



Selection of VOQL Special Type Double Sampling Attribute Plans

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Abstract: In this paper, VOQL plans are derived for the special type double sampling plan, treating the outgoing quality and total inspection as random variables under total rectification and illustrated with the practicability of the determination of the plan.

I. Introduction

While a fixing a sampling plan, it is usual practice to fix the OC curve of the corresponding plan. The OC curve is inturn fixed by suitably chosen parameters. Usually the two points on the OC curve namely (AQL, α) and (LQL, β) are prescribed, where AQL is acceptable quality level, α is producer risk, LQL is limiting quality level and β is consumer risk. Hall (1979) has given a new approach to rectifying inspection plans by introducing Outgoing Quality (OQ) and Total Inspection (TI) as random variables. By considering the standard deviations of these random variables, the extremes in lot-to-lot quality can be revealed. In this paper, VOQL special type double sampling plan following Hall's procedure is derived under total rectification.

II. Special Type Double Sampling Plans

A Specisl Type Double Sampling Plan (STDSP) can be found in the liturture and it can be more useful if one wants to fix the OC curve with an acceptance number as either Zero or One, Hahn (1974) and Kaviyarasu and Devika (2017). Though this plan is not applicable frequently, it can be used for costly or destructive testing, as the acceptance numbers are very low. The STDSP is defined by

- (i) A random sample of n_1 items are selected from the lot and the number of defectives in the sample, d_1 , is observed.
- (ii) If $d_1 \geq 1$, the lot is rejected.
- (iii) If $d_1 = 0$, a second random sample of size n_2 , items are drawn from the lot and number of defectives in the second sample, d_2 , is observed.
- (iv) If the combined number of observed defectives $d_1 + d_2 \leq 1$ the lot is accepted, otherwise the lot is rejected.

Glossary of symbols

N : Lot size

n_i : Sample size at stage i

c_i : Acceptance number at stage i

r_i : Rejection number at stage i

x : Number of defective in the sample

p_{ai} : Probability of acceptance of a lot at stage i

p_{ri} : Probability of rejection of a lot at stage i

M : A constant, whose value is strictly less than one

S_{N-n_1} : Number of defectives in the unsampled portion of a lot before rectification and after a decision is reached at the end of first stage

$S_{N-n_1-n_2}$: Number of defectives in the unsampled portion of a lot before rectification and after a decision is reached at the end of second stage

$b(x; n, p)$: The probability distribution of x based on sample size n from a product under control with process average p .

p_{a1} and p_{a2} are defined by

$$p_{a_1} = \sum_{x=0}^{c_1} \frac{e^{-z} z^x}{x!}; \quad p_{a_2} = \sum_{k=c_1+1}^{c_2} \left[\frac{e^{-z} z^k}{k!} \sum_{x=0}^{c_2-k} \frac{e^{-z} z^x}{x!} \right]$$

and $p_a = p_{a_1} + p_{a_2}; p_r = p_{r_1} + p_{r_2}; p_a + p_r = 1$

III. Distributional properties of OQ and TI

Consider a production process under statistical control has 100p percent defectives in the long run. Lots of size N are made up of this product have varying number of defectives characterized by the density function.

$$P(X; N) = \binom{N}{x} p^x (1-p)^{N-x}; \quad x = 0, 1, \dots, N$$

The output of this process is said to be of quality p . Assume lots of size N of this product to be submitted to a double sampling inspection plan defined by (N, n_1, n_2, c_1, c_2) . The OC curve of this process is characterized by Type B situation of Dodge and Romig (1959).

From each lot of size N , form two portions, one consisting of sampled portion and the other unsampled portion. Unsampled portion of a lot is defined by $(N-n_1)$ when a decision is reached at first stage or $(N-n_1-n_2)$ when a decision is reached at the end of second stage.

Define a random variable X_1 as

$$X_1 = 0 \text{ if lot is rejected at stage 1} \\ = 1 \text{ if go to second sample}$$

When $X_1 = 2$, define another random variable X_2 as

$$X_2 = \begin{cases} 0 : \text{lot is rejected at stage 2} \\ 1 : \text{lot is accepted at stage 2} \end{cases}$$

The following probability models are used.

$$b(X; n, p) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0, 1, \dots, n \quad (1)$$

$$P(S_{N-n} = j) = \binom{N-n}{j} p^j (1-p)^{N-n-j}; \quad n = n_1; j = 0, 1, 2, \dots, (N-n_1) \quad (2) \\ = n_1 + n_2; j = 0, 1, \dots, (N-n_1-n_2)$$

$$\text{Then, } P(X_1 = j) = \begin{cases} p_{r_1} : j=0 \\ p_{r_2} : j=1 \end{cases}$$

Where,

$$p_k = b(k; n_1, p); \quad k = 1, 2, \dots, c_2$$

$$\text{and } P(X_2 = i) = \begin{cases} p_{a_2} : i=0 \\ p_{a_1} : i=1 \end{cases}$$

The above models follow the assumption that the lot of size N contains x defective items where the conditional distribution of the number of defective items in the samples is given by

$$P(x|X) = \frac{\binom{X}{x} \binom{N-X}{n-x}}{\binom{N}{n}}; \quad X = 0, 1, 2, \min(n, x)$$

$$P(X; N) = \binom{N}{x} p^x (1-p)^{N-x}; \quad x = 0, 1, \dots, N \quad (3)$$

Generally, in a double sampling plan, one assumes that $r_1 = r_2 = c_2 + 1$ with $c_1 < c_2$ and $n_2 = kn_1$ ($k = 1, 2$), Duncan (1986).

