



Study of Trapezoidal Fuzzy Linear System of Equations

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Abstract: Herein, we have reported the fuzzy linear system of equations (FLSE) of n th order having matrix form, where A is crisp matrix, B and X are fuzzy matrices with trapezoidal fuzzy numbers and trapezoidal fuzzy variables respectively. To solve these FLSE we have introduced two new methods, Direct method and General method. Out of which, Direct method is a time efficient method and provides conditions for the existence of the solution. However, Direct method is applicable only on all positive or all negative coefficients, this restriction get rid off by General method; it permits all types of coefficients.

Keywords: Fuzzy linear system of equations, Fuzzy matrix, trapezoidal fuzzy number (TrFN).

I. Introduction

A precursor of fuzzy numbers is the foundation of the fuzzy linear system as they have been found to be an adequate in the various fields of sciences, engineering and in recent times in the industry too, as the problem cracking tool. In the practical realization of such problems involves the imprecise and unknown parameters, therefore, appropriate linear system selection to solve the problem becomes crucial one. According to Moore exact numerical data might be unrealistic, but there could be considered uncertain data as more aspects of a real world problem [11]. Fuzzy data have been used as a natural way to describe uncertain data. So, we need to solve those linear systems in which all parameters or some of them are fuzzy numbers. Thus, the crisp system of linear equations becomes a Fuzzy System of Linear Equations (FSLE) or Fully Fuzzy System of Linear Equations (FFSLE). The difference between FLSE and FFLSE is used coefficient matrix; treated as crisp in the FLSE, but in the FFLSE all the parameters and variables are considered to be fuzzy numbers.

Friedman et al. investigated a general fuzzy system using the embedding methodology in 1998[7]. They derived the conditions for the existence of a unique fuzzy solution to $n \times n$ linear system of equations, i.e. $Ax=B$, where, A was non-singular crisp matrix. Moreover, they transferred the $n \times n$ fuzzy linear system into $2n \times 2n$ real linear system. As a result, they solved the $2n \times 2n$ real linear system using the inverse matrix. Rao and Chen proposed the solution of simultaneous linear equations that arise in the analysis of engineering systems involving fuzzy input parameters [12]. The proposed methodology consisted three major aspects, computerized selection of fuzziness, implementation of fuzzy operations and development and execution of a search based algorithm.

In 2006, Dehghan and Hashemi investigated a general fully fuzzy linear system of equations and then extended numerous well-known numerical algorithms such as Richardson, Jacobi, Jacobi Over Relaxation, Gauss-Seidel, Successive Over Relaxation, Accelerated Over Relaxation, Symmetric and Unsymmetrical Successive Over Relaxation and extrapolated the Adomian decomposition method for solving fully fuzzy linear system of equations with triangular coefficients [4-6]. After that Grzegorzewski provided the Trapezoidal approximation of fuzzy numbers persevering the expected interval-Algorithms and properties [8].

In this paper, we have solved trapezoidal FLSE of n th order, i.e. $A \cdot X = B$, where A is crisp matrix. Here, we are introducing two new methods, i.e. Direct method and General method for solving FLSE with trapezoidal fuzzy number. Using these methods we have illustrated the numerical examples.

II. Preliminaries

Fuzzy numbers are one way to describe the data vagueness and imprecision. They can be regarded as an extension of the (crisp) real numbers. A fuzzy number may emerge in a practical application as a description of the variation in our knowledge about the correct value of some measurement when the level of our confidence in that knowledge varies. The theory of fuzzy numbers is based on the theory of fuzzy sets which Zadeh introduced in 1965 [13].

2.1 Fuzzy number

We assume $X = \mathbb{R}$ a fuzzy number A is a fuzzy subset of the set of real numbers $\mathbb{R}$ with membership function $A: \mathbb{R} \rightarrow I$ such that A is normal, convex, upper semi-continuous with bounded support.

