



Heptagonal Fuzzy Number by Singular Value Decomposition

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Abstract: In this article, Singular value decomposition method is used to solve fully fuzzy linear system of equation with heptagonal fuzzy number as inputs has been introduced. An algorithms for $n \times n$ fully fuzzy linear system $\bar{A} \otimes \bar{x} = \bar{b}$ where $\bar{A}$ is a fuzzy matrix, $\bar{x}$ and $\bar{b}$ are fuzzy vectors. Algorithms have been introduced and an numerical example have been solved.

Keywords: Fully Fuzzy Linear Systems (FFLS), Heptagonal Fuzzy Numbers (HFN)

I. INTRODUCTION

In many real life problems, the parameters used to characterize the uncertainty or vagueness or impreciseness in information are either triangular or trapezoidal fuzzy numbers. But it is not always possible to restrict the membership function to either triangular or trapezoidal form. In this paper, a new form of non-normal fuzzy number named as Heptagonal fuzzy number is introduced and its arithmetic operations are defined. L. A. Zadeh [6] introduced the concepts of fuzzy numbers and fuzzy arithmetic. Fuzzy metric spaces are used to solve fuzzy metric spaces, fuzzy differential equations, fuzzy linear and non linear system etc., A. Kumar et. al [1,2,3] introduced fully fuzzy linear system with arbitrary coefficients, A New Approach for Solving Fully Fuzzy Linear Systems, A New Computational Method for Solving Fully Fuzzy Linear Systems of Triangular Fuzzy Numbers. Linear system of equations has widely applied in many areas of science, Economics, Management and Engineering. Fuzzy linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number was first proposed by Friedman et.al. Dehgan et.al [7,8,9] introduced fully fuzzy linear system of the form $Ax + b$, Computational method for solving fully fuzzy linear systems, solving using iterative techniques. G. Malkawi, N. Ahmad and H. Ibrahim [5] investigated, "Solving fully fuzzy linear systems by using implicit gauss cholesky algorithm", A new computational method to solve FFLS by relying on the computation of row reduced echelon form. Ghassan Malkawi, Nazihah Ahmad an Haslinda Ibrahim [4]. V. Vijayalakshmi [10, 11] introduced, "Solving fully fuzzy linear systems by LU decomposition algorithm using Hexagonal and Octagonal fuzzy numbers.

This article mainly consists of preliminary definitions with their diagrammatic representation in section 2. Section 3 proposed a new algorithm solve fuzzy linear system in singular value decomposition. In section 4 numerical example have been illustrated by solving using singular value decomposition method. In section 5, conclusion about the results is established.

II. PRELIMINARIES

A. Definition: Fuzzy Set

Let X be a non empty set. A fuzzy set A is characterized by its membership function $A: X \rightarrow [0,1]$ and $A(x)$ is interpreted as the degree of membership of element in fuzzy A for each $x \in X$. The value 0 represent non membership and the value 1 represent membership in between values represent intermediate degrees of membership.



B. Arithmetic Operations on Heptagonal Fuzzy numbers

Let $AH = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ & $BH = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ be two Heptagonal Fuzzy Numbers then

addition and subtraction can be performed as

$$AH + BH = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$$

$$AH - BH = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$$

C. Definition: Heptagonal Fuzzy Number

Its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2} \left(\frac{x - a_1}{a_2 - a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} & \text{for } a_2 \leq x \leq a_3 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x - a_3}{a_4 - a_3} \right) & \text{for } a_3 \leq x \leq a_4 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{a_5 - x}{a_5 - a_4} \right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} & \text{for } a_5 \leq x \leq a_6 \\ \frac{1}{2} \left(\frac{a_7 - x}{a_7 - a_6} \right) & \text{for } a_6 \leq x \leq a_7 \\ 0 & \text{for } x > a_7 \end{cases}$$

III. PROPOSED METHOD FOR HEPTAGONAL FUZZY NUMBER

A. SINGULAR VALUE DECOMPOSITIONALGORITHM

Any fuzzy linear system of equations in the form of heptagonal fuzzy matrices that can be decomposed into the form such that $A = USV^T$ where A, B, C, D, E, F, G is the rectangular matrix can be decompose into an orthogonal matrix U, Diagonal matrix S and the transpose of an orthogonal matrix V.

