



Total Domination and Inverse Total Domination Number of Some Special Classes of Graphs

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Abstract: Let $G=(V,E)$ be a simple, finite, connected and undirected graph. Let n and m denote the order and size of G . A non-empty set D of vertices in a graph $G=(V,E)$ is a Dominating set, if every vertex in $V-D$ is adjacent to some vertex in D . A total dominating set D_t of G is a dominating set, such that the induced subgraph $\langle D_t \rangle$ has no isolated vertices. Let D_t be the minimum total dominating set of G . If $(V-D_t)$ contains a total dominating set D_t' of G , then D_t' is called an inverse total dominating set with respect to D_t . In this paper, We have obtained the total domination number $\gamma_t(G)$ and inverse total domination number $\gamma_t'(G)$ of some special classes of graphs.

Keywords: Dominating set, Inverse dominating set, Total dominating set, Inverse total dominating set, Total domination number and Inverse total domination number.

I. Introduction

In 1962, Ore used the name “dominating set” and “domination number”. In 1977, Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. The survey paper of Cockayne and Hedetniemi has generated a lot of interest in the study of domination in graphs. Domination has a wide range of applications in Radio station, modeling social networks, coding theory, nuclear power plants problems.

A non-empty subset D of vertices in a graph $G=(V,E)$ is a “**Dominating set**”, if every vertex in $V-D$ is adjacent to some vertex in D . The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set. A **total dominating set** D_t of G is a dominating set such that the induced subgraph $\langle D_t \rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ of G is the minimum cardinality of a minimal total dominating set of G .

Let D be the minimum dominating set of G . If $V-D$ contains a dominating set say D' , then D' is called an **inverse dominating set** with respect to D . The inverse domination number $\gamma'(G)$ of G is the order of the smallest inverse dominating set of G . “Kulli and Sigarkandi” introduced the concept of inverse domination in Graphs.

Let $G=(V,E)$ be a simple graph. Let D_t be the minimum total dominating set of G . If $V-D_t$ contains a total dominating set D_t' of G , then D_t' is called an **inverse total dominating set** with respect to D_t . The inverse total domination number $\gamma_t'(G)$ of G is the minimum number of vertices in an inverse total dominating set of G .

The concept of total domination is first introduced by Cockayne, Dawes and Hedetniemi in 1980. V.R. Kulli and RadhaRajamaniIyer introduced the concept of inverse total domination in graphs. In this paper, we found the total domination number and inverse total domination number of some special classes of graphs.

II. Preliminaries

Definition 2.1

The “**Paley graph**” is a 6-regular graph on 13 vertices and 39 edges. It is also known as the Raymond Paley graph.

Definition 2.2

The “**Shrikhande graph**” is a named graph discovered by S.S.Shrikhande in 1959. It is a strongly regular graph with 16 vertices and 48 edges, with each vertex having degree 6.

Definition 2.3

The “**Clebsch graph**” is a named graph discovered by Alfred Clebsch in 1868. It is a Strongly regular quintic graph on 16 vertices and 40 edges. The Clebsch graph is also known as Greenwood-Gleason graph. It is a 5-regular graph.

Definition 2.4

An “**Andrasfai graph**” is a triangle-free circulant graph named after Bela Andrasfai. It is also known as Wagner graph with each vertex having degree 6.

Definition 2.5

The “**Chvatal graph**” is an undirected graph with 12 vertices and 24 edges, discovered by Vaclav Chvatal in 1970. It is a 4-regular graph.

Definition 2.6

The “**Golomb graph**” is a named graph discovered by Solomon w. Golomb . It is also known as polyhedral graph with 10 vertices and 18 edges.

Definition 2.7

The “**Soifer graph**” is a planar graph on 9 vertices and 20 edges.

Definition 2.8

The “**Herschel graph**” is a bipartite undirected graph with 11 vertices and 18 edges. It is discovered by British astronomer Alexander Stewart Herschel.

Definition 2.9

The “**Poussin graph**” is a planar graph on 15 vertices and 39 edges. It is discovered by Charles Jean de la Vallee-Poussin.

Definition 2.10

The “**Desargues graph**” is a named graph discovered in 17th century by French Mathematician Gerard Desargues. It is a distance-transitive cubic graph with 20 vertices and 30 edges.

Definition 2.11

The “**Franklin graph**” is a 3-regular graph on 12 vertices and 18 edges. It is discovered by Philip Franklin.

Definition 2.12

The “**Mobius-Kantor graph**” is a symmetric bipartite cubic graph on 16 vertices and 24 edges. It is discovered by August Ferdinand Mobius and Seligmann Kantor.

Definition 2.13

The “**Moser graph**” (also called Mosers’ Spindle graph) is an undirected graph with 7 vertices and 11 edges, named after the Mathematician Leo Moser and his brother William.

Definition 2.14

The “**Icosahedral graph**” is the Platonic graph which has 12 vertices and 30 edges. Since the Icosahedral graph is regular and Hamiltonian, it is also called Taylor graph.

Definition 2.15

An “**Octahedron graph**” is a polyhedron graph with 8 faces, 12 edges and 6 vertices. It is a 4-regular graph.