Define the random variable Outgoing Quality (OQ), as the ratio of the number of defectives in the unsampled portion of the lot after inspection and rectification to the total number of items in the lot. Assuming screening to be conducted without error, symbolically one gets,

$$OQ = \frac{1}{N} \left[\left(1 - \frac{X_1(X_1-1)}{2} \right) S_{N-n_1} X_1 + \frac{X_1(X_1-1)}{2} S_{N-n_1-n_2} X_2 \right]; \quad X_1=0,1; \quad X_2=0,1 \quad (4)$$

Directly determining the probability distribution of OQ from equation (4) is very difficult as it involves the sums, differences and products of the following, independent and dependent random variables $X_1, \frac{X_2(X_2-1)}{2}, S_{N-n_1}$ and $S_{N-n_1-n_2}$.

Hence, to determine the probability distribution of OQ, all possible probability values of OQ were enumerated and combined. This leads to compact form, after rearrangement, which is given below

$$P(OQ = j | N) = \begin{cases} p_{a_1} b(j; N - n_1, p) + p_{a_2} b(j; N - n_1 - n_2, p) + p_r : j = 0 \\ p_{a_1} b(j; N - n_1, p) + p_{a_2} b(j; N - n_1 - n_2, p) : j = 1, 2, \dots, (N - n_1 - n_2) \\ p_{a_1} b(j; N - n_1, p) : j = (N - n_1 - n_2) + 1, \dots, (N - n_1) \end{cases} \quad (5)$$

Thus

$$E(OQ) = AOQ = (N - n_1) p_{a_1} + (N - n_1 - n_2) p_{a_2} \left(\frac{p}{N} \right) \quad (6)$$

$$\text{and } Var(OQ) = \frac{1}{N^2} \left[\begin{aligned} & p_{a_1} \left\{ (N - n_1) p (1 - p) + (N - n_1)^2 p^2 \right\} \\ & + p_{a_2} \left\{ (N - n_1 - n_2) p (1 - p) + (N - n_1 - n_2)^2 p^2 \right\} \end{aligned} \right] - (AOQ)^2 \quad (7)$$

The random variable total inspection (TI), is defined as

$$TI = \left[1 - \frac{X_1(X_1-1)}{2} \right] [n_1 X_1 + N(1 - X_1)] + \left[\frac{X_1(X_1-1)}{2} \right] [(n_1 + n_2) X_2 + N(1 - X_2)]; \quad X_1 = 0, 1; \quad X_2 = 0, 1 \quad (8)$$

The probability distribution of TI is defined as

$$P(TI = i) = \begin{cases} p_{a_1} : i = n_1 \\ p_{a_2} : i = n_1 + n_2 \\ 1 - p_a : i = N \end{cases} \quad (9)$$

$$\text{Thus, } E(TI) = ATI = n_1 p_{a_1} + (n_1 + n_2) p_{a_2} + N(1 - p_a) \quad (10)$$

$$Var(TI) = (N - n_1)^2 p_{a_1} + (N - n_1 - n_2)^2 p_{a_2} - \left(\frac{N(AOQ)}{p} \right)^2 \quad (11)$$

IV. VOQL Plans

In this section double sampling plans employing total rectification that have a designated maximum variance of the outgoing quality, at some level of the process average, that minimizes the average total inspection are developed. These plans are, just as sampling plans are determined which yield a designated maximum AOQL, called AOQL. Denote the variance of the outgoing quality, a function of process average, p as VOQ and call the maximum variance attainable, for variations in p , the VQOL of the plan.

To develop a VOQL plan, assume the lot size to be large. This leads to, the coefficients $(N - n_1) / N^2$ and $(N - n_1 - n_2) / N^2$ in $Var(OQ)$ are near zero and the variance of outgoing quality may be approximated as

$$VOQ = \left[\frac{N - n_1}{N} \right]^2 p^2 p_{a_1} (1 - p_{a_1}) + \left[\frac{N - n_1 - n_2}{N} \right]^2 p^2 p_{a_2} (1 - p_{a_2}) \quad (12)$$

