



Estimation of Time Delay in Nonlinear System using Average Mutual Information

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Abstract: Time-delay estimation in nonlinear systems is of critical value in the tasks of system identification, modeling and performance monitoring of processes. Both time domain and frequency domain based methods exist in the literature for identification of delaying systems. However, these methods assume linear relationship between the input signal and output of the process. In this work, we use an estimator (using the concept of average mutual amount of information) which provides a measure of both the linear and nonlinear statistical dependencies between two time series. The estimation of AMI involves estimation of joint probability distribution between the input and output datasets. Thus, AMI looks at all the higher order statistical relationship between the input and output and is a superset of the classical correlation analysis which looks at only the second order relationship between the provided data sets. The above fact brings into light that mutual information is a natural choice for estimating delay in nonlinear systems. Further, an attempt is made to give confidence intervals for these estimates using the idea that the AMI estimate follows beta distribution. The first lag at which the AMI estimate is above the estimated confidence interval (AMI value is significant) is considered as the delay present in the process. This is similar to the delay estimation in processes using the classical correlation analysis, where the first delay above the provided confidence limit is considered as the delay presents in the process. Simulation studies are performed to show the practicality and utility of the proposed method in delay estimation.

Keywords: Average mutual information; delay estimation; nonlinear system; correlation analysis, probability.

I. Introduction

Many real physical systems, such as chemical processes, industrial systems, power and water distribution networks, econometric systems, etc. has inherent time delays present in them. The reason is that realistic systems require a nonzero time to respond to external stimuli. Therefore, the identification of time delays is one of the most important issues in system modelling and identification. The knowledge of this delay is critical to the monitoring, identification and control of such processes. This idea of time delay estimation from measurements has been put into practice for decades now. Several methods that exploit the relationships between the data in both time and frequency domain have been developed to estimate the delay in the processes. Given the complexities underlying the industrial processes and the availability of measured data, it is attractive as well as practical to estimate these delays from the data rather than making an attempt to obtain the same from a physical understanding of the process. Sophisticated methods are now available to estimate time delay in linear systems. However, there are several grey areas in this field, particularly in the context of linear and nonlinear systems. In this present work, a method based on Average Mutual Amount of Information (AMAI) is used to estimate time delay for nonlinear systems. The aim of this study is to use mutual information analysis to measure the transmission of information between various nonlinear systems. AMAI estimation involves computation of joint probability distribution between the input and output datasets. Therefore, AMAI utilizes all the higher order statistical relationship between the data sets which makes it a natural choice for time delay estimation in nonlinear systems.

II. Motivation for this study

Cross correlation function (CCF) is the method being currently used for time-delay estimation but this is applicable only for linear systems. Most of the real time processes are nonlinear in nature and mutual information theory can be applied in these nonlinear systems. Even in linear systems finding delay using AMAI is much better than classical correlation function. The estimation of AMI involves estimation of joint and marginal probability distribution functions between the input and output. This implies, AMI looks at all the higher order statistical relationship between the input and output and is a superset of the classical correlation analysis which looks at only

the second order relationship between the provided data sets. This requires no a priori information of the process model. This is quite natural because in real life we don't have any information about the input and output of the processes. For the analysis of time-delay in chemical processes the use of AMAI is not at reported. This method can work even when the input excitation is a step function which is more realistic in nature. In this work we plan to use AMAI for estimation of time-delay in nonlinear systems.

III. Literature Review

E. J. Wegman et al. (1972) [5] explains the detailed methods of estimation of nonparametric probability density function. Recent literature in statistical theory has provided a large class of nonparametric density estimates, alternatives to the histogram. This paper is an attempt at a complete listing of various estimates together with many of the properties of these estimates. The literature in this area is fairly extensive.

N. J. I. Mars and G. W. Van Arragon et al. (1981) [1] has done detailed study of Average Mutual Amount of Information (AMAI). An estimator for time delay using the concept of average mutual amount of information is described. Although analogous to the cross correlation function, this estimator is not restricted to time delay estimation in linear systems. In many practical situations the joint and marginal probability density functions will not be known and will have to be estimated. Of the different approaches to probability density function estimation described in the literature.