III. Results on Total Domination and Inverse Total Domination in Some Special Classes of Graphs

Theorem 3.1

If G is a Paley graph, then $\gamma_t(G)=4 = \gamma_t'(G)$

Proof:

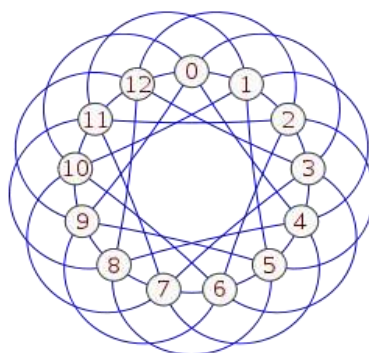


Figure1: Paley Graph

From the figure, $V=\{v_0, v_1, \dots, v_{12}\}$	
$D=\{0, 3, 7\}$	$\Rightarrow \gamma(G)=3$
$D_t=\{0, 1, 6, 7\}$	$\Rightarrow \gamma_t(G)=4$
$D_t'=\{3, 4, 9, 10\}$	$\Rightarrow \gamma_t'(G)=4$

Step(i)

Let $G=(V, E)$ be a Paley graph with 13 vertices of degree 6. Let $V=\{v_0, v_1, \dots, v_{12}\}$ be the vertex set of the graph G . To have a dominating set D , it is found that, the vertices $\{v_0, v_3, v_7\}$ forms a minimum dominating set. Thus, $|D|=3$. Hence $\gamma(G)=3$.

Step(ii)

Now, the total dominating set D_t is a dominating set such that the induced subgraph $\{v_0, v_1, v_6, v_7\}$ has no isolated vertices. Therefore $|D_t| = 4$. Hence $\gamma_t(G) = 4$.

Step(iii)

Also, there exists a set D_t' such that $(V - D_t)$ contains a total dominating set with $|D_t'| = |D_t|$, giving $|D_t'| = 4$. Hence $\gamma_t'(G) = 4$.

Observation 3.1.1

In a Paley graph,

(i) $\gamma(G) < \gamma_t(G)$ and (ii) $\gamma_t(G) = \gamma_t'(G)$

Theorem 3.2

If G is a Shrikhande graph with 16 vertices, then $\gamma(G) = 4 = \gamma_t(G) = \gamma_t'(G)$

Proof:

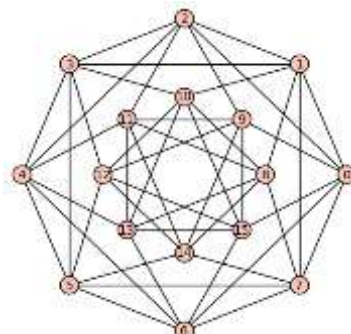


Figure 2: Shrikhande graph

From the figure, $V = \{v_0, v_1, \dots, v_7, v_8, v_9, \dots, v_{15}\}$		
$D = \{11, 6, 10, 14\}$	$\rightarrow$	$\gamma(G) = 4$
$D_t = \{11, 2, 15, 6\}$	$\rightarrow$	$\gamma_t(G) = 4$
$D_t' = \{3, 12, 8, 7\}$	$\rightarrow$	$\gamma_t'(G) = 4$

(i) Consider the Shrikhande graph G with 16 vertices and G is a 6-regular graph. Let $\{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ be the outer vertices and $\{v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$ be the corresponding inner vertices. In the above figure, the vertices v_{11} and v_2 dominate the vertices $\{v_{13}, v_{14}, v_8, v_9, v_4, v_0, v_3, v_1\}$ and similarly the vertices v_{15} and v_6 dominate the vertices $\{v_{10}, v_8, v_7, v_5\}$. Therefore, the minimum dominating set is $\{v_{11}, v_2, v_{15}, v_6\}$. Hence $\gamma(G) = 4$.

(ii) Here, the total dominating set is a dominating set such that the induced subgraph $\{v_{11}, v_2, v_{15}, v_6\}$ has no isolated vertices. Thus, the minimum total dominating set is $\{v_{11}, v_{15}, v_2, v_6\}$. Therefore, the minimum cardinality of the total dominating set is 4. Hence $\gamma_t(G) = 4$.

(iii) There exists a non-empty subset D_t' such that $(V - D_t)$ contains a total dominating set with $|D_t'| = 4$. Hence $\gamma_t'(G) = 4$.

Thus, In a Shrikhande graph, (i) $\gamma(G) = 4$ and (ii) $\gamma(G) = \gamma_t(G) = \gamma_t'(G)$

Theorem 3.3

If G is a Clebsch graph, then $\gamma_t(G) = 5 = \gamma_t'(G)$

Proof:

From the below figure, $V = \{v_1, v_2, \dots, v_{16}\}$		
$D = \{v_{11}, v_7, v_8, v_9\}$	$\rightarrow$	$\gamma(G) = 4$
$D_t = \{v_7, v_{16}, v_8, v_9, v_{10}\}$	$\rightarrow$	$\gamma_t(G) = 5$
$D_t' = \{v_{11}, v_6, v_4, v_{17}, v_5\}$	$\rightarrow$	$\gamma_t'(G) = 5$

