\sqrt{(Q+d)Sc}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Sc(Q+d)t}) + e^{-y\sqrt{(Q+d)Sc}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Sc(Q+d)t}) \right]
\end{aligned} \quad (17)$$

$$\theta = \frac{e^{at}}{2} \left[e^{y\sqrt{\alpha a}} \operatorname{erfc} \left\{ \eta \sqrt{\alpha} + \sqrt{at} \right\} + e^{-y\sqrt{\alpha a}} \operatorname{erfc} \left\{ \eta \sqrt{\alpha} - \sqrt{at} \right\} \right] \quad (18)$$

$$C = \left[\left(\frac{t}{2} + \frac{y\sqrt{Sc}}{4\sqrt{Q}} \right) e^{y\sqrt{ScQ}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Qt}) + \left(\frac{t}{2} - \frac{y\sqrt{Sc}}{4\sqrt{Q}} \right) e^{-y\sqrt{ScQ}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Qt}) \right] \quad (19)$$

$$\text{Where } b = \frac{K}{\alpha-1}, b = -\frac{QSc-K}{Sc-1}, \eta = \frac{y}{2\sqrt{t}}, \alpha = \frac{3Pr}{3+4R}$$

III. Result and Discussion

The problem of flow past an exponential accelerated infinite vertical plate and temperature with variable mass diffusion through porous media has been formulated and solved analytically. The values of velocity are obtained for the physical parameters such as thermal Grashof number Gr , mass Grashof number Gc , Prandtl number Pr , Schmidt number Sc , time t , permeability parameter K , reaction parameter Q , radiation parameter R , acceleration parameter a . Different velocity profiles have been presented in case of heating the plate ($Gr < 0$) and in case of cooling the plate.

The effect of velocity for different values of time t is presented in figure 1.

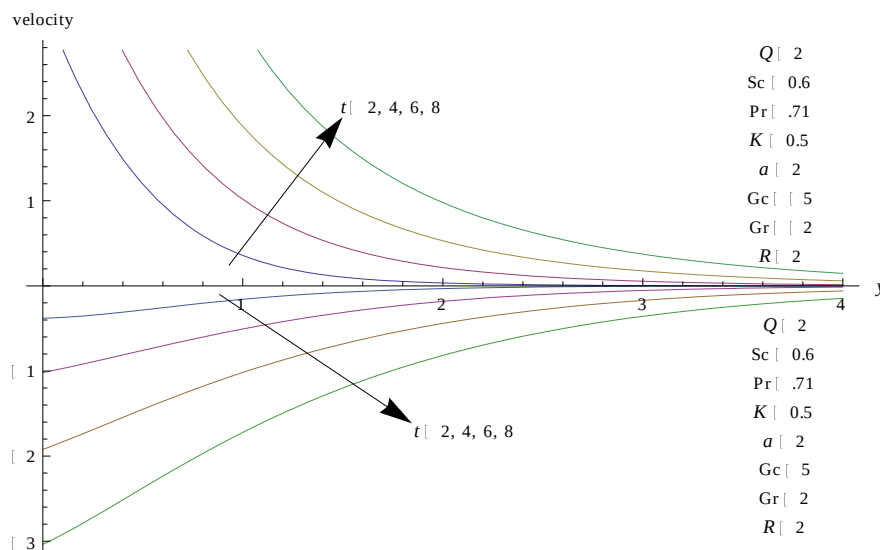


Figure:1 Velocity profile for different values of t

It is observed that the velocity increases as time increases in case of heating the plate but it decreases as time increases in case of cooling the plate.

In figure 2 velocity profile for different values permeability parameter has been presented.