Bernhard Goebel, Zaher Dawy, Joachim Hagenauer and Jakob C. Mueller et al. (2005) [5] has done a detailed study of Average Mutual Amount of Information (AMAI) and given approximate expressions for the distribution of mutual information (MI). These distributions are functions of the available sample size and the number of realisations of the random variables only. Knowledge of the variables pmf is not required. The results are verified numerically for various cases. Exemplary application ideas in statistics and communications engineering are proposed.

Nicolaas J. I. Mars et al. (1982) [12] has implemented Average Amount of Mutual Information (AMAI) in the estimation of time delay between simultaneously recorded EEG (electroencephalographic) signals is important for the determination of the area in the brain which is responsible for the synchronous activity seen during seizures in epileptic patients. Showed that this procedure is an improvement over the use of the cross correlation function. The problem of estimating a probability density function from a number of samples has been discussed extensively in the literature. Wegman [5], [7] and Fryer [10] have given reviews of this literature.

Marcus Hutter et al. (1982) [11] has done detailed study of Average Mutual Amount of Information (AMAI) and gave expressions for expected and variance of AMI value. Suggested that the average of mutual information follows the Dirichlet Posterior distribution (multinomial beta distribution). Numerical issues and the range of validity are discussed.

IV. Scope of the Work

Implementation of AMI in nonlinear process to identify the time delay are shown using a SIMULINK model. Generally CRA is used to identify delay in linear systems. Time delay estimation using CRA fails with step excitation even though the system is linear. It is shown that AMI overcomes this disadvantage of CRA in estimating time delays in both linear and nonlinear systems with step excitation. Further, gave confidence intervals for these estimates using the idea that the AMI value follows beta distribution of first kind. To estimate time delays in nonlinear systems we computed the value of AMAI between the input and output signal for a range of lag values, in analogy to the cross correlation function. The first lag is considered as the delay and the last lag is considered as the order present in the process at which the AMI estimate is above the estimated confidence interval (AMI value is significant). This is similar to the delay estimation in processes using the classical correlation analysis.

V. Results and Discussion

A. Cross Correlation Analysis Vs Average Mutual Information

For the comparison of AMI with CRA, a simple linear system was taken with step excitation which has delay three. CRA function plots the impulse response over the lag 0 to 20 along with two confidence lines. From the CRA function plot, the lag at which the response of impulse function lies outside the confidence interval is considered to be significant. For this example, the system considered is pure delay system with delay three. But from the plot we noticed that for the lag other than three also the impulse response lies outside the confidence limit. CRA failed to estimate delay for this simple system. The same system is considered to estimate delay using AMI method. This method estimated correctly the system delay at lag three. These results are shown below.

B. Confidence interval of AMI Estimate

It is verified that AMI values follow a Beta distribution of first kind. It is also verified using literature. Further 100 realizations of the AMI point estimate at both maximum value and minimum value is obtained individually over the lags 0 to 20 to verify that the point estimates follow a Beta distribution (Usage of dfittool in Matlab).

AMI value is calculated between the input and output. Maximum and minimum value of AMI is noted along with their lags. 100 realizations are performed to obtain the value of the AMI estimate at the maximum value and also at the minimum value. dfittool is used to find out the parameters of the Beta distribution. Beta distribution values lie between zero and one.

Parameters got from dfittool corresponding to minimum value are used to find out the value of confidence line. i.e. We find out the confidence line below which 95% of AMI value pertaining to the minimum AMI value. The values of lag above this line are significant. This estimate is found out to be so robust and all the significant AMI values (various delays) are above this confidence line and insignificant values are below this line.

Parameters got from dfittool corresponding to maximum value are used to find out the value of confidence line. i.e. We find out the confidence line below which 5% of AMI value pertaining to the maximum AMI value. The values of lag above this line are significant. This estimate is not so robust but we have also tried this so that we can concentrate on type II error also. All the cases lower limit line is sufficient. In all the simulations, the lower red line is of interest and provides consistent results.

C. The above procedure is followed for two examples and the results are shown below.

Figure1 A simple non-linear system with lag 3 and its AMI and AMAI Measures.