Consider fully fuzzy linear systems,

Step I

$$\tilde{A} \otimes \tilde{x} = \tilde{b} \text{ where } \tilde{A} = (A, B, C, D, E, F, G),$$

$$\tilde{b} = (p, q, r, s, t, u, v) \geq 0, \quad \tilde{x} = (x, y, z, l, m, n, o) \geq 0$$

$$(A, B, C, D, E, F, G) \otimes (x, y, z, l, m, n, o) = (p, q, r, s, t, u, v)$$

Step II

$$(U_1 S_1 V_1^T, U_2 S_2 V_2^T, U_3 S_3 V_3^T, U_4 S_4 V_4^T, U_5 S_5 V_5^T, U_6 S_6 V_6^T, U_7 S_7 V_7^T) \otimes (x, y, z, l, m, n, o) =$$

$$(p, q, r, s, t, u, v)$$

$$(U_1 S_1 V_1^T x, U_2 S_2 V_2^T y, U_3 S_3 V_3^T z, U_4 S_4 V_4^T l, U_5 S_5 V_5^T m, U_6 S_6 V_6^T n, U_7 S_7 V_7^T o) = (p, q, r, s, t, u, v)$$

Step III

$$U_1 S_1 V_1^T x = p, U_2 S_2 V_2^T y = q, U_3 S_3 V_3^T z = r, U_4 S_4 V_4^T l = s, U_5 S_5 V_5^T m = t,$$

$$U_6 S_6 V_6^T n = u, U_7 S_7 V_7^T o = v,$$

$$\text{Therefore } x = V_1^{T-1} S_1^{-1} U_1^{-1} p$$

$$y = V_2^{T-1} S_2^{-1} U_2^{-1} q$$

$$z = V_3^{T-1} S_3^{-1} U_3^{-1} r$$

$$l = V_4^{T-1} S_4^{-1} U_4^{-1} s$$

$$m = V_5^{T-1} S_5^{-1} U_5^{-1} t$$

$$n = V_6^{T-1} S_6^{-1} U_6^{-1} u$$

$$o = V_7^{T-1} S_7^{-1} U_7^{-1} v$$

This methodology gives more accuracy compared with other methods and gives more accurate result.

IV. Numerical Example

Let $\tilde{x}_i$ denotes the car varieties for $i = 1, 2, 3, 4, 5, 6, 7$. Let $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ denotes the car1, car2, car 3. Let $\tilde{x}_1 = x_1, y_1, z_1, l_1, m_1, n_1, o_1$ denotes the seven parameters are level of satisfaction regarding mileage, Exterior design, Interior design, Technology design, safety, comfort and price for car 1. Let $\tilde{x}_2, \tilde{x}_3$ denotes the same parameter level for car2, car3.

The study would be carried out by comparing competitors in the car Industry for their three popular models in each car company. The relationship between cars and their satisfaction level are given in the form of fully fuzzy linear system of Equations.

$$\begin{bmatrix} (1,3,7,10,11,13,15) & (1,3,5,8,9,12,16) & (3,6,8,12,13,15,18) \\ (1,3,5,8,9,12,15) & (9,6,8,11,14,17,19) & (5,7,8,10,15,17,20) \\ (3,6,8,12,13,16,19) & (5,7,8,10,15,18,21) & (21,23,24,26,31,33,34) \end{bmatrix} * \begin{bmatrix} (x_1, y_1, z_1, l_1, m_1, n_1, o_1) \\ (x_2, y_2, z_2, l_2, m_2, n_2, o_2) \\ (x_3, y_3, z_3, l_3, m_3, n_3, o_3) \end{bmatrix} = \begin{bmatrix} (10,12,14,18,21,24,27) \\ (14,16,17,19,20,22,24) \\ (12,13,15,21,23,25,26) \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 9 & 5 \\ 3 & 5 & 21 \end{pmatrix}, B = \begin{pmatrix} 3 & 3 & 6 \\ 3 & 6 & 7 \\ 6 & 7 & 23 \end{pmatrix}, C = \begin{pmatrix} 10 & 5 & 8 \\ 5 & 8 & 8 \\ 8 & 8 & 24 \end{pmatrix}, D = \begin{pmatrix} 11 & 9 & 12 \\ 9 & 11 & 10 \\ 12 & 10 & 26 \end{pmatrix}$$

