



Shishir Cosmological model Bianchi type VI with use Barotropic in modified theory of general relativity

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Abstract: We Consider Shishir cosmological model Bianchi type VI with use Barotropic in modified theory of general relativity, To solve the Einstein field equation by Shishir assuming $\alpha = \beta^{m_1}$ and $p = \eta\rho$. The Physical Significance of the Cosmological model is discussed.

Keywords: Bianchi type VI, Barotropic space time, Expansion scalar, shear, Hubble parameter, cosmology.

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I. Introduction

Cosmology is the scientific study of large scale properties of the universe as a whole. cosmology is study motion of crystalline objects i.e.study of the early stages of universe.

The model of Universe formed by massive string is initiated by letelier, Which was used as Bianchi type I and 'Kantowski- Sachs' type of Cosmological models [1,2]

Chakraborty et.al [3] investigated string cosmological model in general relativity In LRS Bianchi-I Cosmological model with polytropic equation of state R.kdubey and shishir [4] Bianchi type I and Bianchi type III in westigated type by Bali et.al[5,6] Verma and Ram [7] investigated the solution of Bianchi type VI Bulk Viscous Fluid models with variable Gravitational and Cosmological Constants.

Recently, Bali V Poonia [8] investigated Bianchi type VI. Inflationary Cosmological model in General Relativity, Bali et.al [9] and Bhogar et.al[10] has investigated Bianchi type VI. in general relativity.; In the following paper, We considered 'Shishir Cosmological Bianchi type VI with use Barotropic in modified theory of general relativity.

II. The Metric Equations

We consider Bianchi type VI metric in the form

$$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{-2x} dy^2 + \gamma^2 e^{2x} dz^2 \quad (2.1)$$

Where α and β are function of t only.

The energy momentum tensor for a bulk viscous fluid distribution is given by

$$T_m^n = (\rho + p)v_m v^n + p g_m^n \quad (2.2)$$

Where ρ is the energy density of the cosmic matter and p is its pressure.

v_m is The four velocity vector such that

$$v_m v^n = -1 = -u_m u^n \quad (2.3)$$

$$u^m v_m = 0 \quad (2.4)$$

The vector $u_m u^n$ describes the direction of string or direction or anisotropy.

We take the equation of state

$$p = \eta\rho, 0 \leq \eta \leq 1$$

The Einstein field equation

$$R_{mn} - \frac{1}{2} R g_{mn} = -8\pi G T_{mn} \quad (2.5)$$

Where R_{mn} is Known as Ricci tensor and T_{mn} is the energy momentum tensor for matter

For the line element(2.1) and field Equation (2.5) can be written as

$$\frac{\ddot{\beta}}{\beta} + \frac{\dot{\gamma}}{\gamma} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} + \frac{1}{\alpha^2} = -8\pi G p \quad (2.6)$$

$$\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\gamma}}{\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} - \frac{1}{\alpha^2} = -8\pi G p \quad (2.7)$$

$$\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{1}{\alpha^2} = -8\pi G p \quad (2.8)$$

$$\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} - \frac{1}{\alpha^2} = 8\pi G \rho \quad (2.9)$$

$$\left(\frac{\dot{\beta}}{\beta} - \frac{\dot{\gamma}}{\gamma}\right) = 0 \quad (2.10)$$

Dot(.) denotes the ordinary differentiation with respect to t.

Integrating equation (2.10) we get

$$\log \beta - \log \gamma = \log l_1$$

$$\log \frac{\beta}{\gamma} = \log l_1$$

$$\beta = l_1 \gamma \quad (2.11)$$

Where l_1 is constant of integration.