Now, substituting $n_1 p = z$ and $n_2 = kn_1$ in equation (12), assume the distribution of defectives in the sample to be adequately approximated by the Poisson model, equation (12) becomes

$$VOQ = \left[\frac{N - n_1}{Nn_1} \right]^2 z^2 p_{a_1} (1 - p_{a_1}) + \left[\frac{N - n_1(1+k)}{Nn_1} \right]^2 z^2 p_{a_2} (1 - p_{a_2}) \quad (13)$$

setting $\frac{d}{dz}(VOQ) = 0$ implies;

$$\begin{aligned} & \left[\frac{N - z/p}{p} \right]^2 \left[2p_{a_1} (1 - p_{a_1}) + z(1 - 2p_{a_1}) \frac{d}{dz}(p_{a_1}) \right] \\ & + \left[\frac{N - z/p(1+k)}{p(1+k)} \right]^2 \left[2p_{a_2} (1 - p_{a_2}) + z(1 - 2p_{a_2}) \frac{d}{dz}(p_{a_2}) \right] \end{aligned} \quad (14)$$

where;

$$\begin{aligned} \frac{d}{dz}(p_{a_1}) &= \sum_{x=0}^{C_1} \frac{e^{-z} z^{x-1} (x-z)}{x!} \\ \frac{d}{dz}(p_{a_2}) &= \sum_{k=C_1+1}^{C_2} \frac{1}{k!} \left[\sum_{x=0}^{C_2-k} \frac{e^{-2z} z^{k+x-1} (k+x-2z)}{x!} \right] \end{aligned}$$

Equation (14) is a necessary condition for a maximum VOQ. Therefore,

$$VOQL = MaxVOQ = \left[\frac{N - n_1}{Nn_1} \right]^2 (z^*)^2 p_{a_1}^* (1 - p_{a_1}^*) + \left[\frac{N - n_1(1+k)}{Nn_1} \right]^2 (z^*)^2 p_{a_2}^* (1 - p_{a_2}^*) \quad (15)$$

where, z^* is the value of z which satisfies the equation (14) for a plan (N, n_1, n_2, c_1, c_2) and $p_{a_1}^*, p_{a_2}^*$ are probability of acceptance at stage 1 and stage 2 respectively, calculated at $z = z^*$. Equation (15) can be written as

$$VOQL = \left[\frac{N - n_1}{Nn_1} \right]^2 VOQF_1 + \left[\frac{N - n_1(1+k)}{Nn_1} \right]^2 VOQF_2 \quad (16)$$

$$\text{where, } VOQF_1 = (z^*)^2 p_{a_1}^* (1 - p_{a_1}^*) \quad (17)$$

$$VOQF_2 = (z^*)^2 p_{a_2}^* (1 - p_{a_2}^*) \quad (18)$$

are called VOQ factors.

For each pair of value (c_1, c_2) a distinct $VOQF_1$ and $VOQF_2$ are defined by solving equation (14) for z^* and evaluating the equations (17) and (18). This procedure leads to tables $VOQF_1$ and $VOQF_2$.

To determine a plan for a specified VOQL, equation (16) can be written as a quadratic equation in $(1/n_1)$.

That is,

$$A\left(\frac{1}{n_1}\right)^2 - B\left(\frac{1}{n_1}\right) + C = 0 \quad (19)$$

where;

$$A = VOQF_1 + VOQF_2$$

$$B = \frac{2[VOQF_1 + (1+k)VOQF_2]}{N}$$

$$C = \frac{VOQF_1 + (1+k)^2 VOQF_2}{N} - VOQL$$

Solving equation (19) for n_1 gives,

$$n_1 = \frac{N(VOQF_1 + VOQF_2)}{VOQF_1 + (1+k)VOQF_2 + f} \quad (20)$$

where;

$$f^2 = N^2(VOQF_1 + VOQF_2)VOQL - K^2 VOQF_1 \times VOQF_2$$

Thus, to determine a plan for a specified VOQL, select the (c_1, c_2) value of the plan. Employing the $VOQF_1$ and $VOQF_2$ corresponding to the selected (c_1, c_2) values together with the appropriate lot size, calculate the sample size at stage 1 of the plan by using the relation defined in equation (20). The procedure described above gives many plans, which have the same designated VOQL. It is therefore possible to select a plan, which has the minimum ATI, in addition to the designated VOQL at some designated value of the process average.

V. Example

Consider $N = 1000$, $VOQL = (0.015)^2$, $p = 0.02$ and $n_2 = 2n_1$. To obtain a required plan, for $c_1=0$, $c_2 = 1$ the value for $VOQF_1$ is 0.47686 and $VOQF_2$ is 0.12720. Equation (19) gives $n_1 = 29$. Hence the plan is (1000, 49, 98, 0,1) while ATI equal to 525. Adopting the above procedure by choosing $c_1 = 0$ and $c_2 = 1$ for different values of M , one gets different double sampling plan. The plans thus obtained are given in Table 2, which lists double sampling plans with $c_1 = 0$, $c_2 = 1$; for each plan listed values of p_{a_1} , p_{a_2} , p_a and ATI, for $p = 0.02$. Corresponding to $c_1 = 0$ it is apparent that the ATI has assumed a relatively minimum value of 305 and ATI value shows a rise for the value of c_2 . Hence, for $c_1 = 0$, the plan (1000, 92, 184, 0, 1) has relatively minimum ATI.

VI. Conclusion

In this paper a methodology has been developed to design a special type double sampling attribute plans for a specified Variance and Outgoing Quality limit by treating OQ and TI as well defined random variables and illustrated.

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