



A Study on Hopfield Neural Networks for Its Capacity And Applications

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Abstract: In this paper we have studied the Hopfield neural network and summarize many interesting features, applications and implementations. A broad review of the model is considered which are being used by many researchers that lead to better performance in the field of pattern recognition and storage capacity to overcome previous problems. One of the advantages of the model is that it can be used in both digital and analog form.

Keywords: Hopfield neural network, Pattern recognition, Storage Capacity, Survey.

I. INTRODUCTION

A conventional computer can solve complex mathematical problems very fast but it is not yet efficiently process high intelligent problems such as pattern recognition, categorization and associative memory as human brain. The memorized recalling ability from some significant information of human brain can easily recognize the things even after many years. To satisfy this feature of human brain into the computer system Artificial Intelligence and Neurological approaches was came into the picture by several scientists and researchers in past few decades. To solve such problems many neural network models were developed. But in our study we focused only on a recurrent associative memory network called Hopfield neural network for pattern recognition. In 1982, J.J. Hopfield[1] developed a recurrent network with memory, is a single layer fully connected network which have a symmetric learning weight. He explained the energy with the same Lyapunov function and is gradually decreases with increase in the network movement process and reaches a balanced stable state. One major feature of Hopfield associative memory is to recall a pattern from a partial version or distorted version, for this reason Hopfield model is extensively used in pattern recognition. Hopfield model can be used in discrete time domain as well as in continuous time domain and is easier to implement in hardware also. It has been used in wide range of applications and still a challenging field for researchers in the field of neural network. When the network is at minimum energy it guarantees to converges the entire pattern. In this paper, we studied various applications, capacity and different aspects of Hopfield neural network for the researchers working on pattern recognition with auto-associative memory network.

II. HOPFIELD NEURAL NETWORK

A Hopfield neural network is an artificial recurrent neural network introduced by John Hopfield in 1982 to store and retrieve memory like human brain and usually works by learning number of patterns and returning one that is most likely to input pattern. It is a single layer fully connected network where each neuron or node is connected to all others excluding itself therefore also called a feedback network which means its outputs are redirected to its input. Each neuron of the network has a binary state (i.e activation value) represented as +1 (firing or on) or -1 (not firing or off) which is its output depending on the input it receives from other neurons. The state of neurons converges which means after convinced number of updates the state of each neuron becomes fixed. Initially patterns are trained to store in memory and then recognise any of stored patterns even with by partial or corrupted pattern of the originally stored patterns. Hence like human brain Hopfield model has stability in pattern recognition. In this network there is a connection between any two neurons i and j , and the connectivity weight w_{ij} symmetric ($w_{ij}=w_{ji}$) and a zero weight for self connectivity ($w_{ii}=0$) as shown below:

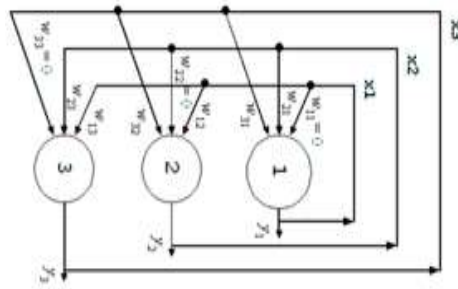


Figure 1: Hopfield Neural Network

Hopfield neural network can be classified into two popular forms: discrete and continuous-time models based on their output function.

2.1 Discrete Hopfield networks

The activation function value (state) of a neuron j is computed in discrete time is illustrated as:

$$x_j(t+1) = \text{sign}\left(\sum_{i=1}^n w_{ij}x_i(t) - \theta_j\right) \quad (1),$$

where $i = 1, 2, \dots, n$ & $j = 1, 2, \dots, n$

OR

We can also write $X = \text{sign}(XW - T)$

Where X , W , T and sign function are:

X is the activation value of n units / neurons : $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

W is the weight matrix: $W = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \dots & \dots & \dots & \dots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{pmatrix}$

where w_{ij} can be interpreted as the influence of neuron i over neuron j and vice versa

T is the externally applied threshold of each unit: $T = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$

And in case of discrete Hopfield neural network the units use a bipolar output function and the sign function is defined as: $\begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{otherwise} \end{cases}$

There are two ways to update a node. One is asynchronously; means update one node at a time by choosing randomly or according to some rule. Other one is synchronously; which means update all nodes together at every time. It has been observed that asynchronous updating is more biologically realistic.

