



Double Diffusive Convection in a Linearly Moving Permeable Vertical Surface under Magnetic Effect with Heat Source in Porous Medium

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Abstract: The present manuscript report the study of double diffusive free convective flow from a linearly moving vertical surface in porous medium. The internal heating is take place during the flow study. The chemical reaction is taken into account in mass transfer. Similarity transformation used for the governing partial differential equations to transformed into a system of ordinary differential equation, which is solved numerically using Finite difference method. Numerical results for local Nussult number are shown graphically for variation in heat transfer. This study also analyzes the effect of velocity, heat transfer in respect to Magnetic Number M , Darcy Number Da , Prandtl number Pr , Schmidt Number Sc and Chemical reaction rate parameter on velocity,, temperature and concentration profiles. Graphical results shows that the thermal boundary layer thickness decreases at and near the wall with the increase in heat flux
Keywords: MHD, Double diffusive, FDM, Porous Medium, Heat Source

I. Introduction

In many fluid flow problems, temperature and mass (or concentration) diffusion act together to induce a buoyancy force, which drives the fluid, is known as a double diffusive or thermo-solutal convection. The main feature of double-diffusive convection is that two components with different rates of diffusion affect the fluid density. Heat and mass transfer fluid flow problem are frequently used in many real life and engineering applications under different types of fluids. Understanding the flow dynamic as well as instability mechanism of double diffusive flow is an active and challenging area of research due to its wide applications in geothermal systems, building thermal insulation, nuclear waste disposal, thermal energy storage, cooling of nuclear reactors during emergency shutdown, etc. The study of double diffusive convection in fluid saturated porous media is a highly active subject of research due to wide applications in science and engineering field. The double diffusive convection is encountered in many various applications such as heat exchangers, solar collectors, boilers, cooling of electronic devices, compact heat exchangers the cooling core of nuclear reactors and in natural environments like as hydrothermal vents, hot springs etc. The foundation of convective flow in porous media has been well-documented in the recent book of Ingham and Pop[1], Nield and Bejan[2]. It is observed from the literature that many researchers have already worked on free convection flow past a vertical plate. Bejan and Khair [3] studied the buoyancy induced heat and mass transfer from a vertical plate embedded in a saturated porous medium. Seddeek [4] was studied the impact of chemical reaction, variable viscosity, on mixed convective heat and mass transfer through porous media. Kapoor et al.[5] have attempted the analytical solution of MHD free convective flow through vertical flat plate in porous medium in addition of that Rawat. et.al [6] shown that effect of MHD flow, heat and mass transfer of a micropolar fluid over a nonlinear stretching sheet with variable micro inertia density, heat flux and chemical reaction in a non-Darcy porous medium were observed. Umamaheshwar [7] numerical investigated MHD free convection flow of a non-Newtonian fluid past an impulsively started vertical plate in the presence of thermal diffusion and radiation absorption. However, Ibrahim et al. [8] studied effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Rawat and Kapoor[9] initiate the numerical solution of linearly moving permeable vertical surface in presence of magnetic field by using FEM, their work is restricted to viscous media. Malaeswari[10] studied the unsteady double diffusive chemically reactive MHD thin fluid layer flow near the surface of an electrically conducting fluid with incompressibility past a flat surfaced plate is investigated by similarity analysis. Keeping in view the above and work an attempt is being taken here to do the numerical investigation of linearly moving permeable vertical surface in presence of magnetic field in porous medium using finite difference method.

II. Mathematical Model

It is assumed that the double- diffusive free convection in steady laminar boundary layer flow of an incompressible viscous fluid in porous medium. We have also taken linearly moving permeable vertical surface under the action of the transverse magnetic field. The velocity of the fluid vanishes as far away from the plate is equal to zero. The variation of surface temperature and concentration are assumed to be linear. It is assumed that the electric field as well as magnetic dissipations. The chemical reaction takes place during mass transfer in the system. The external heat source is also considered. The governing equation under the aforesaid assumptions is as follows.