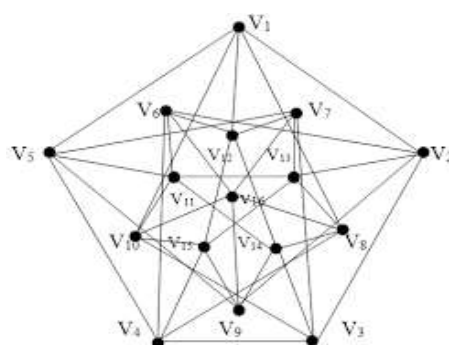


Figure 3: Clebsch graph

By the definition of Clebsch graph, the graph comprises of 16 vertices and G is a 5-regular graph. Here, there are 4 vertices which dominate all the remaining vertices. Suppose $V = \{v_1, v_2, \dots, v_{16}\}$ is the set of all vertices and $D = \{v_{11}, v_7, v_8, v_9\}$ is the minimum dominating set, then there exists the induced subgraph $\langle D_t \rangle = \{v_7, v_{16}, v_8, v_9, v_{10}\}$ which dominate the remaining vertices of V and $\langle D_t \rangle$ has no isolated vertices. Thus D_t is the total dominating set of minimum cardinality 5. Hence $\gamma_t(G) = 5$. Also, there exists another set $D'_t = \{v_{11}, v_6, v_4, v_{12}, v_2\}$ in $(V - D_t)$ which dominates the remaining vertices of V . Thus D'_t is the inverse total dominating set of minimum cardinality 5. Hence $\gamma'_t(G) = 5$.

Note 3.3.1

In a Clebsch graph,

- (i) $\gamma(G) = 4$
- (ii) $\gamma(G) < \gamma_t(G) = \gamma'_t(G)$

Theorem 3.4

Let G be the Andrasfai graph with 17 vertices, then $\gamma(G) = \gamma_t(G) = \gamma'_t(G) = 3$

Proof:

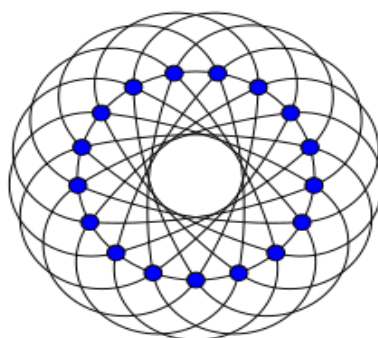


Figure 4: Andrasfai graph

From the figure, $V = \{v_1, v_2, \dots, v_{17}\}$	
$D = \{v_1, v_2, v_3\}$	$\rightarrow \gamma(G) = 3$
$D_t = \{v_1, v_2, v_3\}$	$\rightarrow \gamma_t(G) = 3$
$D'_t = \{v_4, v_5, v_6\}$	$\rightarrow \gamma'_t(G) = 3$

Step(i)

Let G be the Andrasfai graph with 17 vertices and G is a 6-regular graph. Let $V = \{v_1, v_2, \dots, v_{17}\}$ be the vertex set of the graph G . In the above figure, the vertex set $\{v_1, v_2, v_3\}$ forms a minimum dominating set D . Thus $|D| = 3$. Hence $\gamma(G) = 3$.

Step(ii)

Now, the total dominating set D_t be the minimum dominating set such that the induced subgraph $\{v_1, v_2, v_3\}$ has no isolated vertices. Therefore, $|D_t| = 3$, Hence $\gamma_t(G) = 3$.

Step(iii)

Also, there exists a set $D'_t = \{v_4, v_5, v_6\}$ in $(V - D_t)$ which contains a total dominating set with $|D'_t| = |D_t|$, giving $|D'_t| = 3$, Hence $\gamma'_t(G) = 3$. Therefore, $\gamma(G) = \gamma_t(G) = \gamma'_t(G)$.

Theorem 3.5

If G is Chvatal graph, then $\gamma(G) = 4 = \gamma_t(G)$.

Proof:

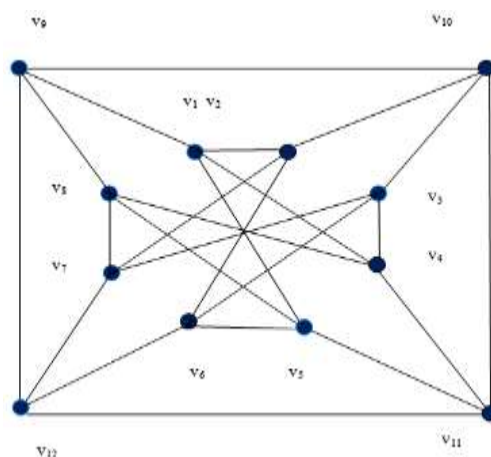


Figure 5: Chvatal graph

From the figure, $V=\{v_1, v_2, \dots, v_{12}\}$			
$D=\{v_1, v_3, v_9, v_{11}\}$	$\rightarrow$	$\gamma(G)=4$	
$D_t=\{v_1, v_2, v_5, v_7\}$	$\rightarrow$	$\gamma_t(G)=4$	
$D_t'=\{v_3, v_4, v_6, v_8\}$	$\rightarrow$	$\gamma_t'(G)=4$	

Consider the Chvatal graph G with 12 vertices and G is a 4-regular graph. Consider the set of all vertices $V=\{v_1, v_2, \dots, v_{12}\}$ and $D=\{v_1, v_3, v_9, v_{11}\}$ is the minimum dominating set, then there exists the induced subgraph $D_t=\{v_1, v_2, v_5, v_7\}$ which dominates the vertices of V and it has no isolated vertices. Thus, D_t is the total dominating set of minimum cardinality 4. Hence $\gamma_t(G)=4$.