Each node in the Hopfield network has an associated energy which is Lyapunov function. To improve the stability of the network, neurons are repetitively updated one at a time in any order which can leads the final state into a stable state. The energy of the network decreases when neuron changes sign otherwise it stays constant. This energy indicates whether network is modified or not when an update occurs and finally reaches to local minima

when it becomes stable that correspond to the energy of the stored patterns. However some local minima may not correspond to a training pattern are called spurious minima.

The energy function is given as:

$$E = -\frac{1}{2} X^T W X + T X^T = -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n x_i w_{ij} x_j + \sum_{i=1}^n \theta_i x_i \dots \dots \dots (2)$$

2.2 Continuous Hopfield networks

The continuous Hopfield network has time as a continuous variable and a generalization and electrical implementation of the discrete one. The general output functions used in this model or architecture are sigmoid and hyperbolic tangent functions. In this case, it is the electrical circuit that imposes the expressions of Equation (3) for the activation function, where g is usually the hyperbolic tangent function, and Equation (4) for the dynamics.

$$s_i(t) = g\left(\frac{u_i(t)}{\beta}\right) \dots \dots \dots (3)$$

$$\frac{du_i}{dt} = -u_i + \sum_j w_{ij} s_j - I_i \dots \dots \dots (4)$$

And the energy function is given by:

$$\frac{du_i}{dt} = -\frac{\partial E}{\partial s_i} = -u_i + \sum_j w_{ij} s_j - I_i \dots \dots \dots (5)$$

$$E = -\frac{1}{2} \sum_i \sum_j w_{ij} s_i s_j + \sum_i I_i s_i + \beta \sum_i \int_0^{s_i} g^{-1}(u_i) du_i \dots \dots \dots (6)$$

It must be emphasized that the Lyapunov function no longer matches the target of a classical optimization problem, but it contains an integral term that must be considered in the dynamical analysis of the evolution.

2.3 Learning Rules

Learning in Hopfield neural network means train the network by adjusting synaptic weight with respect to time. Basically learning is classified into two categories, supervised and unsupervised. In this paper we discussed various learning rules mostly used by Hopfield neural network.

2.3.1 Hebbian Learning rule:

This is the one of the oldest learning algorithm was introduced by Donald Hebb in 1949. According to Hebb. Rule, when two neurons are simultaneously active, the connection between them must be strengthened and when one of them is active, while the other is inactive, the connection strength must be weakened. When an input neuron frequently leads to the firing of the output neuron then a growth process takes place in one or both neurons with a high correlation between them such that input neuron's efficiency is increased to fire output neuron. The formulation is given as:

Suppose we want to store $P = \{P_i\}$ in a network and for this energy E should be minimum at P then we choose:

$$E = -\frac{1}{2N} \left(\sum_i P_i V_i \right)^2 \dots \dots \dots (7)$$

Where E is minimum at $P=V$. Now putting E in the standard form for Hopfield network is given by:

$$E = -\frac{1}{2N} \left(\sum_i P_i V_i \right) \left(\sum_j P_j V_j \right) = -\frac{1}{2} \sum_{ij} \left(\frac{1}{N} P_i P_j \right) V_i V_j \dots \dots \dots (8)$$

Hence the weight matrix to store pattern P is given by:

$$W_{ij} = \frac{1}{N} (P_i P_j) \dots \dots \dots (9)$$

And to store multiple patterns (P^p) we have:

$$W_{ij} = \frac{1}{N} \sum_p P_i^p P_j^p \dots \dots \dots (10)$$

Where N is the number of neurons and p is the number of training patterns.