2.2 Trapezoidal fuzzy numbers (TrFN)

If left hand and right hand curves are straight lines, then the fuzzy numbers are called trapezoidal fuzzy numbers. Among the various shapes of fuzzy numbers TrFN are most popular. A trapezoidal fuzzy number is denoted by $\bar{A}$ and defined as (a_1, a_2, a_3, a_4) , where the membership function is

$$A(x) = \begin{cases} 0 & , \quad x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & , \quad a_1 \leq x \leq a_2 \\ 1 & , \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & , \quad a_3 \leq x \leq a_4 \\ 0 & , \quad x \geq a_4 \end{cases}$$

We denote the above trapezoidal fuzzy number by new notations as $[a, b]_{\varepsilon, \delta}$, where ε and δ are left and right spreads of the fuzzy number [2]. The membership function of ε - δ trapezoidal fuzzy number is given by

$$[a, b](x) = \begin{cases} \frac{x - (a - \varepsilon)}{\varepsilon} & \text{when } a - \varepsilon < x \leq a \\ 1 & \text{when } a \leq x \leq b \\ \frac{(b + \delta) - x}{\delta} & \text{when } b < x \leq b + \delta \\ 0 & \text{otherwise} \end{cases}$$

The α -cut of $[a, b]_{\varepsilon, \delta}$ is given by $([a, b]_{\varepsilon, \delta})_{\alpha} = [a - \varepsilon(1 - \alpha), b + \delta(1 - \alpha)]$, $\alpha \in [0, 1]$.

If the left and right spreads of ε - δ fuzzy number are equals then it is called symmetric trapezoidal fuzzy numbers: denoted by $[a, b]_{\varepsilon, \varepsilon}$ or $[a, b]_{\varepsilon}$. Note that for any fuzzy number $[x, y]_{\varepsilon, \delta}$ $-[x, y]_{\varepsilon, \delta} \neq 0$ and $[x, y]_{\varepsilon, \delta} / [x, y]_{\varepsilon, \delta} \neq 1$.

2.3 Arithmetic operation of TrFN [2]

We defined the arithmetic operation of epsilon-delta TrFN [2],

1. Addition $[x, y]_{\varepsilon_1, \delta_1} + [a, b]_{\varepsilon_2, \delta_2} = [x + a, y + b]_{\varepsilon_1 + \varepsilon_2, \delta_1 + \delta_2}$

2. Negation $-[x, y]_{\varepsilon, \delta} = [-y, -x]_{\varepsilon, \delta}$

3. Subtraction $[x, y]_{\varepsilon_1, \delta_1} - [a, b]_{\varepsilon_2, \delta_2} = [x - b, y - a]_{\varepsilon_1 + \delta_2, \varepsilon_2 + \delta_1}$

4. Scalar multiplication $k.[x, y]_{\varepsilon, \delta} = \begin{cases} [kx, ky]_{k\varepsilon, k\delta}, & k > 0 \\ [ky, kx]_{k\delta, k\varepsilon}, & k < 0 \end{cases}$

5. Trapezoidal multiplication

If $[x, y]_{\varepsilon_1, \delta_1} > 0$ and $[a, b]_{\varepsilon_2, \delta_2} > 0$ then $[x, y]_{\varepsilon_1, \delta_1} \bullet [a, b]_{\varepsilon_2, \delta_2} = [ax, by]_{a\varepsilon_1 + x\varepsilon_2 - \frac{1}{2}\varepsilon_1\varepsilon_2, a\delta_1 + x\delta_2 + \frac{1}{2}\delta_1\delta_2}$

6. Equality of trapezoidal fuzzy numbers, let $[x, y]_{\varepsilon_1, \delta_1}$ and $[a, b]_{\varepsilon_2, \delta_2}$ are equal TrFN if

$$[x, y]_{\varepsilon_1, \delta_1} = [a, b]_{\varepsilon_2, \delta_2} \Rightarrow x = a, y = b, \varepsilon_1 = \varepsilon_2, \delta_1 = \delta_2$$