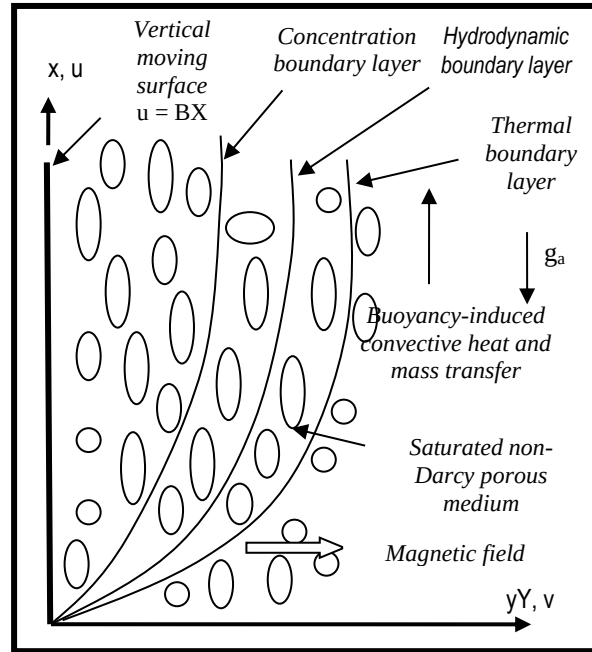


Figure 1 Sketch of physical problem

III. Governing Equations

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \mu \left(\frac{\partial^2 u}{\partial y^2} \right) + g[\beta_T(T - T_\infty) + \beta_S(S - S_\infty)] - \frac{\sigma_0 B_0^2}{\rho} u - \frac{\mu}{K} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + Q_0(T - T_\infty) \quad (3)$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \sigma \frac{\partial^2 S}{\partial y^2} - \Gamma(S - S_\infty) \quad (4)$$

Subject to the boundary conditions

$$y \rightarrow \infty: S = S_\infty, T = T_\infty, u = 0 \quad (5)$$

$$x = 0: v = -v_w, S = S_\infty + bx, T = T_\infty + ax, u = Bx \quad (6)$$

where u and v are velocity components in X and Y directions respectively, T denotes fluid temperature, g is acceleration due to gravity, μ is kinematic viscosity, C is species concentration, σ is electrical conductivity, B_0 is externally imposed magnetic field in the Y -direction. K is permeability, Γ is chemical reaction parameter.

IV. Similarity Transformation

Using the normalized variables:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \eta = \sqrt{\frac{B}{\mu}} Y, f(\eta) = \frac{\psi}{X\sqrt{B\mu}}, \theta(\eta) = \frac{T - T_\infty}{T_w(X) - T_\infty}, c(\eta) = \frac{C - C_\infty}{C_w(X) - C_\infty} \quad (7)$$

V. Non-Dimensional Governing Equation

$$F' = U, \quad (8)$$

$$U'' + FU' - \left(M + \frac{1}{Da} + U\right)U + \frac{Gr_T}{Re^2}[\theta + N_c] = 0, \quad (9)$$

$$\theta'' + Pr(F\theta' - U\theta + Q\theta) = 0, \quad (10)$$

$$C'' + Sc(FC' - UC + \chi C) = 0 \quad (11)$$

$$\text{With the boundary conditions: } \eta = 0, F = F_w, U = 1, \theta = 1, \varphi = 1; \eta \rightarrow \infty, U = 0, \theta = 0, \varphi = 0 \quad (12)$$

where, Magnetic parameter is given by $M = \frac{\sigma B_0^2}{\rho B}$, Prandtl number is given by $= \frac{\mu}{\alpha}$, Chemical reaction parameter is given by $= \frac{k_c}{B}$, Buoyancy parameter is given by $\frac{Gr_T}{Re^2} = \frac{g\beta_T a}{B^2}$, Buoyancy ratio is given by $N_r = \frac{\beta_c b}{\beta_T a}$, which is also known as the Eckert Number, Schmidt number is given by $= \frac{\mu}{D}$, here D is coefficient of diffusion in the mixture and χ is known as chemical reaction parameter.