Also, there exists another set $D_t'=\{v_3, v_4, v_6, v_8\}$ in $(V-D_t)$ which is the inverse total dominating set of minimum cardinality 4. Thus $\gamma_t'(G)=4$.

Note 3.5.1

In a Chvatal graph,

(i) $\gamma(G)=4$

(ii) $\gamma(G)=\gamma_t(G)=\gamma_t'(G)$

Theorem 3.6

If G be the Soifer graph, then $\gamma_t(G)=3=\gamma_t'(G)$

Proof:

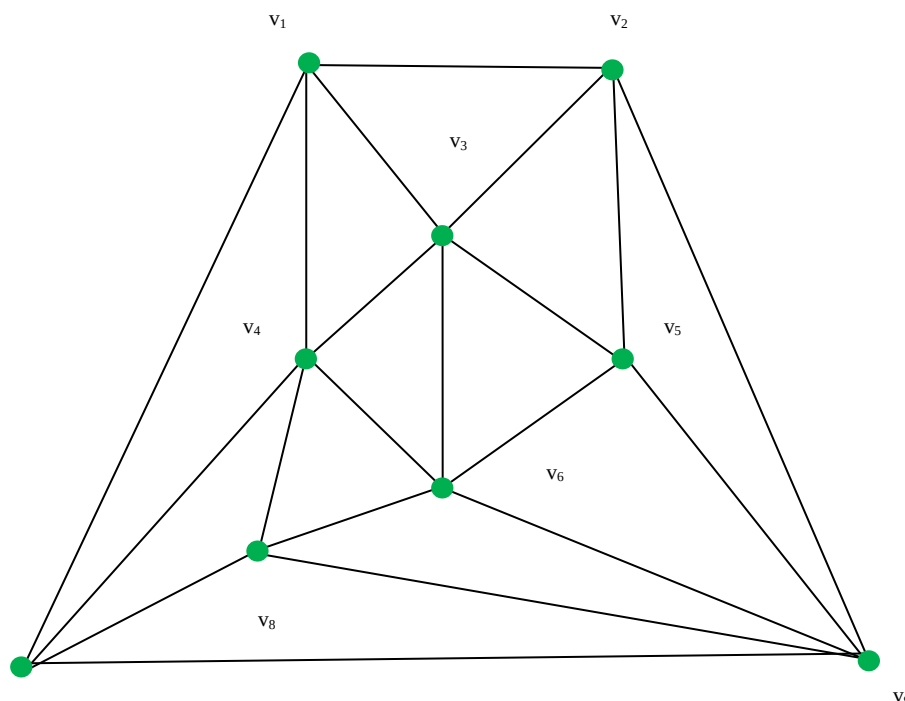


Figure 6: Soifer graph

From the figure, $V=\{v_1, v_2, \dots, v_9\}$			
$D=\{v_1, v_6\}$	$\rightarrow$	$\gamma(G)=2$	
$D_t=\{v_3, v_6, v_7\}$	$\rightarrow$	$\gamma_t(G)=3$	
$D_t'=\{v_1, v_2, v_4\}$	$\rightarrow$	$\gamma_t'(G)=3$	

Let G be the Soifer graph with 9 vertices. Let $V=\{v_1, v_2, \dots, v_9\}$ be the vertex set of the graph G . To have a dominating set D , we found at least two vertices $\{v_1, v_6\}$ which dominate the remaining vertices of G . Hence $\gamma(G)=2$.

Now, the total dominating set D_t is a dominating set such that the induced subgraph $\{v_3, v_6, v_7\}$ has no isolated vertices. Hence $\gamma_t(G)=3$.

Also, there exists a set $D_t'=\{v_1, v_2, v_4\}$ such that $(V-D_t)$ contains a total dominating set. Hence $\gamma_t'(G)=3$.

Note 3.6.1

In a Soifer graph, (i) $\gamma(G)<\gamma_t(G)=\gamma_t'(G)$

Theorem 3.7

If G be the Moser graph with 7 vertices, then $\gamma_t(G)=3$

Proof:

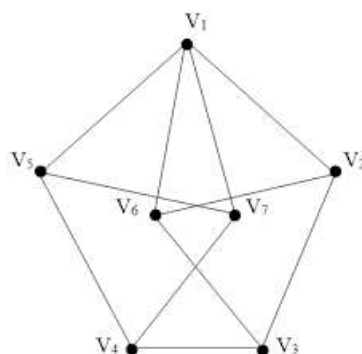


Figure 7: Moser Graph

From the figure, $V = \{v_1, v_2, \dots, v_7\}$	
$D = \{v_2, v_5\} \rightarrow \gamma(G) = 2$	
$D_t = \{v_1, v_2, v_5\} \rightarrow \gamma_t(G) = 3$	

Let G be the Moser graph with 7 vertices and 11 edges of degree 3. Let $V = \{v_1, v_2, \dots, v_7\}$ be the set of all vertices in G . Now, the vertex v_2 is adjacent to the vertices $\{v_1, v_3, v_6\}$ and the vertex v_5 is adjacent to the vertices $\{v_4, v_7\}$.

Therefore, the two vertices $\{v_2, v_5\}$ dominate the entire vertex set V . But it is not a total dominating set, because $\{v_2, v_5\}$ are not adjacent in D .

So, we include one more vertex v_1 . So that $D_t = \{v_1, v_2, v_5\}$ is a total dominating set. Therefore, Every vertex in V is adjacent to atleast one vertex in D_t .

