



Intuitionistic Fuzzy Ideals in BE-Algebras

P.Ayesha Parveen¹ and M.Himaya Jaleela Begum²

¹M.Phil Scholar and ²Assistant Professor,

Department of Mathematics, SadakathullahAppa College,
ManonmaniamSundaranar University,
Tirunelveli, Tamil Nadu, India.

Abstract: In this paper, we discuss the concept of intuitionistic fuzzy ideals of BE-Algebras. The homomorphic behaviors of intuitionistic fuzzy ideals have been obtained. We study the properties of intuitionistic fuzzy ideals of BE-Algebras with the help of (α, β) cut sets.

Keywords: Intuitionistic fuzzy set (IFS), intuitionistic fuzzy ideal (IFI), (BE-algebra)-Homomorphism, (α, β) -cut set

I. Introduction

The concept of fuzzy sets was first initiated by Zadeh[8] 1965. The idea of Intuitionisticfuzzysets[2]was introduced by Atanassov as a generalization of the notion of fuzzysets.The notion of BE-algebras[3] was first introduced by HeeSik Kim and Young Hee Kim. The notion of fuzzy ideals in BE-algebras[5] was introduced by SeokZun Song, Young Bae Jun,KyoungJa Lee. In this paper, we study the intuitionistic fuzzy ideals of BE-algebras and give some of its properties. We discuss homomorphic behaviors of intuitionistic fuzzy ideals of BE-algebras. We also examine (α, β) cut setsin BE-algebras

II. Preliminaries

Definition 2.1:[3] Let $k(\tau)$ be the class of all algebras of type $\tau = (2,0)$. By a BE-algebra we mean a system $(X;*,1) \in k(\tau)$ in which the following axioms hold.

- (i) $x*x=1 \ \forall x \in X$
- (ii) $x*1=1 \ \forall x \in X$
- (iii) $1*x=x \ \forall x \in X$
- (iv) $x*(y*z)=y*(x*z)$ (exchange)

Definition 2.2:[3] A relation $\leq$ on a BE-algebra X is defined by $x \leq y \Leftrightarrow x*y=1 \ \forall x,y \in X$.

Definition 2.3:[1] A non empty subset I of a BE-algebra X is called an **Ideal** of X if it satisfies

- (i) $x*a \in I \ \forall x \in X, \forall a \in I$
- (ii) $(a*(b*x))*x \in I \ \forall x \in X \forall a,b \in I$

Definition 2.4:[5] A fuzzy set μ in X is called a **fuzzy ideal** of X if it satisfies

- (i) $\mu(x*y) \geq \mu(y) \ \forall x,y \in X$
- (ii) $\mu(x*(y*z))*z \geq \min \{ \mu(x), \mu(y) \} \forall x,y,z \in X$

Definition 2.5:An intuitionistic fuzzy set (IFS) in a non empty set X is an object having the form IFS $A=\{x, \mu_A(x), \lambda_A(x) / x \in X\}$ where the function $\mu_A: X \rightarrow [0, 1]$ and $\lambda_A: X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$.In simple, $A=(\mu_A, \lambda_A)$ for the IFS $A=\{x, \mu_A(x), \lambda_A(x) / x \in X\}$

Definition 2.6:[4] Let A be an IFS of X . Then (α, β) cut of A is a **crisp set** $C_{(\alpha, \beta)}(A)$ of IFS A is given by $C_{(\alpha, \beta)}(A) = \{x/x \in X \text{ such that } \mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta\}$ where $\alpha, \beta \in [0, 1]$ with $\alpha+\beta \leq 1$

Definition 2.7:[6]Let $(X_1, *, 1)$ and $(X_2, \circ, 1)$ be two BE-algebras. Then a mapping $f: X_1 \rightarrow X_2$ is called a homomorphism if $f(x*y)=f(x) \circ f(y) \ \forall x,y \in X_1$.(It is clear that if $f: X_1 \rightarrow X_2$ is homomorphism , then $f(1)=1$)

Definition 2.8:[7]Let $f: X \rightarrow Y$ be a homomorphism of X for any IFS $A=(\mu_A, \lambda_A)$ in Y . Define an IFS $A^f=(\mu_A^f, \lambda_A^f)$ in X by $\mu_A^f(x)=\mu_A(f(x)), \lambda_A^f(x)=\lambda_A(f(x))$

III. Intuitionistic Fuzzy Ideals

In what follows, let X denote a BE-algebra unless otherwise specified.