VI. Numerical Computation

The dimensionless equations (8) – (11) together with the boundary conditions (12) are solved numerically by means of the second order accurate finite difference technique

$$F'' = \frac{F(i+1)-2F(i)+F(i-1)}{h^2} + O(h^2), \quad \theta'' = \frac{\theta(i+1)-2\theta(i)+\theta(i-1)}{h^2} + O(h^2), \quad \varphi'' = \frac{\varphi(i+1)-2\varphi(i)+\varphi(i-1)}{h^2} + O(h^2)$$

$$F' = \frac{F(i+1)-F(i-1)}{2h} + O(h^2), \quad \theta' = \frac{\theta(i+1)-\theta(i-1)}{2h} + O(h^2), \quad \varphi' = \frac{\varphi(i+1)-\varphi(i-1)}{2h} + O(h^2),$$

with boundary conditions: Hence the system of equations is:

$$\frac{f(i+1)-f(i-1)}{2h} = U(i) \quad (13)$$

$$\frac{U(i+1)-2U(i)+U(i-1)}{h^2} + F(i) \frac{U(i+1)-U(i)}{2h} - \left[M + U\left(i + \frac{1}{Da}\right)U(i) + \frac{Gr_{Tc}}{Re^2}\right][\theta(i) + N_r(i)] = 0 \quad (14)$$

$$\frac{\theta(i+1)-2\theta(i)+\theta(i-1)}{h^2} + Pr \left[F(i) \frac{\theta(i+1)-\theta(i-1)}{2h} - U(i)\theta(i) + Q\theta(i)\right] = 0 \quad (15)$$

$$\frac{\varphi(i+1)-2\varphi(i)+\varphi(i-1)}{h^2} + Sc \left[F(i) \frac{\varphi(i+1)-\varphi(i-1)}{2h} - U(i)\varphi(i) + \chi\varphi(i)\right] = 0 \quad (16)$$

With boundary conditions:

$$U(x+1) = \theta(x+1) = \varphi(x+1) = 1, \quad U(0) = \theta(0) = \varphi(0) = 0 \quad (17)$$

The linearized system of equations $AX = B$ for finding out the value of $F(i)$, $\theta(i)$ and $\varphi(i)$ solved using Gauss-seidel method with order of accuracy 0.0005.

VII. Results and Discussion

In order to understand the flow dynamics (Velocity, temperature and concentration distributions) of double diffusive free convective flow from a linearly moving permeable vertical surface in porous medium. The rigorous numerical computation is performed by using FDM. The influence of the thermal Grashof number (Gr), solutal Grashof number (Gc), magnetic parameter (M), Prandtl number Pr, heat source parameter (Q), Schmidt number (Sc) and Darcy Number (Da) on fluid flow is presented.

Magnetic number(M)	M=0.0	M=0.5	M=1.0	M=1.5
Takhar	-1.0	-1.22	-1.41	-1.57
Yih	-1.0	-1.2247	-1.412	-1.501
Adala	-1.0	-1.2356	-1.4142	-1.497
Present work	-1.0	-1.23573	-1.4136	-1.5011

Table 1 Comparison of non-dimensional wall velocity gradient for the different value of Hartman number (M)

Before discussing the effect of different parameters, the validation of numerical is made for the particular case. The Comparison of our numerical solution with the published result of M.M Abdelkhalek (2009) is presented in tables (3.1) and (3.2) for different value of magnetic parameter (M = 0, 0.5, 1.0, 1.5 & 2.0) and Pr (0.72, 1.0, 3.0, 10.0, 20.0 & 100) with published results in terms of velocity as well as temperature gradient $F''(0)$ and $\theta''(0)$ respectively. It has been observed from the above table that our numerical results are matched with published results and the relative error found less than 10^{-4} . It is a significant achievement of our applied technique.