$$E = \begin{pmatrix} 11 & 9 & 13 \\ 9 & 14 & 15 \\ 13 & 15 & 31 \end{pmatrix}, F = \begin{pmatrix} 13 & 12 & 15 \\ 12 & 17 & 17 \\ 16 & 18 & 33 \end{pmatrix}, G = \begin{pmatrix} 15 & 16 & 18 \\ 15 & 19 & 20 \\ 19 & 21 & 34 \end{pmatrix}$$

The matrix A, B, C, D, E, F, G can be decomposed into $A = U_1 S_1 V_1^T, B = U_2 S_2 V_2^T, C = U_3 S_3 V_3^T, D = U_4 S_4 V_4^T, E = U_5 S_5 V_5^T,$

$$F = U_6 S_6 V_6^T, G = U_7 S_7 V_7^T$$

Consider the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 9 & 5 \\ 3 & 5 & 21 \end{pmatrix}, A = U_1 S_1 V_1^T$ where $U_1 = \begin{pmatrix} -0.141 & 0.012 & -0.990 \\ -0.336 & 0.941 & 0.037 \\ -0.931 & 0.338 & 0.136 \end{pmatrix}$

$$S_1 = \begin{pmatrix} 23.259 & 0 & 0 \\ 0 & 7.191 & 0 \\ 0 & 0 & 0.55 \end{pmatrix}, V_1^T = \begin{pmatrix} -0.141 & -0.336 & -0.931 \\ 0.012 & 0.941 & 0.338 \\ -0.990 & 0.037 & 0.136 \end{pmatrix}$$

$$x = V_1^T S_1^{-1} U_1^{-1} p$$

$$x = \begin{pmatrix} -0.141 & 0.012 & -0.990 \\ -0.336 & 0.941 & 0.037 \\ -0.931 & 0.338 & 0.136 \end{pmatrix} \begin{pmatrix} 0.043 & 0 & 0 \\ 0 & 0.139 & 0 \\ 0 & 0 & 1.818 \end{pmatrix} \begin{pmatrix} -0.141 & -0.336 & -0.931 \\ 0.012 & 0.941 & 0.338 \\ -0.990 & 0.037 & 0.136 \end{pmatrix} \begin{pmatrix} 10 \\ 14 \\ 12 \end{pmatrix}$$

$$x = \begin{pmatrix} 14.038 \\ 0.6896 \\ -1.664 \end{pmatrix}$$

V. Results and Discussions

The other parameters for car1, car2, car3 are found as follows by the same algorithm,

$$x = \begin{pmatrix} 14.038 \\ 0.6896 \\ -1.664 \end{pmatrix}, y = \begin{pmatrix} 4.019 \\ 1.996 \\ 0.989 \end{pmatrix}, z = \begin{pmatrix} -4.011 \\ 3.432 \\ -0.206 \end{pmatrix}, l = \begin{pmatrix} 0.577 \\ 1.163 \\ 0.084 \end{pmatrix}, m = \begin{pmatrix} 1.731 \\ 0.614 \\ -0.317 \end{pmatrix}$$

$$n = \begin{pmatrix} 2.023 \\ 0.107 \\ 0.222 \end{pmatrix}, o = \begin{pmatrix} 3.774 \\ -1.214 \\ -0.617 \end{pmatrix}$$

On comparison, the parameters which are outstanding and those which need improvement can be identified for each model of car. The unknown value $\bar{x}_1, \bar{x}_2, \bar{x}_3$ with seven heptagonal values can be found. The level of satisfaction can now be represented as a heptagonal fuzzy number. Higher level indicates higher level of satisfaction for each model of car. Lower level indicates needs improvement in the customer satisfaction level. Positive and negative values can be found.

VI. Conclusion

In this article, new methodology to solve fully fuzzy linear system of equations using singular value decomposition method is adopted. Practical example of comparison study of three different types of cars with seven parameters has been taken. A rejuvenate algorithm has been introduced. Solving the problem gives the level of satisfaction of customers, positive values indicates the positive opinion and the negative value indicates needs some improvement.

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