Definition 3.1: An IFS $A = (\mu_A, \lambda_A)$ of X is called an **intuitionistic fuzzy ideal** of X if $\forall x, y, z \in X$

- (i) $\mu_A(x*y) \geq \mu_A(y)$
- (ii) $\mu_A((x*(y*z))*z) \geq \min \{ \mu_A(x), \mu_A(y) \}$
- (iii) $\lambda_A(x*y) \leq \lambda_A(y)$
- (iv) $\lambda_A((x*(y*z))*z) \leq \max \{ \lambda_A(x), \lambda_A(y) \}$

Example 3.2: Let $X = \{1, a, b, c, d, 0\}$ be a set with the following Cayley table

*	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Then $(X; *, 1)$ is a BE-algebra[3]

Let $A = (\mu_A, \lambda_A)$ be an IFS in X is defined by

$$\mu_A(x) = \begin{cases} 0.7 & \text{if } x \in \{1, a, b\} \\ 0.2 & \text{if } x \in \{c, d, 0\} \end{cases}$$

$$\lambda_A(x) = \begin{cases} 0.2 & \text{if } x \in \{1, a, b\} \\ 0.6 & \text{if } x \in \{c, d, 0\} \end{cases}$$

Then $A = (\mu_A, \lambda_A)$ is an IFI of X

- (i) Let $A = (\mu_A, \lambda_A)$ be an IFS in X is defined by

$$\mu_A(x) = \begin{cases} 0.6 & \text{if } x \in \{1, a, b, c\} \\ 0.4 & \text{if } x \in \{d, 0\} \end{cases}$$

$$\lambda_A(x) = \begin{cases} 0.3 & \text{if } x \in \{1, a, b, c\} \\ 0.2 & \text{if } x \in \{d, 0\} \end{cases}$$

Then $A = (\mu_A, \lambda_A)$ is not an IFI of X . Since $\lambda_A(a) \leq \lambda_A(0)$ (i.e.,) $0.3 \not\leq 0.2$

Similarly, $\lambda_A(d*(0*1))*1 \leq \max \{ \lambda_A(d), \lambda_A(0) \} \Rightarrow \lambda_A(1) \leq \lambda_A(0) \Rightarrow 0.3 \leq \max \{ 0.2, 0.2 \} \Rightarrow 0.3 \not\leq 0.2$

Theorem 3.3: If $A = (\mu_A, \lambda_A)$ be an IFI of X , then $\mu_A(x) \leq \mu_A(1)$ and $\lambda_A(x) \geq \lambda_A(1) \quad \forall x \in X$

Proof: We know that $\mu_A(1) = \mu_A(x*x) \geq \mu_A(x)$ and $\lambda_A(1) = \lambda_A(x*x) \leq \lambda_A(x)$

Since x is arbitrary, $\mu_A(x) \leq \mu_A(1)$ and $\lambda_A(x) \geq \lambda_A(1) \quad \forall x \in X$

Theorem 3.4: Let $A = (\mu_A, \lambda_A)$ be an IFS of X . Then A is an IFI of X iff the fuzzy sets μ_A, λ_A^c are fuzzy ideals of X .

Proof: Clearly, μ_A is a fuzzy ideal of X . (i.e.) $\mu_A(x*y) \geq \mu_A(y)$ and $\mu_A((x*(y*z))*z) \geq \min \{ \mu_A(x), \mu_A(y) \}$ for all $x, y, z \in X$

For $x, y, z \in X$

$$\begin{aligned} \text{(i)} \quad \lambda_A^c(x*y) &= 1 - \lambda_A(x*y) \\ &\geq 1 - \lambda_A(y) \\ &= \lambda_A^c(y) \end{aligned}$$

Therefore, $\lambda_A^c(x*y) \geq \lambda_A^c(y)$

$$\begin{aligned} \text{(ii)} \quad \lambda_A^c((x*(y*z))*z) &= 1 - \lambda_A((x*(y*z))*z) \\ &\geq 1 - \max \{ \lambda_A(x), \lambda_A(y) \} \\ &= \min \{ (1 - \lambda_A(x)), (1 - \lambda_A(y)) \} \\ &= \min \{ \lambda_A^c(x), \lambda_A^c(y) \} \end{aligned}$$

Therefore, $\lambda_A^c((x*(y*z))*z) \geq \min \{ \lambda_A^c(x), \lambda_A^c(y) \}$

Conversely,

Let μ_A, λ_A are fuzzy ideal of X. Then clearly by definition $\mu_A(x*y) \geq \mu_A(y)$ and $\mu_A((x*(y*z))*z) \geq \min\{\mu_A(x), \mu_A(y)\}$ are satisfied.