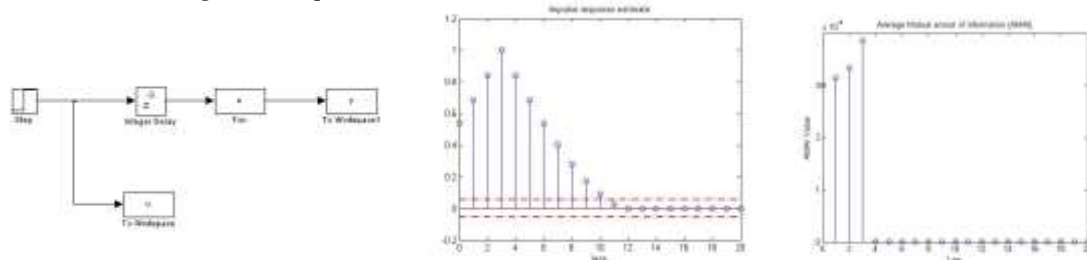
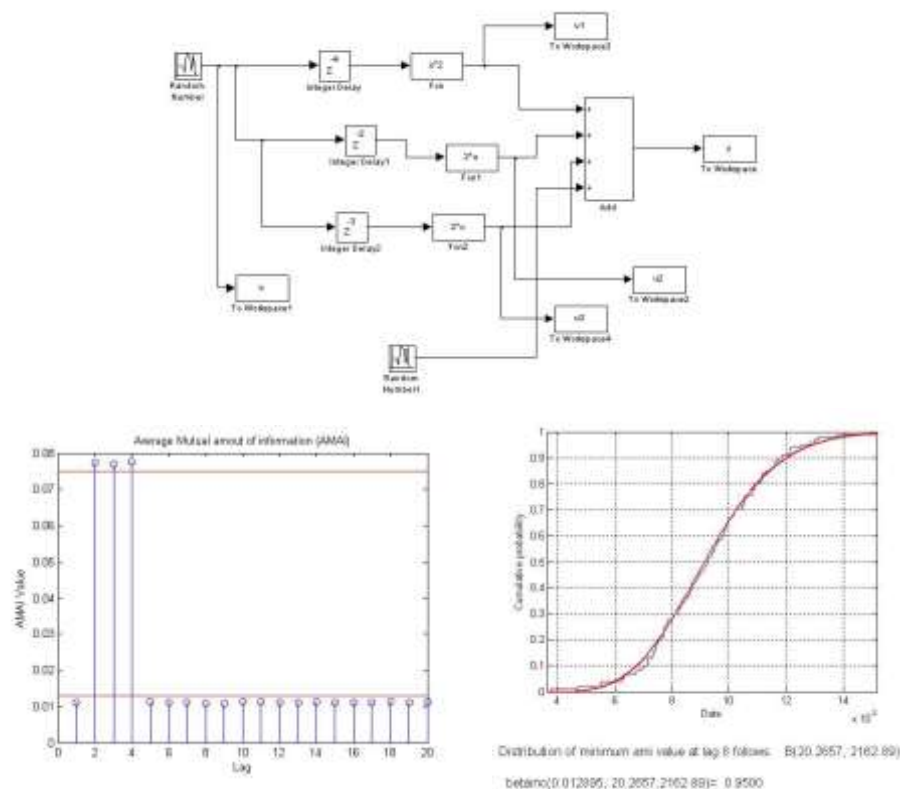


Figure2 A non-linear system with lag 4 and its AMI, AMAI Measures and its approximation distribution.



VI. Conclusions

AMI has a versatile feature for estimating the time delay and order the process. It is compared with the well-known technique CRA for a least excited signal (step input) given to a simple linear system of delay three. The results obtained for delay estimation from CRA and AMI are 1 and 3 respectively. It implies that with the least efforts the delay can be estimated for linear systems.

A novel method is proposed for estimating the time delay and order in nonlinear systems using AMI. To estimate these parameters the confidence limits of AMI values are determined based on Beta and Gamma distribution. AMI is applied on different class of nonlinear systems to demonstrate its robustness. Statistical analysis is performed on AMI values and the Beta distribution is proved to be better choice in estimating the confidence intervals.

VII. References

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