2.4 Fuzzy Matrix

The matrix A is called a fuzzy matrix, if each element of A is a fuzzy number. A fuzzy matrix A is called non-negative, if each element of A is non-negative fuzzy number.

i) Definition If $A = [a_{ij}], 1 \leq i, j \leq n$ is a crisp matrix, then we define a matrix $A = \left[[a_{ij}] \right], 1 \leq i, j \leq n$, That is A^+ a matrix whose all elements are non-negative.

$$\text{Let } A_1 = \frac{1}{2}(A + A^+) \text{ and } A_2 = \frac{1}{2}(A^+ - A) \Rightarrow A_1 - A_2 = \frac{1}{2}(A + A^+) - \frac{1}{2}(A^+ - A) = A$$

Note that every matrix A can be written as $A_1 - A_2 = A$.

ii) Conditional distributive property

Matrices $A = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ and $B = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ said to be conditional distributive property hold if

$$\left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} - \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \right\} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix} - \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

III. Trapezoidal Fuzzy linear system of equation

Here, we have solved the FLSE with TrFNs of order n. Moreover, they transformed the n x n FLSE to 2n x 2n system of linear equation. This can be solved by inverse matrix or any algebraic methods.

Definition:- A fuzzy linear system of equations of order n is given by $A \bullet [X, Y]_{P,Q} = [C, D]_{R,S}$ where,

$$A = [a_{ij}], 1 \leq i, j \leq n, \text{ and } [X, Y] = [x_1, y_1][x_2, y_2] \dots [x_n, y_n]^T, P = [\varepsilon_1, \varepsilon_2 \dots \varepsilon_n]^T, Q = [\delta_1, \delta_2 \dots \delta_n]^T, \text{ and}$$

$$[C, D] = [c_1, d_1][c_2, d_2] \dots [c_n, d_n]^T, R = [\alpha_1, \alpha_2 \dots \alpha_n]^T, S = [\beta_1, \beta_2 \dots \beta_n]^T$$

that is,

$$\begin{aligned} a_{11}[x_1, y_1]_{\varepsilon_1, \delta_1} + \dots + a_{1n}[x_n, y_n]_{\varepsilon_n, \delta_n} &= [c_1, d_1]_{\alpha_1, \beta_1} \\ \vdots & \\ a_{n1}[x_1, y_1]_{\varepsilon_1, \delta_1} + \dots + a_{nn}[x_n, y_n]_{\varepsilon_n, \delta_n} &= [c_n, d_n]_{\alpha_n, \beta_n} \end{aligned}$$

3.1 Direct Method for FLSE

In this method we have solved the FLSE with TrFN of order 2. In which all coefficients are either positive or negative. This theorem gives the conditions for the existence of the solutions of a FLSE.

Theorem:- The system of fuzzy linear equations $A \bullet [X, Y]_{P,Q} = [C, D]_{R,S}$ of order 2 has solution exists, if either all scalars are positive or negative and $|A| \neq 0$ that is

$$1. \text{ if } |A| > 0 \text{ we have } \frac{a_{12}}{a_{22}} \leq \frac{\alpha_1}{\alpha_2}, \frac{\beta_1}{\beta_2} \leq \frac{a_{11}}{a_{21}}.$$

$$2. \text{ if } |A| < 0 \text{ we have } \frac{a_{11}}{a_{21}} \leq \frac{\alpha_1}{\alpha_2}, \frac{\beta_1}{\beta_2} \leq \frac{a_{12}}{a_{22}}.$$

$$\text{Proof :- Let } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} [x_1, y_1] \\ [x_2, y_2] \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}, \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} [c_1, d_1] \\ [c_2, d_2] \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

FLSE can be written as $A \bullet [X, Y]_{P,Q} = [C, D]_{R,S}$, we have solved in two cases on the basis of the signs of the coefficients as follows,