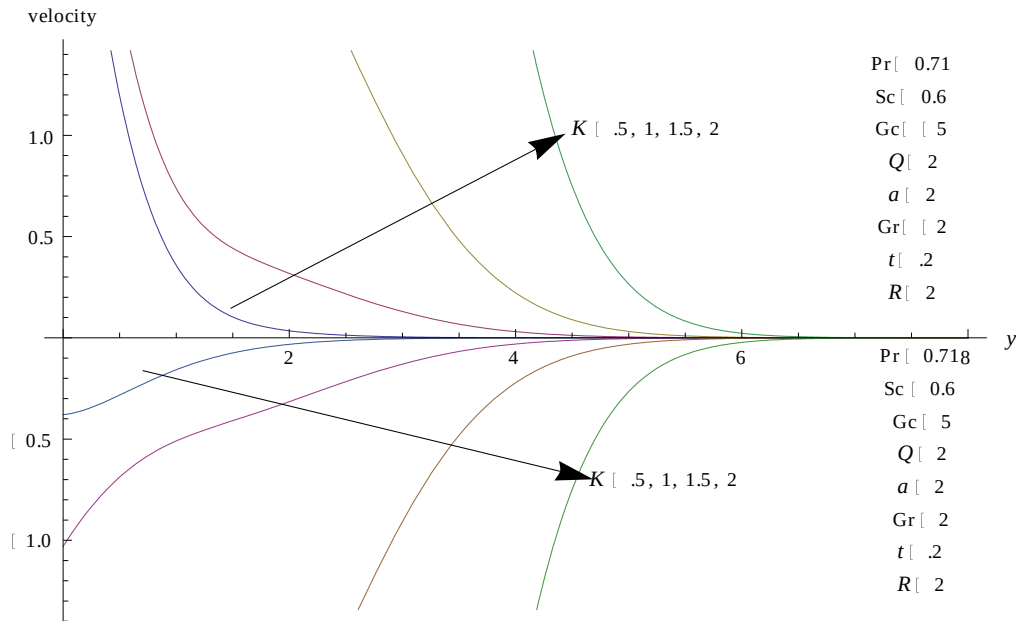


Figure:2 Velocity profile for different values of K

It is observed that the velocity increases as K increases in case of heating the plate but velocity decreases as K increases in case of cooling the plate.

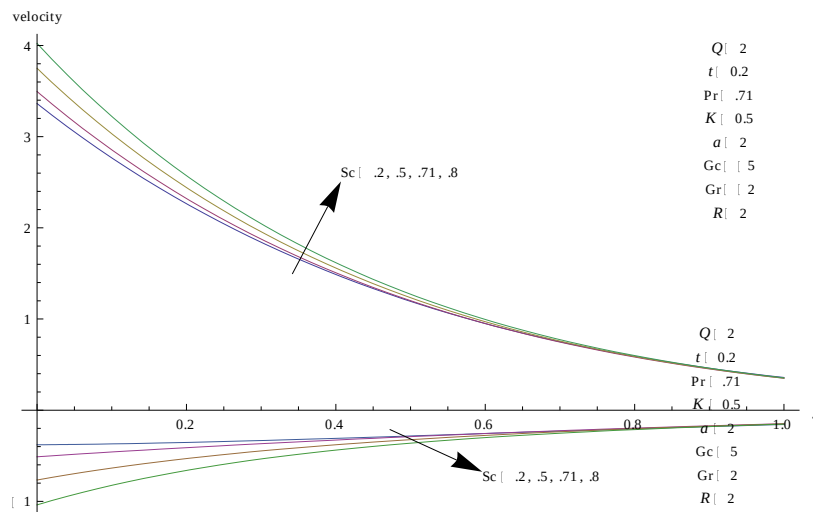


Figure:3 Velocity profile for different values of Sc

Figure 3 represents the velocity profile for different values of Sc . It is observed that the velocity increases as Sc increases in case of heating the plate while the reverse trend in case of cooling the plate.