Thus, $D_t = \{v_1, v_2, v_5\}$ is the minimum total dominating set with cardinality 3. Hence $\gamma_t(G) = 3$.

Note 3.7.1

In a Moser graph, (i) $\gamma(G) = 2$

(ii) $\gamma_t'(G) = 4$

(iii) $\gamma(G) < \gamma_t(G) < \gamma_t'(G)$

Theorem 3.8

If G be the Franklin graph, then $\gamma_t(G) = 6 = \gamma_t'(G)$

Proof:

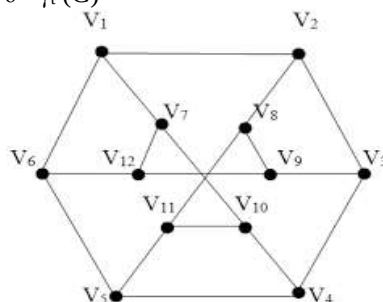


Figure 8: Franklin graph

From the figure, $V = \{v_1, v_2, \dots, v_6, v_7, v_8, \dots, v_{12}\}$	
$D = \{v_1, v_4, v_7, v_8\} \rightarrow \gamma(G) = 4$	
$D_t = \{v_1, v_2, v_3, v_4, v_5, v_6\} \rightarrow \gamma_t(G) = 6$	
$D_t' = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\} \rightarrow \gamma_t'(G) = 6$	

Let G be the Franklin graph with 12 vertices and G is a Cubic graph. Let $V = \{v_1, v_2, \dots, v_6\}$ and $U = \{v_7, v_8, \dots, v_{12}\}$ be the two subset of vertices of G . Here, the set of 4 vertices $\{v_1, v_4, v_7, v_8\}$ dominates the entire vertex set without isolated vertices.

Here, the set $D_t = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the total dominating set and $D_t' = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ is the inverse total dominating set. Hence $\gamma_t(G) = 6 = \gamma_t'(G)$.

Note 3.8.1

In a Franklin graph, (i) $\gamma(G) = 4$

(ii) $\gamma(G) < \gamma_t(G) = \gamma_t'(G)$

Theorem 3.9

If G be the Golomb graph, then $\gamma_t(G) = 3$ and $\gamma_t'(G) = 4$.

Proof:

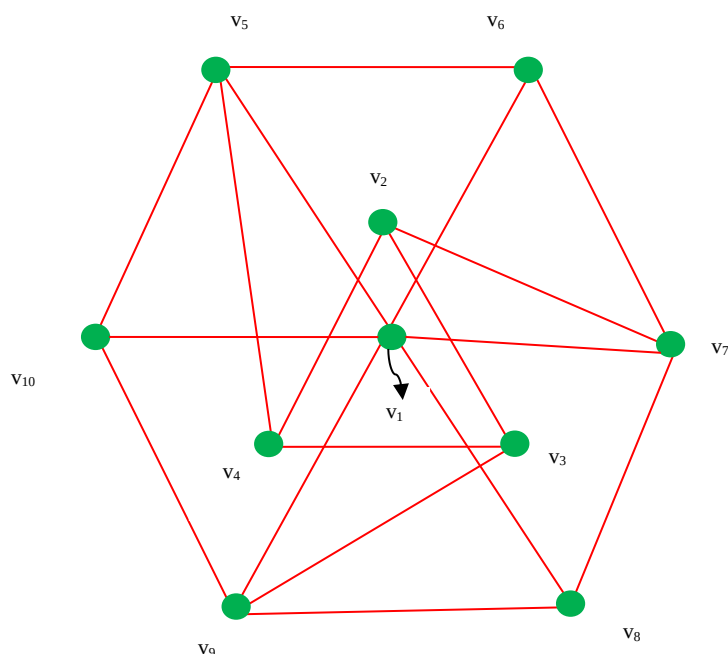


Figure 9: Golomb graph

From the figure, $V = \{v_1, v_2, \dots, v_{10}\}$			
$D = \{v_1, v_4, v_5\}$	$\Rightarrow$	$\gamma(G) = 3$	
$D_t = \{v_1, v_5, v_4\}$	$\Rightarrow$	$\gamma_t(G) = 3$	
$D'_t = \{v_7, v_2, v_9, v_{10}\}$	$\Rightarrow$	$\gamma'_t(G) = 4$	

By the definition of Golomb graph, the graph G comprises of 10 vertices. The set $D = \{v_1, v_4, v_5\}$ dominates all the remaining vertices of V . Suppose $V = \{v_1, v_2, \dots, v_{10}\}$ is the set of all vertices and $D = \{v_1, v_4, v_5\}$ is the minimum dominating set of G . Then, $\gamma(G) = 3$. This dominating set is the minimum total dominating set such that the induced subgraph $\langle D_t \rangle = \{v_1, v_5, v_4\}$ is adjacent to every vertex of G and it has no isolated vertices. Hence $\gamma_t(G) = 3$. Also, there exists an inverse total dominating set $D'_t = \{v_2, v_7, v_9, v_{10}\}$ which dominates the vertices of V with minimum cardinality 4. Thus $\gamma'_t(G) = 4$.