Case I: - All scalars are positive

$$A \bullet [X, Y]_{P,Q} = [C, D]_{R,S} \quad (\text{by positive scalar multiplication})$$

$$\Rightarrow [A \bullet X, A \bullet Y]_{AP, AQ} = [C, D]_{R,S} \quad (\text{by equality of fuzzy numbers})$$

$$\Rightarrow A \bullet X = C, A \bullet Y = D \text{ and } A \bullet P = R, A \bullet Q = S$$

$$A \bullet X = C \Rightarrow X = A^{-1} \bullet C$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \bullet \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22}c_1 - a_{12}c_2 \\ -a_{21}c_1 + a_{11}c_2 \end{bmatrix} \Rightarrow x_1 = \frac{(a_{22}c_1 - a_{12}c_2)}{(a_{22}a_{11} - a_{12}a_{21})}, x_2 = \frac{(-a_{21}c_1 + a_{11}c_2)}{(a_{22}a_{11} - a_{12}a_{21})}.$$

$$\text{And } A \bullet Y = D \Rightarrow Y = A^{-1} \bullet D$$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \bullet \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22}d_1 - a_{12}d_2 \\ -a_{21}d_1 + a_{11}d_2 \end{bmatrix} \Rightarrow y_1 = \frac{(a_{22}d_1 - a_{12}d_2)}{(a_{22}a_{11} - a_{12}a_{21})}, y_2 = \frac{(-a_{21}d_1 + a_{11}d_2)}{(a_{22}a_{11} - a_{12}a_{21})}.$$

$$\text{Next } A \bullet P = R \Rightarrow P = A^{-1} \bullet R$$

$$\Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \bullet \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22}\alpha_1 - a_{12}\alpha_2 \\ -a_{21}\alpha_1 + a_{11}\alpha_2 \end{bmatrix} \Rightarrow \varepsilon_1 = \frac{(a_{22}\alpha_1 - a_{12}\alpha_2)}{(a_{22}a_{11} - a_{12}a_{21})}, \varepsilon_2 = \frac{(-a_{21}\alpha_1 + a_{11}\alpha_2)}{(a_{22}a_{11} - a_{12}a_{21})}.$$

$$\text{Finally } A \bullet Q = S \Rightarrow Q = A^{-1} \bullet S$$

$$\Rightarrow \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \bullet \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22}\beta_1 - a_{12}\beta_2 \\ -a_{21}\beta_1 + a_{11}\beta_2 \end{bmatrix} \Rightarrow \delta_1 = \frac{(a_{22}\beta_1 - a_{12}\beta_2)}{(a_{22}a_{11} - a_{12}a_{21})}, \delta_2 = \frac{(-a_{21}\beta_1 + a_{11}\beta_2)}{(a_{22}a_{11} - a_{12}a_{21})}.$$

Since $\varepsilon_i, \delta_i, i = 1, 2$ are non-negative we must have.

$$\text{If } |A| > 0, \text{ then } (a_{22}\alpha_1 - a_{12}\alpha_2) \geq 0 \text{ and } (-a_{21}\alpha_1 + a_{11}\alpha_2) \geq 0 \Rightarrow \frac{a_{12}}{a_{22}} \leq \frac{\alpha_1}{\alpha_2} \leq \frac{a_{11}}{a_{21}},$$

$$\text{similarly } (a_{22}\beta_1 - a_{12}\beta_2) \geq 0 \text{ and } (-a_{21}\beta_1 + a_{11}\beta_2) \geq 0 \Rightarrow \frac{a_{12}}{a_{22}} \leq \frac{\beta_1}{\beta_2} \leq \frac{a_{11}}{a_{21}}$$

$$\text{If } |A| < 0, \text{ then } (a_{22}\alpha_1 - a_{12}\alpha_2) \leq 0 \text{ and } (-a_{21}\alpha_1 + a_{11}\alpha_2) \leq 0 \Rightarrow \frac{a_{11}}{a_{21}} \leq \frac{\alpha_1}{\alpha_2} \leq \frac{a_{12}}{a_{22}},$$

similarly $(a_{22}\beta_1 - a_{12}\beta_2) \leq 0$ and $(a_{11}\beta_2 - a_{21}\beta_1) \leq 0 \Rightarrow \frac{a_{11}}{a_{21}} \leq \frac{\beta_1}{\beta_2} \leq \frac{a_{12}}{a_{22}}$