2.3.2 Pseudo-inverse rule

It is a generalization of Hebb rule when the prototype vectors are not orthogonal and an efficient method for feedback network model. As compared to Hebbian rule it is neither incremental nor local which means pattern cannot be incrementally added to the network and the update does not depend on either side of the connection. The pseudoinverse rule is given as:

Let input patterns P of vectors $\{p_1, p_2, \dots, p_N\}$ and target pattern T of $\{t_1, t_2, \dots, t_N\}$ then we can write:

$$WP = T \dots \dots \dots (11)$$

Where W is the weight matrix. If the P matrix has an inverse then we can write:

$$W = PT^{-1} \dots \dots \dots (12)$$

Again if P is not a square matrix then no exact inverse will exist. But it has been shown that it will minimize $F(W) = \sum_{n=1}^N ||t_n - Wp_n||^2$ is obtained by using the pseudoinverse matrix and given by pseudoinverse rule:

$$W = TP^+ \dots \dots \dots (13)$$

Where P^+ is the Moore Penrose pseudoinverse, then the pseudoinverse of real matrix P is the unique matrix that satisfies: $PP^+P = P$, $P^+PP^+ = P^+$, $P^+P = (P^+P)^T$ and $PP^+ = (PP^+)^T$.

So the pseudoinverse weight matrix can be calculated as:

$$W_{pinv} = W^T * (W * W^T)^{-1} \dots \dots \dots (14)$$

Where W^T is the transpose of weight matrix.

If the patterns are linearly independent then pseudoinverse learning rule works well and will be able to store up to N patterns in an N unit network. This rule finds a set of orthogonal vectors and calculates output weight matrix by pseudoinverse solution.

2.3.3 Storkey Learning Rule

Amos Storkey in 1997 proposed a new learning rule named as Storkey learning rule. It is also both local and incremental and provides greater storage capacity ($N/2 * \sqrt{\ln N}$) than Hebbian learning rule ($N/2 \ln N$). It is local because it only considers neurons at either side. The learning rule is mathematically defined as:

$$w_{ij}^0 = 0 \text{ for all } i, j$$

$$\text{And } w_{ij}^p = w_{ij}^{p-1} + \frac{1}{n} \xi_i^p \xi_j^p - \frac{1}{n} \xi_i^p h_{ji}^p - \frac{1}{n} h_{ij}^p \xi_j^p \dots \dots \dots (15)$$

where w_{ij}^p is the weight between i and j after p^{th} pattern has been learnt and

ξ^p is the new learning pattern

$$h_{ij}^p = \sum_{k=1, k \neq i, j}^n w_{ik}^{p-1} \xi_k^p \text{ is a form of local field.}$$

III. LITERATURE SURVEY

In this study, we focused on the detail survey of Hopfield Network for its applications and capacity. Both discrete and analog Hopfield network has been widely used in both hardware and software implementation in various applications.

Taylor[2], initiated the work of Hopfield network. Then after many researchers made contribution to associative memory including Anderson, Kohonen and Nakano who proposed correlation matrix based learning rule. Amari, S. I[3], proposed an autocorrelator to analyze the stability of above models by using statistical neurodynamics. Lippmann[4], made a comparison between Hopfield and other neural network models for pattern classification. Young, S. S., Scott, P. D., Nasrabadi, N. M[5], proposed a multilayer Hopfield neural network for pattern or object recognition that converges to the single layer model. Bhartikar, S., Mendel, J. M[6], proposed a hysteretic Hopfield model to prove Lyapunov stability to solve a combinatorial optimization problem as N -queen problem. Sun, Y., Li, J. G., Yu, S. Y[8], developed a modified Hopfield neural network for restoration of blurred image. Roth, M. W[9], implemented Hopfield network on all optical design by using computer generated holograms. Rietman, E[10], in his book, describes how neural circuit can be implemented by Hopfield network. Many researchers also worked on continuous time Hopfield neural network. Cohen & Grossberg [11] and Hopfield[12], was first introduced continuous Hopfield network and the earlier work done by Cowan & Grossberg. Hopfield, J. J., Tank, D. W[14], analysed the analog Hopfield network strength with the ability to handle data in continuous time. Kosko B 1987, [15], proposed an adaptive bidirectional associative memory as an extension work of Hopfield and Grossberg. Somesh Kumar, Manu Pratap Singh[16], evaluated performance of Hopfield neural network by