Pr	Abdelkhalek MM(2009)	Present work FDM	Relative Error
0.72	0.8086	0.8088	0.00024734
1.0	1.0000	1.0015	0.00150000
3.0	1.9246	1.9237	-0.00046763
10.0	3.7216	3.7225	0.0002418
20.0	5.88792	5.887009	0.000201
100.0	12.294	12.294	0

Table 2 Comparison of Non-Dimensional temperature gradient $\theta''(0)$ at Different Values of M at $F_w = 0, Q = 0, Kr = 0, Da = 10^7$

To study the effect of Pr on flow profile at $Gr = 2.0$, $Gc = 2.0$, $M = 1$, $Da = 0.05$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$ is depicted in figure (2). From figure (2), it is observed the fluid flow in term velocity, temperature and concentration is decreased on enhancing the value of Pr . Further, a common feature is observed that velocity, temperature and concentration converge to its asymptotic value for the in the region $\eta \in (5, 7)$. Since the thermal boundary-layer becomes thinner for the higher values of Pr . Therefore, above results are expected.

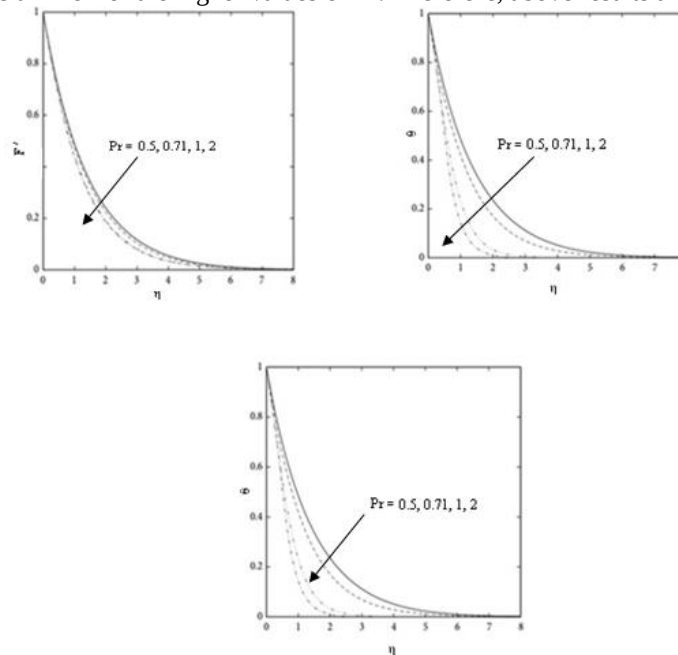


Figure 2 Effect of Prandtl number (Pr) on velocity, temperature and concentration at $Gr = 2.0$, $Gc = 2.0$, $M = 1$, $Da = 0.05$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$.

In order to study the dependency of fluid flow on Magnetic number ($M=0.5, 1, 1.5, 2, 2.5, 3$) on keeping other parameters fixed as, $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.05$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$ is shown in figure (3). It has been observed the figure (3) that the influence of M on fluid flow is not significant at the assumed fixed parameters. This has been observed that the effect of M is not much significant, the profile was smooth. This has also been encountered that the velocity is increased in a very low scale on increasing values of M . Whereas, both temperature and concentration are decreasing (at very small magnitude) for the higher values of M .

The influence of Buoyancy ratio (N_r) on fluid flow at $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.01$, $\chi = 0.5$ and $Sc = 0.6$ is plotted in figure (4). It can be visualized from the figure (4) that the temperature and concentration are increasing on enhancing the value of Buoyancy ratio (N_r), while, the velocity is decreasing. The velocity, temperature and concentration for different value of Da , 0.5, 0.1, 0.01, 0.001 at fixed values of $Gr = 2.0$, $Gc = 2.0$, $M=0.5$, $Pr = 0.7$, $M=1$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$ are plotted in figure (5). From figure (5) a notable feature is observed that the velocity, temperature and concentration are decreased on decreasing value of Da . As, we know that, Darcy number is directly proportional to the permeability of the porous media, as a consequence of it the permeability is decreasing on decreasing Da , in general high permeability induced more flow in the porous system and low permeability induced weak flow in the system. Therefore, above results are admissible. Further, it is also observed from figure (5) that a point of inflection appears near in the profiles of velocity and temperature. This can be visualize from that an increase in buoyancy lead to a decrease in velocity on the other hand the reverse profile can captured in case of temperature and concentration profile. With the figure 4, it is also can be conclude that an increase in buoyancy ratio give rise in velocity profile. Therefore, velocity profile results in an increase in heat and concentration transport thus leading to a decreased temperature and concentration profile. To study the behavior of fluid flow profile in respect to the Darcy (Da) figure (5) display results for the velocity U and temperature and concentration for different values of Darcy number Da . The effect of chemical reacting parameter on velocity as well as on concentration is presented in figure (6). It can be seen from figure (6) that the velocity profile possess a point of inflection, which indicates the reverse effect chemical reaction parameter, it may be due to the motion in the surface of boundary layer. Further, it is also observed from the figure (6) that concentration reduces on increasing chemical reaction parameter. Since, the more chemical reaction carry more concentration in the system as a consequence of it concentration reduces.