Note 3.9.1

- In a Golomb graph, (i) $\gamma(G) = \gamma_t(G)$
(ii) $\gamma_t(G) < \gamma'_t(G)$

Theorem 3.10

If G be the Herschel graph, then $\gamma_t(G) = 2$ and $\gamma'_t(G) = 4$

Proof:

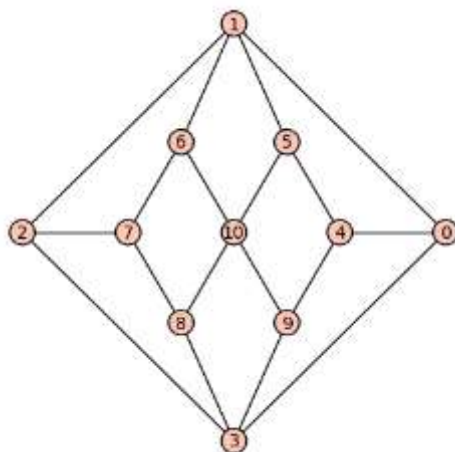


Figure 10: Herschel graph

From the figure, $V=\{v_0, v_1, \dots, v_{10}\}$		
$D=\{v_{10}, v_2, v_0\}$	$\rightarrow$	$\gamma(G)=3$
$D_t=\{v_{10}, v_5, v_8, v_3\}$	$\rightarrow$	$\gamma_t(G)=4$
$D'_t=\{v_6, v_7, v_4, v_9\}$	$\rightarrow$	$\gamma'_t(G)=4$

Let G be the Herschel graph of 11 vertices. Let $V=\{v_1, v_2, \dots, v_{11}\}$ be the vertices of G . The subset $D=\{v_0, v_2, v_{10}\}$ dominates the entire vertex set V , and the set $\{v_{10}, v_2, v_0\}$ forms the minimum dominating set of G . Thus $\gamma(G)=3$.

In the above figure, the vertex subset $\{v_{10}, v_5, v_8, v_3\}$ forms the minimum total dominating set D_t of G . Thus $\gamma_t(G)=4$.

Further, there exists at least one $D'_t=\{v_6, v_7, v_4, v_9\}$ such that $(V-D_t)$ contains a total dominating set. Hence $\gamma'_t(G)=4$.

Note 3.10.1

In a Herschel graph, (i) $\gamma(G) < \gamma_t(G)$

(ii) $\gamma_t(G) = \gamma'_t(G)$

Theorem 3.11

Let G be the Poussin graph, then $\gamma_t(G) = \gamma'_t(G) = 4$

Proof:

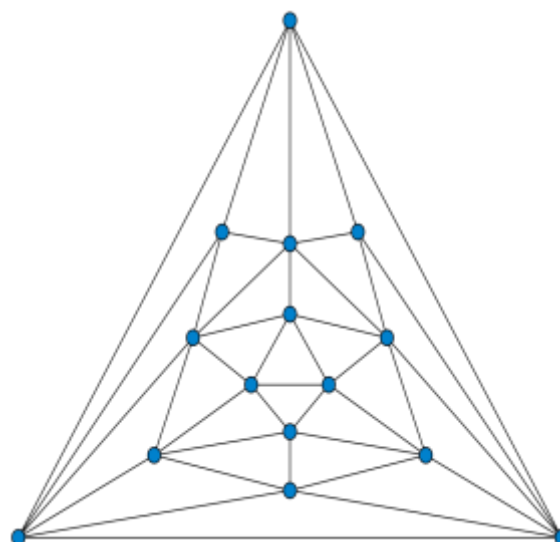


Figure 11: Poussin Graph

From the figure, $V=\{v_1, v_2, \dots, v_{15}\}$		
$D=\{v_1, v_{10}, v_6\}$	$\rightarrow$	$\gamma(G)=3$
$D_t=\{v_3, v_6, v_{10}, v_{14}\}$	$\rightarrow$	$\gamma_t(G)=4$
$D'_t=\{v_7, v_{13}, v_{13}, v_9\}$	$\rightarrow$	$\gamma'_t(G)=4$

Case (i)

Let $G=(V,E)$ be the Poussin graph with 15 vertices. Let $V=\{v_1, v_2, \dots, v_{15}\}$ be the vertex set of the graph G . The vertex subset D is a dominating set, since every vertex in $(V-D)$ is adjacent to some vertex in D .

In the above figure, it is found that the set $\{v_1, v_{10}, v_6\}$ forms the minimum dominating set. Thus $|D|=3$, Hence $\gamma(G)=3$.

Case (ii)

In Poussin graph, the total dominating set D_t is a dominating set such that the induced subgraph $\langle D_t \rangle = \{v_3, v_6, v_{10}, v_{14}\}$ has no isolated vertices. Therefore, $|D_t|=4$. Hence $\gamma_t(G)=4$.

Case (iii)

Also, there exists a set D'_t such that $(V-D_t)$ contains a total dominating set. In the above figure, the another minimum total dominating set $D'_t=\{v_1, v_7, v_{13}, v_9\}$ with minimum cardinality $|D'_t|=4$. Hence $\gamma'_t(G)=4$.

Note 3.11.1

In a Poussin graph, (i) $\gamma_t(G) = \gamma'_t(G)$

Theorem 3.12

Let G be the Mobius-Kantor graph with 16 vertices, then $\gamma_t(G) = \gamma'_t(G) = 8$.