using Hebb. Rule and genetic algorithm for recalling stored English character patterns. Donq-Liang Lee[17], proposed a continuous time Hopfield network for pattern recognition. He implemented a time varying Hopfield network by using conventional matrix encoding and proved many advantages over existing neural models such as higher memory capacity and stronger pattern recognition capability. Kussay Nugamesh Mutter, Zubir Mat Jafri and Azlan Abdul Aziz[18], proposed an algorithm by using run-length encoding for fingerprint identification with Hopfield neural network for saving the capacity of network memory where the weight data is reduced by encoding scheme. Chia-Hsin Cheng, Guo-Jun Wen, Yung-Fa Huang[19], proposed a multi-user detection in direct sequence ultra wide band(DS-UWB), by maximum likelihood feature to reduce the computational complexity of detector. Kushan Ahmadian and Marina Gavrilova[20], proposed an algorithm of Hopfield neural network for Delaunay Triangulation by using duality of fingerprint and showed a significant improvement in false rejection rate with a reliable matching process. Qun Liu, Supratik Mukhopadhyay[21], proposed a framework for CNN model for feature extraction with Hopfield neural network as an associative memory for storing and classification purpose. Christopher Hillar, Ram Mehta, Kilian Koepsel[22], implemented Hopfield neural network by using minimum probability flow learning technique on discretizations of gray scale digital photographs and the result shown the local structure of the image is remarkably well modelled. Many researchers worked on capacity of Hopfield neural network to improve the memory for storing patterns and we analysed some such papers and our focus is to present those research works to the readers or researchers for their better understanding and for their future work. Amit, D.J., H. Gutfrund, H. Sompolinsky, 1985[23], Bhartikar, S. and J.M. Mendel, 2000[24], Calvert, B.D. and C.A. Marinov[25], Hammerstrom, D[26], Hopfield, J.J. and D.W. Tank[27], Huang, H., D.W.C. Ho and J. Lam[28], Shen, Y.J. and M.S. Wang[29], were try to improve the capacity of the Hopfield neural network. Venkatesh, S.S[30] and Hassoun, M.H[31] proposed the upper and lower bound on the capacity of Hopfield neural network respectively. Hassoun, M.H[31] and Kakeya, H. and T. Kindo[32] improve the capacity of Hopfield network by some different combinations. Pao, Y.H[33], worked to improve the capacity of Hopfield network. McEliece, R.J., E.C. Posner, E.R. Rodemich and S.S. Venkatesh[34], proposed that the number of pattern can be recovered no more than $(n/4 \log n)$, where n is the number of neurons. Storkey, A. J. and Valabregue, R.[35] proposed a new learning rule called storky learning rule to increase the storage capacity of the Hopfield neural network against to the Hebb. Rule as $(n/\sqrt{2 \ln n})$. Kubota, T.[36], state that the capacity of order m associative memory is $O(n^m/\ln n)$. Mohammad Reza Rajati, Mohammad Bagher Menhaj[37], classified Hopfield neural networks with higher order nonlinearity with the stability of equilibrium points and proposed a significant improvement on storage capacity and performance of Hopfield networks with Hebbian learning. Lippmann, R. P[38], compares the Hopfield neural network with other neural network models for pattern classification.

IV. CONCLUSION

In our review process, we focused on the capacity, dynamics, learning rules, and application of Hopfield neural network. In our study it has been observed that Hopfield neural network was implemented in various fields like pattern classification and recognition, speech processing, signal processing, control systems, database retrieval, fault tolerance computing, knowledge processing, object detection, combinatorial optimization and many more. It has also been observed that many researchers worked on improvement of capacity with different perspective of the Hopfield network. It is still a challenging area for improvement of storage capacity of the Hopfield network.

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