



Square Difference Labeling of Triangular Belt Graph and Finding the Sum of the Edges

S.Sriram¹, Dr.R.Govindarajan²

¹Assistant Professor, Department of Mathematics,
Patrician College of Arts and Science, Adyar, Chennai-20, India
Dr.R.Govindarajan²

²Associate Professor and Head (Retd.)

P.G and U.G, Department of Mathematics, D.G.Vaishnav College, Arumbakkam, Chennai, India

Abstract: Let $G = (V, E)$ be a graph with p vertices and q edges. A square difference labelling is defined as a bijection $f : V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \rightarrow \mathbb{N}$ given by $f^*(u, v) = \left| (f(u))^2 - (f(v))^2 \right|$ for every $uv \in E(G)$ and all are distinct. A graph which admits square difference labelling is called square difference graph. In this paper we discuss the square difference labelling of triangular belt graph and also try to analyse on the generalised nature of triangular belt graph $TB(n)(\downarrow^n)$ by choosing one part of triangular belt graph and adding successively 1 part at every point of time to find the general triangular belt graph $TB(n)(\downarrow^n)$ and also study on the sum of edges of each part and have initiated to find the sum of edges of any triangular belt graph $TB(n)(\downarrow^n)$ for which the size n is known. Also we intend to study some properties by which we construct the triangular belt graph $TB(n)(\downarrow^n)$. In further we prove that sum of the edges of ladder graph $L_n = P_n \times P_2$ $n \geq 2$ can be obtained from the triangular belt graph $TB(n)(\downarrow^n)$.

Keywords: Triangular belt graph, square difference labelling, square difference labelling graph, Ladder graph.

I. Introduction

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. J.A Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. Different labelling techniques on different graphs make the graph labelling papers interesting. One interesting labelling type is square difference labelling introduced by J.Shiamma.[2,3,4] and have analysed on various graph to be called as square difference labelling graph, G.Amuda and S.Meena[5] studied on square difference labelling of graphs in their paper. Motivated on the square difference labelling techniques we try to study further on the square difference labelling of graphs. Labelling of graphs is as such is to identify a label for vertices or edges or both so as to satisfy some condition so as to call the graph a particular labelling graph. It is also interesting that the number of vertices and number of edges that are required to be labelled and the way in which it is labelled. So in this connection we have taken the graph triangular belt $TB(n)(\downarrow^n)$ and proved that the graph is square difference labelling graph and also tried to generalise the nature of obtaining a triangular belt graph $TB(n)(\downarrow^n)$ by considering a part of the graph which is systematically increased one part at a time to give the required triangular belt graph $TB(n)(\downarrow^n)$ of desired size. n In this study we also have started thinking of finding the sum of edges of each part so as to find the sum of the edges of any triangular graph $TB(n)(\downarrow^n)$. Also we have intended to study some properties connecting the construction of triangular belt graph $TB(n)(\downarrow^n)$. In further we have obtained the sum of the edges of ladder graph $L_n = P_n \times P_2$ $n \geq 2$ in terms of sum of the edges of triangular belt graph $TB(n)(\downarrow^n)$

II. Preliminaries

Definition 2.1: Let $L_n = P_n \times P_2$ ($n \geq 2$) be the ladder graph with vertex set u_i and v_i , $i=1,2,\dots,n$. The Triangular Belt is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \leq i \leq n-1$. This graph is denoted by $TB(n)(\downarrow^n)$.

III. Main Results

Theorem 3.1 : The Triangular Belt graph $TB(n)(\downarrow^n)$ is square difference labelling graph

Proof: Let $G = TB(n)(\downarrow^n)$ be the triangular belt graph. Let $L_n = P_n \times P_2$ ($n \geq 2$) be the ladder graph with vertex set u_i and v_i , $i=1,2,\dots,n$. The Triangular Belt graph $G = TB(n)(\downarrow^n)$ is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \leq i \leq n-1$. This graph is denoted by $TB(n)(\downarrow^n)$. The vertex set is $V = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and the edge set is $E = \{u_i u_{i+1}, v_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i, 1 \leq i \leq n\}$. Now to label the vertices let us consider the bijective function $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function

$f^*: E(G) \rightarrow N$ given by $f^*(u, v) = \left| (f(u))^2 - (f(v))^2 \right|$ for every $uv \in E(G)$ and all are distinct.