Case II:- All scalars are negative then, from negative scalar multiplication then we get,

$$A \cdot X = D \Rightarrow X = A^{-1} \cdot D \Rightarrow x_1 = \frac{(-a_{21}d_1 + a_{11}d_2)}{(a_{22}a_{11} - a_{12}a_{21})}, x_2 = \frac{(-a_{21}d_1 + a_{11}d_2)}{(a_{22}a_{11} - a_{12}a_{21})},$$

$$A \cdot Y = C \Rightarrow Y = A^{-1} \cdot C \Rightarrow y_1 = \frac{(a_{22}c_1 - a_{12}c_2)}{(a_{22}a_{11} - a_{12}a_{21})}, y_2 = \frac{(-a_{21}c_1 + a_{11}c_2)}{(a_{22}a_{11} - a_{12}a_{21})}.$$

$$\text{Next } A^+ \cdot Q = S \Rightarrow Q = (A^+)^{-1} \cdot R = -A^{-1} \cdot R \Rightarrow \delta_1 = \frac{(-a_{22}\alpha_1 + a_{12}\alpha_2)}{(a_{22}a_{11} - a_{12}a_{21})}, \delta_2 = \frac{(a_{21}\alpha_1 - a_{11}\alpha_2)}{(a_{22}a_{11} - a_{12}a_{21})}$$

$$\text{Finally } A^+ \cdot P = S \Rightarrow P = (A^+)^{-1} \cdot S = -A^{-1} \cdot S \Rightarrow \varepsilon_1 = \frac{(-a_{22}\beta_1 + a_{12}\beta_2)}{(a_{22}a_{11} - a_{12}a_{21})}, \varepsilon_2 = \frac{(a_{21}\beta_1 - a_{11}\beta_2)}{(a_{22}a_{11} - a_{12}a_{21})}$$

Since $\varepsilon_i, \delta_i, i = 1, 2$ are non-negative we must have

$$1. \text{ If } |A| < 0 \text{ then, } \frac{a_{11}}{a_{21}} \leq \frac{\alpha_1}{\alpha_2} \leq \frac{a_{12}}{a_{22}} \text{ and } \frac{a_{11}}{a_{21}} \leq \frac{\beta_1}{\beta_2} \leq \frac{a_{12}}{a_{22}}.$$

$$2. \text{ If } |A| > 0, \text{ then } \frac{a_{12}}{a_{22}} \leq \frac{\alpha_1}{\alpha_2} \leq \frac{a_{11}}{a_{21}} \text{ and } \frac{a_{12}}{a_{22}} \leq \frac{\beta_1}{\beta_2} \leq \frac{a_{11}}{a_{21}}.$$

Note: Above propositions evidently is so restrictive which is applicable only when all scalars are either positive or negative in FLSE of order 2. So, we introduced a general method for solving FLSE of order n with no such restriction.