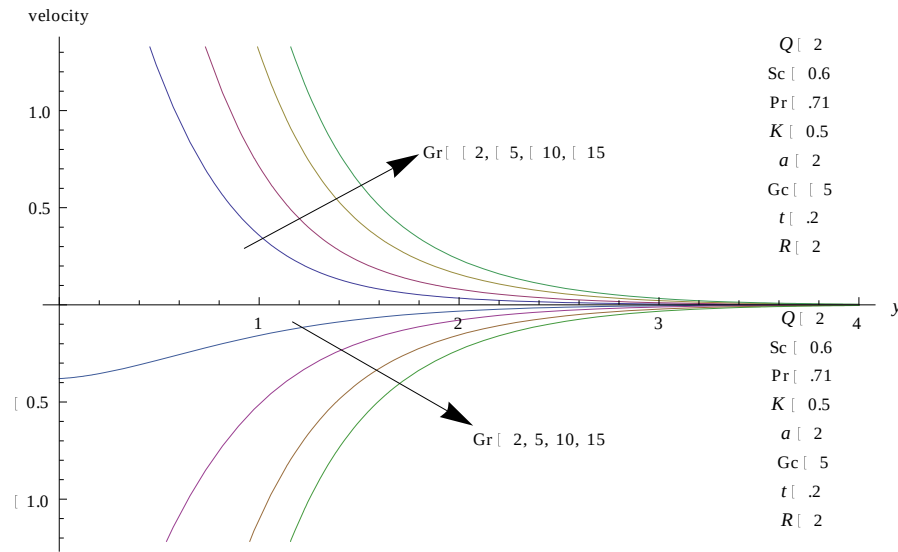


Figure:4 Velocity profile for different values of Gr

In figure 4 the velocity profile for different values of thermal Garshof number Gr has been presented and it is observed that the velocity increases as value of Gr decreases in case of heating the plate but in case of cooling the plate the reverse trend has been observed that the velocity decreases as Gr increases.

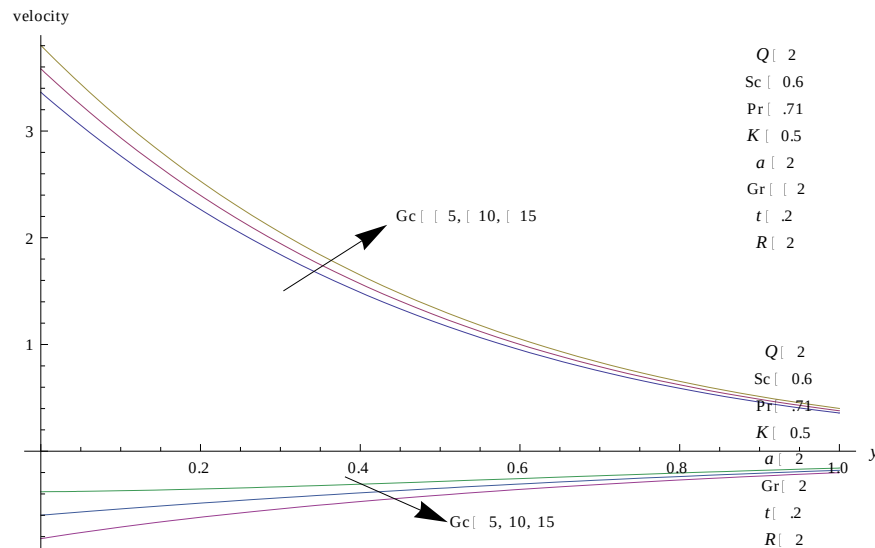


Figure:5 Velocity profile for different values of Gc

Velocity profile for different values of mass Garshof number Gc has been presented in figure:5

It is observed that the velocity decreases as Gc increases in case of cooling the plate . It is also observed that the velocity increases as Gc decreases in case of heating the plate .