A graph which admits square difference labelling is called square difference graph. We define the labelling of vertices $u_1, u_2, \dots, u_n$ and for $v_1, v_2, \dots, v_n$ as follows

$$f(u_1) = 0$$

$$f(u_i) = 2(i-1) \text{ for } 2 \leq i \leq n$$

$$f(v_i) = 2i-1 \text{ for } 1 \leq i \leq n$$

Then the induced edge labelling for the triangular belt $TB(n)(\downarrow^n)$ are

$$f^*(u_i u_{i+1}) = 8i-4 \text{ for } 1 \leq i \leq n-1$$

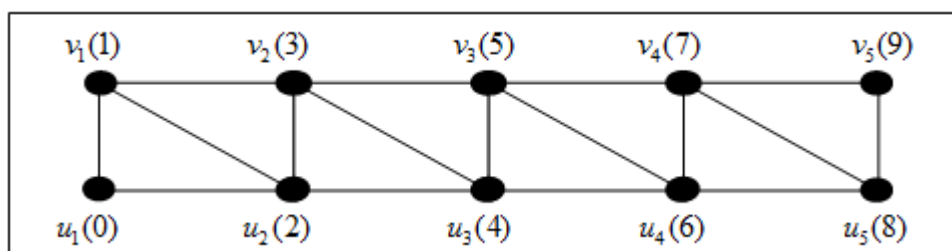
$$f^*(v_i v_{i+1}) = 8i \text{ for } 1 \leq i \leq n-1$$

$$f^*(u_i v_i) = 4i-3 \text{ for } 1 \leq i \leq n$$

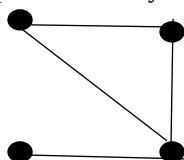
$$f^*(v_i u_{i+1}) = 4i-1 \text{ for } 1 \leq i \leq n-1$$

Noticing the induced edge labelling we find that the edges are all distinct and they are assigned with N. Therefore the triangular belt $TB(n)(\downarrow^n)$ graph is a square difference labelling graph.

Figure1A Square difference labelling of triangular belt graph $TB(5)(\downarrow^5)$



Definition 3.3: One part in Triangular Belt graph $TB(n)(\downarrow^n)$: For a triangular belt graph $TB(n)(\downarrow^n)$ we define one part denoted by $T(F)$ as shown below where each part consists of 2 vertices and 4 edges.



The above is the one part of triangular belt graph

We construct the basic triangular belt graph $TB(2)(\downarrow^2)$ and from it we understand that it is possible to obtain $TB(3)(\downarrow^3)$ by adding one part $T(F)$. Similarly we can construct any order triangular belt graph $TB(n)(\downarrow^n)$.

Definition.3.4: Let us define the number of vertices in a triangular belt graph as $V(TB(n))$ and number of edges in a triangular belt graph as $E(TB(n))$, the number of vertices in one part $T(F)$ as $V(T(F))$ and number of edges in one part $T(F)$ as $E(T(F))$.

Theorem.3.5: For a triangular belt graph $TB(n)(\downarrow^n)$ we have the following results

$$V(TB(n)) = V(TB(2)) + 2(n-2); E(TB(n)) = V(TB(2)) + 4(n-2) \text{ for } n \geq 3.$$

Proof: Let us consider the triangular belt graph $TB(n)(\downarrow^n)$ with the labelling procedure adopted in theorem.3.1. Now let us prove the results on the size of the triangular belt graph on the number of vertices and on the number of edges by method of mathematical induction. Consider for $n=3$. On substituting $V(TB(n)) = V(TB(2)) + 2(n-2)$ in we find that $V(TB(3)) = V(TB(2)) + 2(3-2)$

On simplifying we have $V(TB(3)) = V(TB(2)) + 2$ which implies

$$V(TB(3)) = 4 + 2 = 6.$$

Which is true since the number of vertices in triangular belt graph $TB(2)(\downarrow^2)$ is 4 and each part of triangular part consists of 2 vertices. Hence the result is true for $n=3$

Now let us assume that it is true for $n=k$

$$\text{i.e } V(TB(k)) = V(TB(2)) + 2(k-2)$$

Now let us claim the result is true for $n=k+1$

$$\text{i.e to prove } V(TB(k+1)) = V(TB(2)) + 2(k-1)$$

$$\text{Consider } V(TB(k+1)) = V(TB(k)) + V(TB(1))$$

We have assume that the result is true for $n=k$ and $V(TB(1))$ refers to one part of number of vertices which is 2.

$$\text{Hence } V(TB(k)) = V(TB(2)) + 2(k-2) + 2$$

On simplifying we have $V(TB(k+1)) = V(TB(2)) + 2(k-1)$. Hence the theorem is true for the number of vertices.