3.2 General Method for FLSE

A fuzzy linear system of equations $A \cdot [X, Y]_{P,Q} = [C, D]_{R,S}$ where $A = [a_{ij}]$, is scalar matrix,

$[X, Y]_{P,Q} = \left[[x_i, y_i]_{\varepsilon_i, \delta_i} \right]$ and $[C, D]_{R,S} = \left[[c_i, d_i]_{\alpha_i, \beta_i} \right]$ are matrices whose elements are TrFN. Since

$A = [a_{ij}], 1 \leq i, j \leq n$ crisp matrix and $A^+ = [a_{ij}^+], 1 \leq i, j \leq n$ all positive crisp coefficients. Let $A_1 = \frac{1}{2}(A + A^+)$

and $A_2 = \frac{1}{2}(A^+ - A)$ clearly A_1, A_2 contains all positive scalars. Clearly we get

$$A_1 - A_2 \Rightarrow \frac{1}{2}(A + A^+) - \frac{1}{2}(A^+ - A) = A \Rightarrow A_1 - A_2 = A$$

Therefore FLSE $A \cdot [X, Y]_{P,Q} = [C, D]_{R,S}$ becomes

$$(A_1 - A_2) \cdot [X, Y]_{P,Q} = [C, D]_{R,S} \quad (\text{by definition of } A)$$

$$\Rightarrow A_1 \cdot [X, Y]_{P,Q} - A_2 \cdot [X, Y]_{P,Q} = [C, D]_{R,S} \quad (\text{by conditional distributive property})$$

$$\Rightarrow [A_1 \cdot X, A_1 \cdot Y]_{A_1 P, A_1 Q} - [A_2 \cdot X, A_2 \cdot Y]_{A_2 P, A_2 Q} = [C, D]_{R,S}$$

$$\Rightarrow [A_1 \cdot X, A_1 \cdot Y]_{A_1 \cdot P, A_1 \cdot Q} + [-A_2 \cdot Y, -A_2 \cdot X]_{A_2 \cdot Q, A_2 \cdot P} = [C, D]_{R,S}$$

$$\Rightarrow [A_1 \cdot X - A_2 \cdot Y, A_1 \cdot Y - A_2 \cdot X]_{A_1 \cdot P + A_2 \cdot Q, A_2 \cdot P + A_1 \cdot Q} = [C, D]_{R,S}$$

$$\Rightarrow A_1 \cdot X - A_2 \cdot Y = C, A_1 \cdot Y - A_2 \cdot X = D, A_1 \cdot P + A_2 \cdot Q = R \text{ and } A_2 \cdot P + A_1 \cdot Q = S$$

Now $A_1 \cdot X - A_2 \cdot Y = C, A_1 \cdot Y - A_2 \cdot X = D$ gives a system of 2n equations with 2n variables and also $A_1 \cdot P + A_2 \cdot Q = R, A_2 \cdot P + A_1 \cdot Q = S$ gives 2n equation in 2n variables. These systems can be solved by any known algebraic method. If we get a non-negative solution for system $A_1 \cdot P + A_2 \cdot Q = R, A_2 \cdot P + A_1 \cdot Q = S$, then given FLSE has a solution. Otherwise, the solution for FLSE does not exist.

IV. Numerical Examples

Example 5.1 consider the FLSE,

$$5[x_1, y_1]_{\varepsilon_1, \delta_1} + 3[x_2, y_2]_{\varepsilon_2, \delta_2} = [3, 5]_{2,3}$$

$$2[x_1, y_1]_{\varepsilon_1, \delta_1} + 4[x_2, y_2]_{\varepsilon_2, \delta_2} = [4, 7]_{1,2}$$

Given FLSE contains all positive coefficients, so we have used case I of the direct method

thus $|A| = 14 > 0$ and $\frac{a_{11}}{a_{21}} = \frac{5}{2} \geq \frac{\alpha_1}{\alpha_2} = 2, \frac{\beta_1}{\beta_2} = \frac{3}{2} \geq \frac{a_{12}}{a_{22}} = \frac{3}{4}$.