Proof:

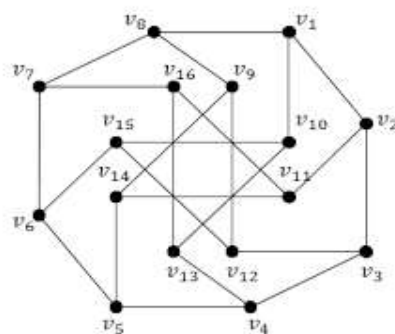


Figure 12: Mobius-Kantor graph

From the figure, $V = \{v_1, v_2, v_3, \dots, v_{16}\}$
 $D = \{v_8, v_5, v_{12}, v_{11}, v_{10}\} \rightarrow \gamma(G) = 5$
 $D_t = \{v_1, v_2, \dots, v_8\} \rightarrow \gamma_t(G) = 8$
 $D'_t = \{v_9, v_{10}, \dots, v_{16}\} \rightarrow \gamma'_t(G) = 8$

Let G be a Mobius-Kantor graph with 16 vertices and G is a Cubic graph. Let $V = \{v_1, v_2, \dots, v_{16}\}$ be the set of vertices of the graph. The vertex set V can be partitioned into two vertex subsets V_1 and V_2 .

(i.e.) $V_1 = \{v_1, v_2, \dots, v_8\}$

$V_2 = \{v_9, v_{10}, \dots, v_{16}\}$ each one comprising of 8 vertices and the degree of each vertex being three. In the above figure, V_1 is taken as the minimum total dominating set $D_t = \{v_1, v_2, \dots, v_8\}$ whose vertices dominates the remaining vertices, giving $\gamma_t(G) = 8$.

Also, there exists a set $D'_t = \{v_9, v_{10}, \dots, v_{16}\}$ such that $(V - D_t)$ contains a total dominating set V_2 which dominates the remaining vertices of $(V - D_t)$. Thus, the vertex subset V_2 is the inverse total dominating set with minimum cardinality 8. Hence $\gamma'_t(G) = 8$.

Note 3.12.1

In a Mobius-Kantor graph, (i) $\gamma(G) = 5$

(ii) $\gamma_t(G) = \gamma'_t(G)$

(iii) $\gamma(G) < \gamma_t(G) = \gamma'_t(G)$

Theorem 3.13

Let G be the Icosahedral graph, then $\gamma_t(G) = 4 = \gamma'_t(G)$

Proof:

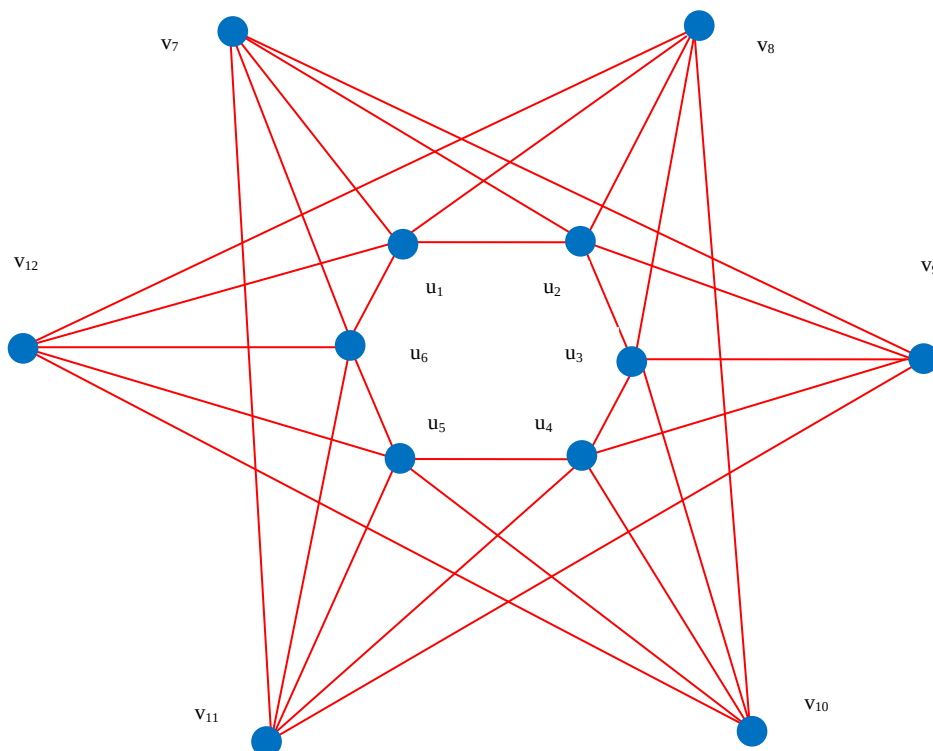


Figure 13: Icosahedral graph

From the figure, $V = \{u_1, u_2, \dots, u_6, v_7, v_8, \dots, v_{12}\}$			
$D = \{7, 10\}$	$\rightarrow$	$\gamma(G) = 2$	
$D_t = \{7, 1, 4, 10\}$	$\rightarrow$	$\gamma_t(G) = 4$	
$D'_t = \{2, 8, 5, 11\}$	$\rightarrow$	$\gamma'_t(G) = 4$	

Consider the Icosahedral graph G with 12 vertices of degree 5. Let $\{u_1, u_2, \dots, u_6\}$ be the inner vertices and $\{v_7, v_8, \dots, v_{12}\}$ be the corresponding outer vertices. Now, the vertex v_7 is adjacent to the vertices $\{u_1, u_2, u_6, v_9, v_{11}\}$ and the vertex v_{10} is adjacent to the vertices $\{u_3, u_4, u_5, v_8, v_{12}\}$.