Now similarly let us claim for the number of edges of triangular belt graph

Let us prove for $n=3$ the result $E(TB(n)) = V(TB(2)) + 4(n-2)$ is true. On substituting we have

$$E(TB(3)) = V(TB(2)) + 4(3-2) \text{ and on simplifying we have } E(TB(3)) = V(TB(2)) + 4. \text{ Which is true as number of edges in } TB(2)(\downarrow^2) \text{ is 5 and one part of triangular belt graph consists of 4 edges. Hence } E(TB(3)) = 5 + 4 = 9.$$

$$\text{Now let us assume that it is true for } n=k \text{ i.e } E(TB(k)) = V(TB(2)) + 4(k-2)$$

$$\text{Now let us prove that it is true for } n=k+1 \text{ i.e } E(TB(k+1)) = V(TB(2)) + 4(k-1)$$

$$\text{Consider } E(TB(k+1)) = E(TB(k)) + E(TB(1))$$

Which is $E(TB(k+1)) = V(TB(2)) + 4(k-2) + 4$ and on simplifying we have the required result

$$E(TB(k+1)) = V(TB(2)) + 4(k-1). \text{ Hence the theorem.}$$

Definition.3.6: Sum of edges in the basic triangular belt graph is obtained by the formula

$S = 2[f^*(u_1u_2) + f^*(v_1v_2)] - d(f^*(v_1v_2), f^*(v_2u_2))$ Where $d(f^*(v_1v_2), f^*(v_2u_2))$ is the positive difference between $f^*(v_1v_2)$ and $f^*(v_2u_2)$

Definition.3.7: Sum of edges in the i th part of triangular belt graph is obtained by the formula

$$S(i) = 2[f^*(v_{i+1}v_{i+2})] + f^*(u_{i+1}u_{i+2})$$

Theorem.3.8: For a triangular belt graph $TB(n)(\downarrow^n)$ the sum of the triangular belt graph

$$S(n) = S + \sum_{i=1}^{n-1} S(i)$$

Proof: Let us consider the triangular belt graph $TB(n)(\downarrow^n)$ with the labelling procedure adopted in theorem.3.1. Now as we have defined the sum of edges in basic triangular belt graph and sum of edges in the i th part of triangular belt graph we will try to compute the induced edge labelling of the graph for which we intend to find the sum. Now for proving the same let us use method of mathematical induction on n the size of the triangular edge.

If $n=2$ then $S(2) = S$. Which is true and it can be verified by the labelling procedure. Now let us assume for

$$n=k \text{ i.e it is true that } S(k) = S + \sum_{i=1}^{k-1} S(i)$$

$$\text{Now let us prove for } n=k+1 \text{ i.e to prove } S(k+1) = S + \sum_{i=1}^k S(i)$$

$$\text{For that consider } S(k+1) = S + \sum_{i=1}^{k-1} S(i) + S(1)$$

$$\text{Which reduces to } S(k+1) = S + \sum_{i=1}^k S(i). \text{ Hence the theorem.}$$

Theorem.3.9: For a triangular belt graph $TB(n)(\downarrow^n)$ we have $f^*(v_iu_i) + f^*(v_iu_{i+1}) = f^*(u_iu_{i+1})$

$$f^*(v_iu_{i+1}) + f^*(v_{i+1}u_{i+1}) = f^*(v_iv_{i+1})$$

Proof: The proof is trivial as we find the labelling procedure adopted in theorem.3.1

The induced edge labelling are

$$f^*(u_iu_{i+1}) = 8i - 4 \text{ for } 1 \leq i \leq n-1$$

$$f^*(v_iv_{i+1}) = 8i \text{ for } 1 \leq i \leq n-1$$

$$f^*(u_iv_i) = 4i - 3 \text{ for } 1 \leq i \leq n$$

$$f^*(v_iu_{i+1}) = 4i - 1 \text{ for } 1 \leq i \leq n-1$$

$$\text{Now consider the L.H.S of first result } f^*(v_iu_i) + f^*(v_iu_{i+1}) = 4i - 3 + 4i - 1$$

$$\text{On simplifying we have } f^*(v_iu_i) + f^*(v_iu_{i+1}) = 8i - 4 = f^*(u_iu_{i+1}). \text{ Hence it is true}$$

Similarly consider the L.H.S of second result

$$f^*(v_iu_{i+1}) + f^*(v_{i+1}u_{i+1}) = 4i - 1 + 4(i+1) - 3 \text{ on simplifying we have}$$

$$f^*(v_iu_{i+1}) + f^*(v_{i+1}u_{i+1}) = 8i = f^*(v_iv_{i+1}). \text{ Hence the result is true.}$$

Hence the proof.