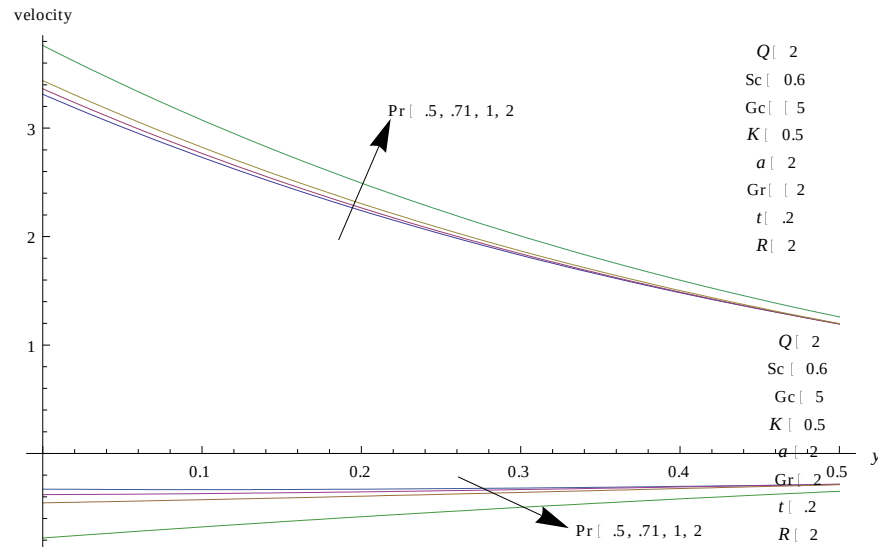


Figure:6 Velocity profile for different values of Pr

Figure 6 represents the velocity profile for different Prandlt numbers. It is observed that the velocity increases as Pr increases in case of heating the plate but reversed trend is observed in case of cooling the plate. The velocity decreases as Pr increases in case of cooling the plate.

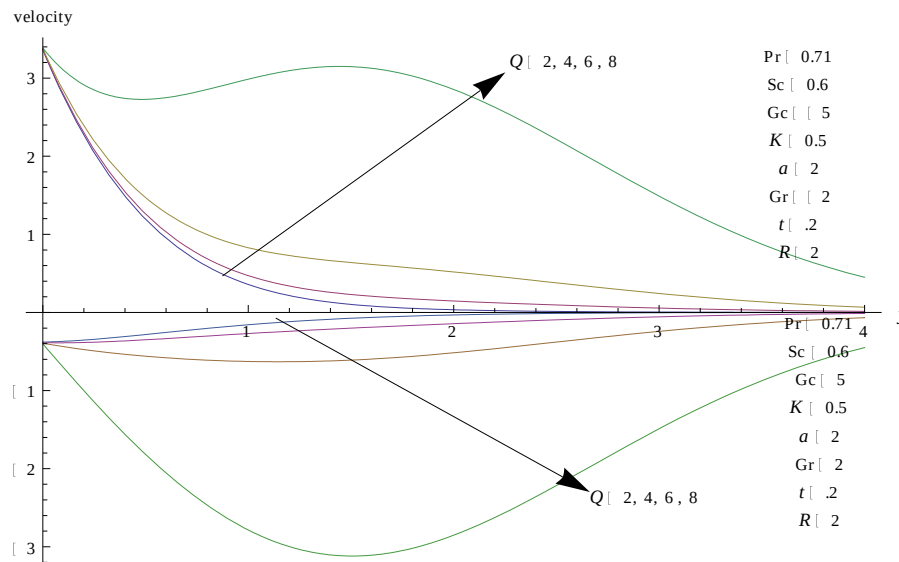


Figure:7 Velocity profile for different values of Q

The effect in velocity due to the reaction parameter has been presented in figure 7. The velocity increases with the increase of reaction parameter in case of heating the plate. In case of cooling the plate velocity decreases with the increase of reaction parameter.

IV. Conclusion

The following results have been concluded from the study:

The velocity increases as thermal Grashof number (Gr), mass Grashof number (Gc) decreases, and time (t), Permeability parameter (K), radiation parameter (Q), Schmidt number (Sc) increases in case of heating the plate.