Clearly all conditions holds. Thus solution of the FLSE is exists which is as follows,

$$x_1 = \frac{(a_{22}c_1 - a_{12}c_2)}{(a_{22}a_{11} - a_{12}a_{21})} = 0, x_2 = \frac{(-a_{22}c_1 + a_{11}c_2)}{(a_{22}a_{11} - a_{12}a_{21})} = 1, y_1 = \frac{(a_{22}d_1 - a_{12}d_2)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{1}{14}, y_2 = \frac{(-a_{21}d_1 + a_{11}d_2)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{25}{14},$$

$$\varepsilon_1 = \frac{(a_{22}\alpha_1 - a_{12}\alpha_2)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{5}{14}, \varepsilon_2 = \frac{(a_{11}\alpha_2 - a_{21}\alpha_1)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{1}{14} \text{ and } \delta_1 = \frac{(a_{22}\beta_1 - a_{12}\beta_2)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{3}{7}, \delta_2 = \frac{(a_{11}\beta_2 - a_{21}\beta_1)}{(a_{22}a_{11} - a_{12}a_{21})} = \frac{2}{7}.$$

The required solution is $[x_1, y_1]_{\varepsilon_1, \delta_1} = [0, \frac{1}{14}]_{\frac{5}{14}, \frac{3}{7}}$ and $[x_2, y_2]_{\varepsilon_2, \delta_2} = [1, \frac{25}{14}]_{\frac{1}{14}, \frac{2}{7}}$.

Example 5.2 consider the FLSE as

$$2[x_1, y_1]_{\varepsilon_1, \delta_1} - 9[x_2, y_2]_{\varepsilon_2, \delta_2} = [5, 11]_{4, 10}$$

$$-8[x_1, y_1]_{\varepsilon_1, \delta_1} + 7[x_2, y_2]_{\varepsilon_2, \delta_2} = [7, 17]_{10, 11}$$

Given FLSE contains mixed sign coefficients. Here, we have used general method as follows,

$$A = \begin{bmatrix} 2 & -9 \\ -8 & 7 \end{bmatrix}, A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} 0 & 9 \\ 8 & 0 \end{bmatrix} \text{ from above the method we have } A = A_1 - A_2 \text{ and}$$

$$A_1 \cdot X - A_2 \cdot Y = C \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x_1 - 9y_2 \\ 7x_2 - 8y_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix},$$

$$A_1 \cdot Y - A_2 \cdot X = D \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 2y_1 - 9x_2 \\ 7y_2 - 8x_1 \end{bmatrix} = \begin{bmatrix} 11 \\ 17 \end{bmatrix}$$

Then we get $[x_1, y_1] = [\frac{-268}{29}, \frac{-28}{29}]$, $[x_2, y_2] = [\frac{-37}{29}, \frac{-31}{29}]$

$$\text{Similarly } A_1 \cdot P + A_2 \cdot Q = R \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} 0 & 9 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 2\varepsilon_1 + 9\delta_2 \\ 7\varepsilon_2 + 8\delta_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

$$\text{and } A_2 \cdot P + A_1 \cdot Q = S \Rightarrow \begin{bmatrix} 0 & 9 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 2\delta_1 + 9\varepsilon_2 \\ 7\delta_2 + 8\varepsilon_1 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

From above system we get $[\varepsilon_1, \delta_1] = [\frac{142}{29}, \frac{10}{29}]$, $[\varepsilon_2, \delta_2] = [\frac{30}{29}, \frac{5}{29}]$.

Thus, the required solution is $[x_1, y_1]_{\varepsilon_1, \delta_1} = [\frac{-268}{29}, \frac{-28}{29}]_{\frac{142}{29}, \frac{10}{29}}$, $[x_2, y_2]_{\varepsilon_2, \delta_2} = [\frac{-37}{29}, \frac{-31}{29}]_{\frac{30}{29}, \frac{5}{29}}$

V. Conclusion

In this article, fuzzy linear systems of equations (FLSE) are investigated, direct method and general method are applied for solving them. Here, we found the solution of FLSE by the direct method along with the condition for the existence of the solutions. The general method for finding the solution of FLSE offers no such restrictions on the selection of coefficients and variables. Finally, we illustrated some numerical examples.

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