Now, for all $x, y, z \in X$

$$\begin{aligned} \text{(i)} \quad 1 - \lambda_A(x*y) &= \lambda_A^c(x*y) \\ &\geq \lambda_A^c(y) \\ &= 1 - \lambda_A(y) \end{aligned}$$

Therefore, $\lambda_A(x*y) \leq \lambda_A(y)$

$$\begin{aligned} \text{(ii)} \quad \lambda_A((x*(y*z))*z) &= \lambda_A^c((x*(y*z))*z) \\ &\geq \min\{\lambda_A^c(x), \lambda_A^c(y)\} \\ &= \min\{(1 - \lambda_A(x)), (1 - \lambda_A(y))\} \\ &= 1 - \max\{\lambda_A(x), \lambda_A(y)\} \end{aligned}$$

Therefore, $\lambda_A((x*(y*z))*z) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

Hence A is an IFI of X.

Theorem 3.5: If $A = (\mu_A, \lambda_A)$ be an IFS in X, then A is an IFI of X iff $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of X.

Proof: If $A = (\mu_A, \lambda_A)$ is an IFI of X, then $\mu_A = (\mu_A^c)^c$ and λ_A^c are fuzzy ideal of X.

By theorem(3.4), $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of X.

Conversely, Let $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of X.

Then the fuzzy set μ_A, λ_A^c are fuzzy ideal of X.

Therefore, $A = (\mu_A, \lambda_A)$ is an IFI of X.

Theorem 3.7: Let $f: X \rightarrow Y$ be a homomorphism of X. If an IFS $A = (\mu_A, \lambda_A)$ is an IFI of Y, then an IFS $A^f = (\mu_A^f, \lambda_A^f)$ in X is an IFI of X.

Proof: For $x, y \in X$

$$\begin{aligned} \text{(i)} \quad \mu_A^f(x*y) &= \mu_A(f(x*y)) \\ &= \mu_A(f(x)*f(y)) \\ &\geq \mu_A(f(y)) \\ &= \mu_A^f(y) \end{aligned}$$

Therefore, $\mu_A^f(x*y) \geq \mu_A^f(y)$

$$\begin{aligned} \text{(ii)} \quad \mu_A^f((x*(y*z))*z) &= \mu_A(f((x*(y*z))*z)) \\ &= \mu_A(f(x*(y*z))*f(z)) \\ &= \mu_A((f(x)*f(y*z))*f(z)) \\ &= \mu_A((f(x)*(f(y)*f(z)))*f(z)) \\ &\geq \min\{\mu_A(f(x)), \mu_A(f(y))\} \\ &= \min\{\mu_A^f(x), \mu_A^f(y)\} \end{aligned}$$

Therefore, $\mu_A^f((x*(y*z))*z) \geq \min\{\mu_A^f(x), \mu_A^f(y)\}$

$$\begin{aligned} \text{(iii)} \quad \lambda_A^f(x*y) &= \lambda_A(f(x*y)) \\ &= \lambda_A(f(x)*f(y)) \\ &\leq \lambda_A(f(y)) \\ &= \lambda_A^f(y) \end{aligned}$$

Therefore, $\lambda_A^f(x*y) \leq \lambda_A^f(y)$

$$\begin{aligned} \text{(iv)} \quad \lambda_A^f((x*(y*z))*z) &= \lambda_A(f((x*(y*z))*z)) \\ &= \lambda_A(f(x*(y*z))*f(z)) \\ &= \lambda_A((f(x)*f(y*z))*f(z)) \\ &= \lambda_A((f(x)*(f(y)*f(z)))*f(z)) \\ &\leq \max\{\lambda_A(f(x)), \lambda_A(f(y))\} \\ &= \max\{\lambda_A^f(x), \lambda_A^f(y)\} \end{aligned}$$

Therefore, $\lambda_A^f((x*(y*z))*z) \leq \max\{\lambda_A^f(x), \lambda_A^f(y)\}$

Theorem 3.8: Let $f: X \rightarrow Y$ be an epimorphism of X. Let $A = (\mu_A, \lambda_A)$ is an IFS of Y. If $A^f = (\mu_A^f, \lambda_A^f)$ is an IFI of X then $A = (\mu_A, \lambda_A)$ is an IFI of Y.