Therefore, these two vertices $\{v_7, v_{10}\}$ dominate the entire vertex set. But it is not a total dominating set, because $\{v_7, v_{10}\}$ are not adjacent in D .

Consider another subset $D_t = \{u_1, u_4, v_7, v_{10}\}$ which is a total dominating set. Therefore, Every vertex in V is adjacent to atleast one vertex in D .

Thus, $D_t = \{u_1, u_4, v_7, v_{10}\}$ is the minimum total dominating set with cardinality 4. Hence $\gamma_t(G) = 4$

Note 3.13.1

In a Icosahedral graph,

- (i) $\gamma(G) = 3$
- (ii) $\gamma_t(G) = \gamma'_t(G) = 4$
- (iii) $\gamma(G) < \gamma_t(G) = \gamma'_t(G)$

Theorem 3.14

If G be the Octahedron graph, then $\gamma(G) = \gamma_t(G) = \gamma'_t(G) = 2$

Proof:

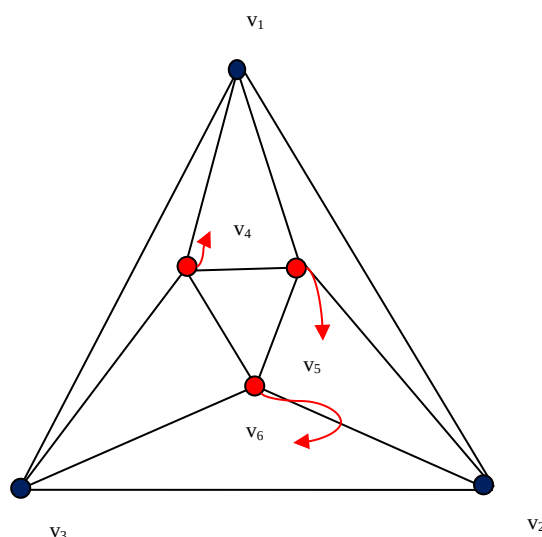


Figure 14: Octahedron graph

From the figure, $V = \{v_1, v_2, \dots, v_6\}$			
$D = \{v_1, v_4\}$	$\rightarrow$	$\gamma(G) = 2$	
$D_t = \{v_1, v_4\}$	$\rightarrow$	$\gamma_t(G) = 2$	
$D'_t = \{v_2, v_5\}$	$\rightarrow$	$\gamma'_t(G) = 2$	

Let G be the Platonic Solid Octahedron. Then G is a 4-regular graph with 6 vertices and 12 edges. Let $D = \{1, 4\}$ be the minimum dominating set and $D_t = \{1, 4\}$ be the minimum total dominating set and also $D'_t = \{2, 5\}$ be the minimum inverse total dominating set of G . In the above figure, the domination number as well as total domination number and the inverse total dominating number, are the same. Hence $\gamma(G) = 2$; $\gamma_t(G) = 2$; $\gamma'_t(G) = 2$.

Theorem 3.15

If G be the Desargues graph with 20 vertices, then $\gamma_t(G) = 9$ and $\gamma'_t(G)$ does not exist.

Proof:

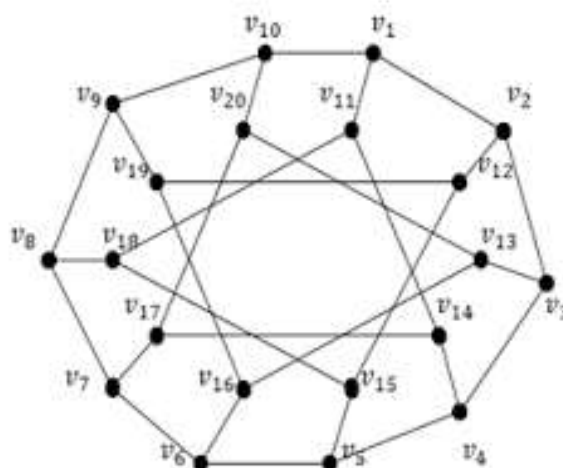


Figure 15: Desargues Graph

From the figure, $V = \{v_1, v_2, \dots, v_{20}\}$	
$D = \{v_1, v_4, v_7, v_9, v_5, v_3\}$	$\Rightarrow \gamma(G) = 6$
$D_t = \{v_{11}, v_{18}, v_{19}, v_{16}, v_3, v_4, v_{17}, v_7, v_{20}\}$	$\Rightarrow \gamma_t(G) = 9$

Let G be the Desargues graph with 20 vertices and G be the Cubic graph. In the above figure, it is found that

- (i) The minimum dominating set $D = \{v_1, v_4, v_7, v_9, v_5, v_3\}$.
- (ii) The minimum total dominating set $D_t = \{v_{11}, v_{18}, v_{19}, v_{16}, v_3, v_4, v_{17}, v_7, v_{20}\}$.
- (iii) There is no inverse total dominating set in the Desargues graph.

Hence, $\gamma(G) = 6$; $\gamma_t(G) = 9$.

Note 3.15.1

The inverse total dominating number does not exist for the above graph.

IV. Conclusion

In this paper, we have investigated the Total domination number and the Inverse total domination number of some special classes of graphs. We further study on some more domination parameters of these special graphs.

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