The velocity decreases as thermal Grashof number (Gr), mass Grashof number (Gc), time (t), Permeability parameter (K), radiation parameter (Q), Schmidt number (Sc) increases in case of heating the plate.

REFERENCES

- 1.Soundalgekar V.M., Free convection effects on the stokes problem for an infinite vertical plate, *ASME Journal of Heat Transfer*, (99), pp. 499-501, 1977.
- 2 Soundalgekar VM and Patil MR., On flow past a vertical oscillating plate with variable temperature. *Latin American Journal of Heat and Mass Transfer* (4), pp.143–148, 1980a.
- 3 Soundalgekar VM and Patil MR., Stokes problem for a vertical plate with constant heat flux. *Astrophysics Space Science* (70),pp. 179–182. 1980b
- 4 Muthucumaraswamy R.,Sundar Raj M. and Subramanian VSA., Unsteady flow past an accelerated infinite vertical plate with variable temperature and mass diffusion, *International Journal of Applied Mathematics and Mechanics*, 5(6), pp.51-56 , 2009
5. Muthucumaraswamy R. and Valliamal V., First order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion, *International Journal of Engineering*, TUME VII, fascicule I (ISSN 1584-2665), 2009.
- 6 Muthucumaraswamy R.,Sathappan K.E and Nataragan R. ,Heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature, *Theoretical Applied Mechanics*, 35(4) , pp. 323-333 , 2008
- 7 Jha B.K., MHD free-convection and mass transform flow through a porous medium. *Astrophysics and Space science* 175,pp.283-289,1991.
- 8 Muthucumaraswamy R, and Vijayalakshmi A ,effect of heat and mass transfer on flow past an oscillating vertical plate with variable temperature ,*Int. J. of Appl. Math. and Mech.* 4(1), pp.59-65, 2008.
- 9 V. Rajesh,MHD effect of free convection and mass transform flow through a porous medium with variable temperature *Int. J. of Appl. Math and Mech.* 6 (14),pp. 1-16, 2010.
- 10 Muthucumaraswamy R.,Sathappan K.E and Nataragan R.. Diffusion and Heat transfer effects on exponentially accelerated vertical plate with variable temperature, *Theoretical Applied Mechanics*, Vol. 14(1), Pp. 73-77 , 2010.
- 11Muthucumaraswamy R.,Sundar Raj M. and Subramanian VSA. Unsteady flow past an accelerated infinite vertical plate with variable temperature and mass diffusion, *International Journal of Applied Mathematics and Mechanics*, Vol. 5(6), Pp. 51-56 , 2009
- 12 Asogwa et al. Flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion, *International Journal of Computer Applications*(0975-8887),Vol.45-No.2, 2012.

APPENDIX

Nomenclature

C^* Concentration in the fluid far away from the plate

C_w^* Concentration of the plate

y^* Coordinate axis normal to the plate

t^* Time

u^* Velocity of the fluid in the x^* -direction

u_0 Velocity of the plate

C Dimensionless concentration

y Dimensionless coordinate axis normal to the plate

u Dimensionless velocity

G_m Mass Grashof number

Gr Thermal Grashof number

Pr Prandtl number

Sc Schmidt number

C^* Species concentration in the fluid

C_p Specific heat at constant pressure

T_∞^* Temperature of the fluid far away from the plate

T^* Temperature of the fluid near the plate

T_w^* Temperature of the plate

κ Thermal conductivity of the fluid
D Chemical Molecular diffusivity
g Acceleration due to gravity
K permeability parameter
M Magnetic field parameter
t Dimensionless time
Q chemical Reaction Parameter
q radiative heat flux
R Radiation parameter

Greek symbols

μ Coefficient of viscosity
erfc Complementary error function
 ρ Density of the fluid
 τ Dimensionless skin friction
 θ Dimensionless temperature
 σ Electric conductivity
erf Error function
 ν Kinematic viscosity
 α Thermal diffusivity

Subscripts

w Conditions on the wall
 ∞ Free stream conditions