Theorem.3.10: For a triangular belt $TB(n)(\downarrow^n)$ we have the following results

$$(i) \quad f^*(u_iu_{i+1}) = f^*(u_{i-1}u_i) + 8$$

$$(ii) \quad f^*(v_iv_{i+1}) = f^*(v_{i-1}v_i) + 8$$

$$(iii) \quad f^*(v_iu_i) = f^*(v_{i-1}u_{i-1}) + 4$$

$$(iv) \quad f^*(v_i u_{i+1}) = f^*(v_{i-1} u_i) + 4$$

Proof: Let us consider the triangular belt graph $TB(n)(\downarrow^n)$ with the labelling procedure adopted in theorem.3.1. The induced edge labelling for the triangular belt graph is given below

$$f^*(u_i u_{i+1}) = 8i - 4 \text{ for } 1 \leq i \leq n-1$$

$$f^*(v_i v_{i+1}) = 8i \text{ for } 1 \leq i \leq n-1$$

$$f^*(u_i v_i) = 4i - 3 \text{ for } 1 \leq i \leq n$$

$$f^*(v_i u_{i+1}) = 4i - 1 \text{ for } 1 \leq i \leq n-1$$

$$\text{Now for (i) L.H.S } f^*(u_i u_{i+1}) = 8i - 4$$

$$\text{R.H.S of (i) can be computed as } f^*(u_{i-1} u_i) + 8 = 8(i-1) - 4 + 8$$

$$\text{On simplifying we have } f^*(u_{i-1} u_i) + 8 = 8i - 4$$

Hence (i) is proved

$$\text{Similarly L.H.S of (ii) } f^*(v_i v_{i+1}) = 8i$$

$$\text{Now for R.H.S of (ii) } f^*(v_{i-1} v_i) + 8 = 8(i-1) + 8$$

$$\text{On simplifying we have } f^*(v_{i-1} v_i) + 8 = 8i$$

Hence (ii) is proved

$$\text{Similarly we have L.H.S of (iii) } f^*(u_i v_i) = 4i - 3$$

$$\text{Now R.H.S of (iii) is } f^*(v_{i-1} u_{i-1}) + 4 = 4(i-1) - 3 + 4$$

$$\text{Hence on simplifying } f^*(v_{i-1} u_{i-1}) + 4 = 4i - 3$$

Hence (iii) is proved

$$\text{Similarly we have L.H.S of (iv) } f^*(v_i u_{i+1}) = 4i - 1$$

$$\text{Now for R.H.S of (iv) } f^*(v_{i-1} u_i) + 4 = 4(i-1) - 1 + 4$$

$$\text{On simplifying we have } f^*(v_{i-1} u_i) + 4 = 4i - 1$$

Hence (iv) is proved.

Hence the proof of the theorem.

We now obtain from the triangular belt graph the ladder graph $L_n = P_n \times P_2$ $n \geq 2$ by eliminating the edge between the vertices v_i and u_{i+1} .

Hence we can use the same labelling procedure adopted in theorem.3.1 except computing the edge label between v_i and u_{i+1} . Also we obtain the sum of the edges of a ladder graph $L_n = P_n \times P_2$ $n \geq 2$.

Theorem.3.11: For the Ladder graph $L_n = P_n \times P_2$ $n \geq 2$ the sum of the edges is given by

$$S_L(n) = S(n) - \sum_{i=1}^n (4i - 1), \text{ Where } S_L(n) \text{ is the sum of the edges of ladder graph}$$

Proof: Consider the triangular belt graph $TB(n)(\downarrow^n)$. To find the sum of the edges of ladder graph $L_n = P_n \times P_2$ $n \geq 2$ we try to compute by removing the edge between v_i and u_{i+1} for each of the triangular belt graph $TB(n)(\downarrow^n)$. We know to compute $S(n)$ from theorem.3.8 and hence on computing $\sum_{i=1}^n (4i - 1)$ for each

part and subtracting we find the desired result $S_L(n) = S(n) - \sum_{i=1}^n (4i - 1)$. Hence the proof of the theorem.

IV. Results

In this paper we have considered triangular belt graph $TB(n)(\downarrow^n)$ and proved that it is square difference labelling graph and also computed the sum of the edges of the triangular belt graph and by removing the diagonal edge corresponding to the triangular belt graph we have obtained the original ladder graph and using the similar construction of labels for the vertices we have found out the sum of ladder graph using the sum obtained for the triangular belt graph.

V. Conclusion

We wish to identify some more graphs which can be labelled and proved in a similar nature to be Square difference labelling graph.

VI. References

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