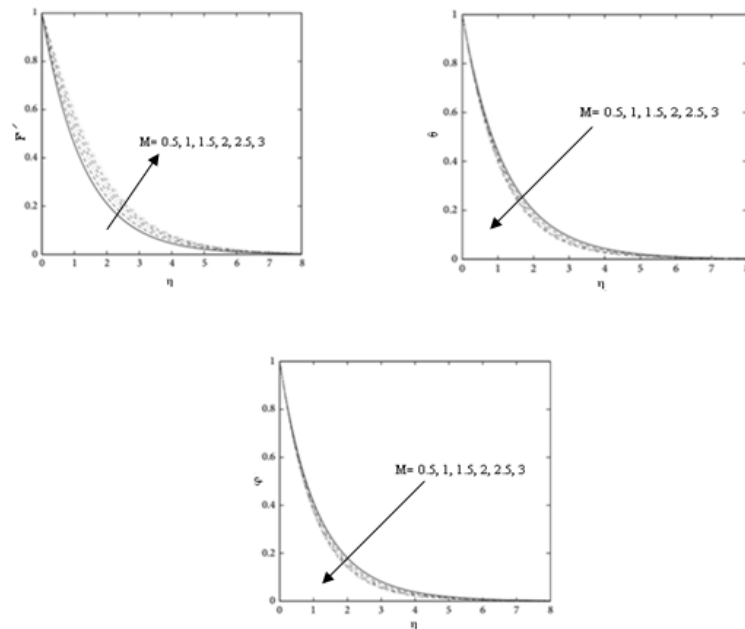


Figure 3 Effect of Magnetic number (M) on velocity, temperature and concentration at $Gc = 2.0$, $Pr = 0.7$, $Da = 0.05$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$.

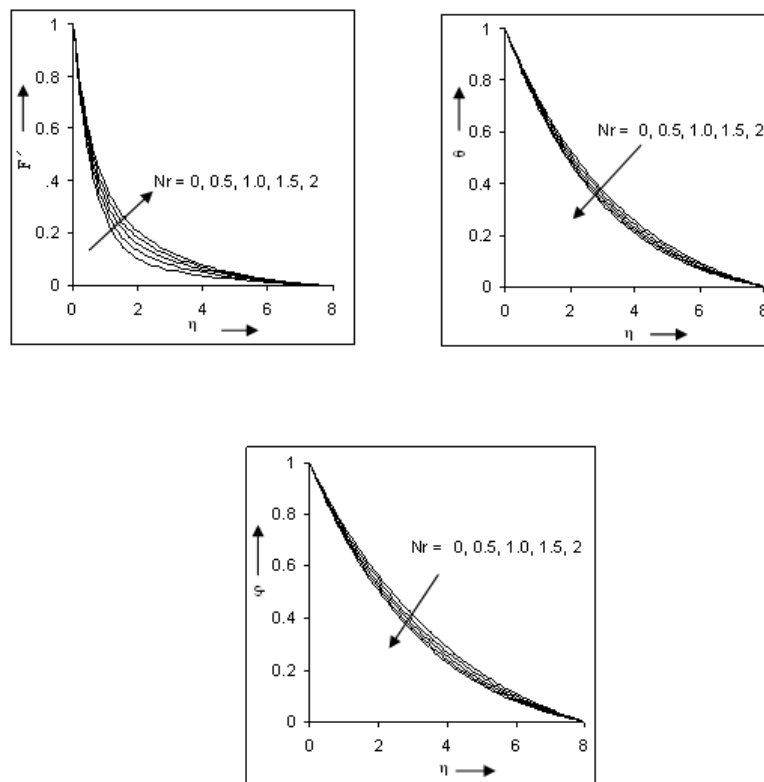


Figure 4 Effect of buoyancy ratio (N_r) on velocity, temperature and concentration at $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.01$, $M = 1$, $\chi = 0.5$ and $Sc = 0.6$.