Proof: For any $x, y, z \in Y$, there exists $a, b, c \in X$ such that $f(a) = x, f(b) = y, f(c) = z$.

$$\begin{aligned} \text{(i)} \quad \mu_A(x*y) &= \mu_A(f(a)*f(b)) \\ &= \mu_A(f(a*b)) \\ &= \mu_A^f(a*b) \\ &\geq \mu_A^f(b) \\ &= \mu_A(f(b)) \\ &= \mu_A(y) \end{aligned}$$

Therefore, $\mu_A(x*y) \geq \mu_A(y)$

$$\begin{aligned} \text{(ii)} \quad \mu_A^f((x*(y*z))*z) &= \mu_A((f(a)*(f(b)*f(c)))*f(c)) \\ &= \mu_A((f(a)*f(b*c))*f(c)) \end{aligned}$$

$$\begin{aligned}
 &= \mu_A((f(a*(b*c))*f(c)) \\
 &= \mu_A((f(a*(b*c))*c)) \\
 &= \mu_A^f((a*(b*c))*c) \\
 &\geq \min \{ \mu_A^f(a), \mu_A^f(b) \} \\
 &= \min \{ \mu_A(f(a)), \mu_A(f(b)) \} \\
 &= \min \{ \mu_A(x), \mu_A(y) \}
 \end{aligned}$$

Therefore, $\mu_A^f((x*(y*z))*z) \geq \min \{ \mu_A(x), \mu_A(y) \}$

$$\begin{aligned}
 \text{(iii)} \quad \lambda_A(x*y) &= \lambda_A(f(a)*f(b)) \\
 &= \lambda_A(f(a*b)) \\
 &= \lambda_A^f(a*b) \\
 &\leq \lambda_A^f(b) \\
 &= \lambda_A(f(b)) \\
 &= \lambda_A(y)
 \end{aligned}$$

Therefore, $\lambda_A(x*y) \leq \lambda_A(y)$

$$\begin{aligned}
 \text{(iv)} \quad \lambda_A^f((x*(y*z))*z) &= \lambda_A((f(a)*(f(b)*f(c))*f(c)) \\
 &= \lambda_A((f(a)*(b*c))*f(c)) \\
 &= \lambda_A((f(a*(b*c))*f(c)) \\
 &= \lambda_A((f(a*(b*c))*c) \\
 &= \lambda_A^f((a*(b*c))*c) \\
 &\leq \max \{ \lambda_A^f(a), \lambda_A^f(b) \} \\
 &= \max \{ \lambda_A(f(a)), \lambda_A(f(b)) \} \\
 &= \max \{ \lambda_A(x), \lambda_A(y) \}
 \end{aligned}$$

Therefore, $\lambda_A^f((x*(y*z))*z) \leq \max \{ \lambda_A(x), \lambda_A(y) \}$

Theorem 3.9: If $A=(\mu_A, \lambda_A)$ is an IFI of X , then $C_{(\alpha, \beta)}(A)$ is an ideal of X if $\mu_A(1) \geq \alpha$ and $\lambda_A(1) \leq \beta$

Proof: Let $A=(\mu_A, \lambda_A)$ is an IFI of X .

Let $\mu_A(1) \geq \alpha$ and $\lambda_A(1) \leq \beta$.

Clearly $C_{(\alpha, \beta)}(A) \neq \emptyset$

Let $x, y \in C_{(\alpha, \beta)}(A) \Rightarrow \mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha$ and $\lambda_A(x) \leq \beta, \lambda_A(y) \leq \beta$

Now, $\mu_A(x*y) \geq \mu_A(y) \geq \alpha$ and $\lambda_A(x*y) \leq \lambda_A(y) \leq \beta$

Therefore, $x*y \in C_{(\alpha, \beta)}(A)$

Let $x, y \in C_{(\alpha, \beta)}(A)$ and $z \in X$. Then $\mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha$ and $\lambda_A(x) \leq \beta, \lambda_A(y) \leq \beta$

$\mu_A(x*(y*z))*z \geq \min \{ \mu_A(x), \mu_A(y) \} \geq \alpha$

$\lambda_A(x*(y*z))*z \leq \max \{ \lambda_A(x), \lambda_A(y) \} \leq \beta$

Therefore, $(x*(y*z))*z \in C_{(\alpha, \beta)}(A)$

Hence $C_{(\alpha, \beta)}(A)$ is an ideal of X .

Theorem 3.10: If $A=(\mu_A, \lambda_A)$ is an IFS of X , then A is an IFI of X iff $C_{(\alpha, \beta)}(A)$ is an ideal of $X \forall \alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(1) \geq \alpha$ and $\lambda_A(1) \leq \beta$.

Proof: If $A=(\mu_A, \lambda_A)$ is an IFI of X , then $C_{(\alpha, \beta)}(A)$ is an ideal of $X \forall \alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(1) \geq \alpha, \lambda_A(1) \leq \beta$ (from theorem 3.9)

Conversely,

Let $A=(\mu_A, \lambda_A)$ is an IFS of X , such that $C_{(\alpha, \beta)}(A)$ is an ideal of $X \forall \alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(1) \geq \alpha, \lambda_A(1) \leq \beta$

Suppose that $x, y \in X, \mu_A(x*y) < \mu_A(y)$ and $\lambda_A(x*y) > \lambda_A(y)$.

Choose α, β be such that $\mu_A(x*y) < \alpha < \mu_A(y)$ and $\lambda_A(x*y) > \beta > \lambda_A(y)$.

We get $x, y \in C_{(\alpha, \beta)}(A)$ but $x*y \notin C_{(\alpha, \beta)}(A)$ which is a contradiction.

Therefore, $\mu_A(x*y) \geq \mu_A(y)$ and $\lambda_A(x*y) \leq \lambda_A(y)$

Similarly for $x, y \in C_{(\alpha, \beta)}(A)$ and $z \in X$

$\mu_A(x*(y*z))*z < \alpha < \min \{ \mu_A(x), \mu_A(y) \}$

$\lambda_A(x*(y*z))*z > \beta > \max \{ \lambda_A(x), \lambda_A(y) \}$

We get $x, y \in C_{(\alpha, \beta)}(A)$ but $(x*(y*z))*z \notin C_{(\alpha, \beta)}(A)$ which is a contradiction.

Therefore, $\mu_A(x*(y*z))*z \geq \min \{ \mu_A(x), \mu_A(y) \}$, $\lambda_A(x*(y*z))*z \leq \max \{ \lambda_A(x), \lambda_A(y) \}$

Hence $A=(\mu_A, \lambda_A)$ is an IFI of X

Theorem 3.11: If $A=(\mu_A, \lambda_A)$ is an IFI of X , then the set $M = \{ x \in X \mid \mu_A(x) = \mu_A(1) \text{ and } \lambda_A(x) = \lambda_A(1) \}$ is an ideal of X

Proof: Let $A=(\mu_A, \lambda_A)$ is an IFI of X .

Let $x, y \in M$. Then $\mu_A(x) = \mu_A(1), \lambda_A(x) = \lambda_A(1)$ and $\mu_A(y) = \mu_A(1), \lambda_A(y) = \lambda_A(1)$

Since $A=(\mu_A, \lambda_A)$ is an IFI of X , $\mu_A(x*y) \geq \mu_A(y) = \mu_A(1)$

$\Rightarrow \mu_A(x*y) \geq \mu_A(1)$

By theorem 3.3, $\mu_A(x*y) = \mu_A(1)$

Also, $\lambda_A(x*y) \leq \lambda_A(y) = \lambda_A(1)$

$\Rightarrow \lambda_A(x*y) \leq \lambda_A(1)$

By theorem 3.3, $\lambda_A(x*y) = \lambda_A(1)$

So, $x*y \in M$

Let $x, y \in M, z \in X$. Then $\mu_A(x) = \mu_A(1)$, $\lambda_A(x) = \lambda_A(1)$ and $\mu_A(y) = \mu_A(1)$, $\lambda_A(y) = \lambda_A(1)$

Since $A = (\mu_A, \lambda_A)$ is an IFI of X , we get $\mu_A((x*(y*z))*z) \geq \min\{\mu_A(x), \mu_A(y)\} = \mu_A(1)$

By theorem 3.3, we get $\mu_A((x*(y*z))*z) = \mu_A(1)$

Similarly, $\lambda_A((x*(y*z))*z) = \lambda_A(1)$

Therefore, $((x*(y*z))*z) \in M$

Thus M is an ideal of X .

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