In figure (7) the impact of suction/blowing parameter on the velocity and temperature profile has been discussed. As expected, sucking decelerated fluid particles through the porous wall leading to a reduction in the velocity and temperature boundary layers. The Figure (8) shows the effect of Schmidt number, the concentration profile is depicted. The Sc is taken 0.2, 0.5, 1.2 while fixing the other parameters at $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.05$, 5, $M = 0.5$ for two different value of chemical reaction parameter $\chi = 0$ and 1. That is for non reactive and reactive chemical. This is to be observed that Sc increased the mass transport decreased. Furthermore, we can say that boundary layer concentration is decreased. It is observed that as χ increases from 0 to 1, the concentration profile change noticeably.

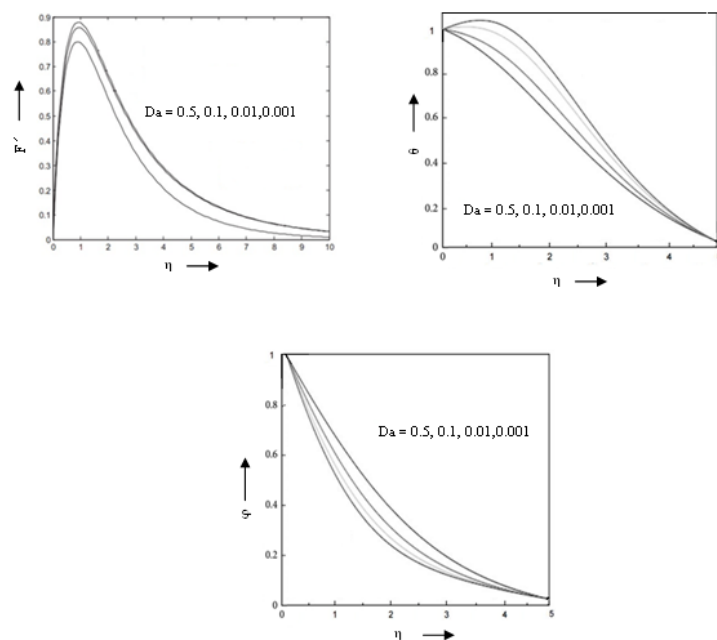


Figure 5 Effect of Darcy number (Da) on velocity, temperature and concentration at $Gr = 2.0$, $Gc = 2.0$, $M=0.5$, $Pr = 0.7$, $M=1$, $\chi = 0.5$, $N_r = 0.25$ and $Sc = 0.6$.

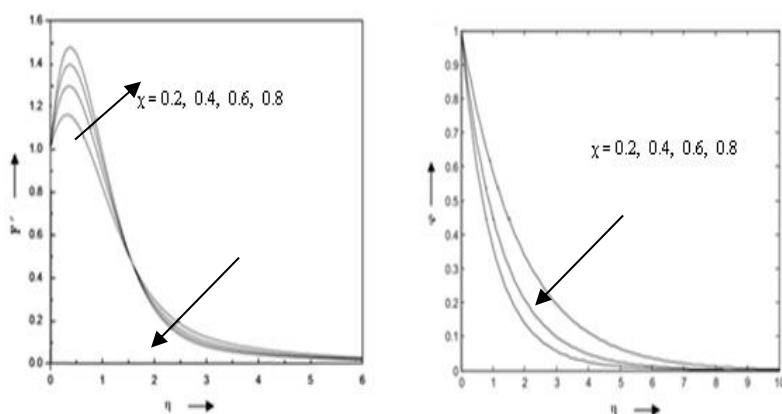


Figure 3.6 Effect of chemical reacting parameter (χ) on velocity, and concentration at $Gr = 2.0$, $Gc = 2.0$, $M=0.5$, $Pr = 0.7$, $M=1$, $Da = 0.01$, $N_r = 0.25$ and $Sc = 0.6$.

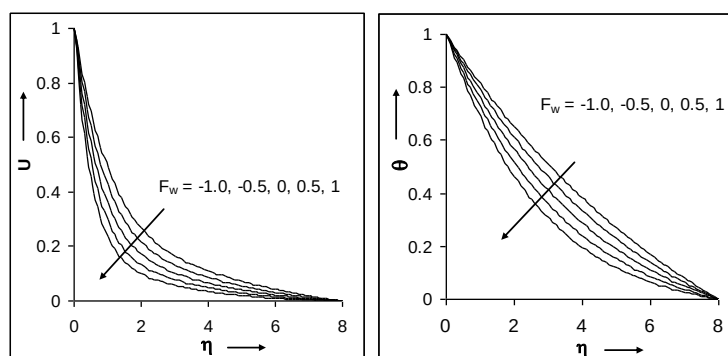


Figure 7. Effect suction parameter (F_w) on velocity and temperature at $Gr = 2.0$, $Gc = 2.0$, $M=0.5$, $Pr = 0.7$, $M=1$, $Da = 0.01$, $N_r = 0.25$ and $Sc = 0.6$

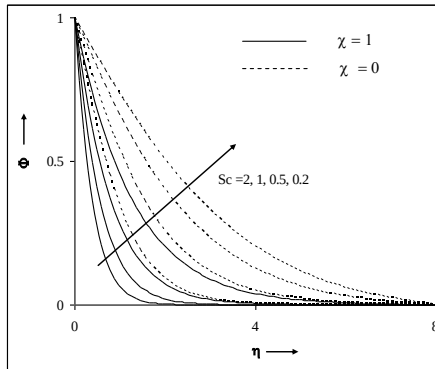


Fig. 8: Effects of Schmidt number (Sc) on concentration (Φ), both in the presence and absence of chemical reaction parameter

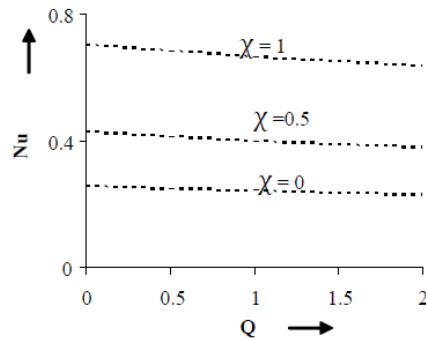


Figure 9 Local Nusslet Number for different value of Chemical reaction parameter with internal heating

This is at ($\eta = 0$) to zero at the free stream (i.e. at $\eta = 8$), these end values. As χ is increased, the profiles become more monotonic in nature; this leads physically that to *faster* rates of reaction. Finally the figure (9) shows the variation of the local Nusselt Number with heat source intensity this variation of heat flux is leads along with chemical reaction, keeping all other parameter fixed. It is observed that heat transfer at the surface of the plate increases with the increase in chemical reaction and internal heating

VIII. Conclusion

In this paper the finite difference method is used for solving all coupled differential equations of double diffusive free convective flow over a linearly moving permeable vertical surface under magnetic field. The similarity transformation is adopted to transform mathematical model into dimensionless form. All the results are presented in terms of velocity, temperature and concentration.

From, this study the following concluding remarks can be made:

- i) On increasing Prandtl number (Pr) the flow strength (velocity, temperature and concentration) is reducing.
- ii) On increasing the value of Hartmann number (M) the velocity is increasing while temperature and concentration are reducing.
- iii) On enhancing the value of buoyancy ratio (Nr) the temperature and concentration are increasing in contrast of that velocity is reducing.
- iv) On reducing media permeability via Darcy number (Da) from 0.5 to 0.001 the fluid flow is reducing.
- v) Concentration reduces on increasing chemical reacting parameter, whereas the velocity profile possess point of inflection near the boundary layer. The Schmidt number diminish mass transfer
- vi) Buoyancy parameter up lift the velocity, However, its decelerate the temperature and concentration.
- vii) Heat transfer at the surface of the plate increases with the increase in chemical reaction and internal heating

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