We choose the value $l_1 = 1$ then

$$\beta = \gamma \quad (2.12)$$

The spatial volume for the model given by

$$\xi^3(t) = (\alpha\beta\gamma) \quad (2.13)$$

$$V = \xi^3(t) = (\alpha\beta\gamma) \quad (2.14)$$

$$\xi(t) = (\alpha\beta\gamma)^{\frac{1}{3}} \quad (2.15)$$

Where $\xi(t)$ as the average scale factor,

The expression for scalar expansion, shear scalar

$$\theta = v_{;m}^m = \left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\beta}}{\beta} + \frac{\dot{\gamma}}{\gamma} \right) \quad (2.16)$$

$$\sigma^2 = \sigma_{mn}\sigma^{mn} = \frac{1}{3} \left(\frac{\dot{\alpha}^2}{\alpha^2} + \frac{\dot{\beta}^2}{\beta^2} + \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} - \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} \right) \quad (2.17)$$

The Hubble parameter and deceleration parameter are respectively defined as

$$H = \frac{\dot{\xi}}{\xi} = \frac{1}{3} \left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\beta}}{\beta} + \frac{\dot{\gamma}}{\gamma} \right) \quad (2.18)$$

$$H = \frac{1}{3} (H_1 + H_2 + H_3)$$

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \quad (2.19)$$

III. Solution of field Equation

The system of five field equation (2.6) to (2.10) are in five unknown parameter $(\alpha, \beta, \gamma, p, \rho)$. we assume that expansion scalar is proportional to shear scalar $\theta \propto \sigma$. This condition leads to

$$\alpha = \beta^{m_1} \quad (2.20)$$

Where m_1 is any real constant.

From equation (2.8)

$$(m_1 + 1) \frac{\ddot{\beta}}{\beta} + [m_1^2 - m_1 + m_1] \frac{\dot{\beta}^2}{\beta^2} - \frac{1}{\beta^{2m_1}} = -8\pi G p$$

$$(m_1 + 1) \frac{\ddot{\beta}}{\beta} + m_1^2 \frac{\dot{\beta}^2}{\beta^2} - \frac{1}{\beta^{2m_1}} = -8\pi G p \quad (2.21)$$

From equation (2.9)

$$m_1 \frac{\ddot{\beta}^2}{\beta^2} + m_1 \frac{\dot{\beta}^2}{\beta^2} + \frac{\dot{\beta}^2}{\beta^2} - \frac{1}{\beta^{2m_1}} = 8\pi G \rho$$

$$(2m_1 + 1) \frac{\dot{\beta}^2}{\beta^2} - \frac{1}{\beta^{2m_1}} = 8\pi G \rho \quad (2.22)$$

To get determinate solution we need extra condition we assume

$$p = \eta \rho \quad (2.23)$$

i.e. fluid is barotropic fluid.

From equation (2.21), (2.22), (2.23) we get

$$\ddot{\beta} + \frac{[m_1^2 + (2m_1 + 1)\eta]}{(m_1 + 1)} \frac{\dot{\beta}^2}{\beta} = \left[\frac{1 + \eta}{m_1 + 1} \right] \beta^{1-2m_1} \quad (2.24)$$

we select the value $m_1 = 2$

$$\ddot{\beta} + c_1 \frac{\dot{\beta}^2}{\beta} = c_2 \beta^{-3} \quad (2.25)$$

Where $c_1 = \frac{4+5\eta}{3}$ and $c_2 = \frac{1+\eta}{3}$

$$\frac{d}{dt} (\alpha_1^2 \beta^{2c_1}) = 2c_2 \beta^{2c_1-3} \dot{\beta} \quad (2.26)$$

Where $\alpha_1 = \dot{\beta}$

Integrating equation (2.26) we get

$$\alpha_1 = \frac{\sqrt{c_3}}{\beta} \quad (2.27)$$

$$\text{Where } c_3 = \frac{c_2}{c_1 - 1} = \frac{1 + \eta}{1 + 5\eta}$$

$$\beta^2 = 2\sqrt{c_3} t + 2d_1 \quad (2.28)$$

$$\beta^2 = h_1 t + h_2$$

$$\beta = (h_1 t + h_2)^{\frac{1}{2}}$$

$$\beta = \sqrt{h_1 t + h_2} \quad (2.29)$$

Where h_2 is integrating constant

From equation (2.20)

$$\alpha = [(h_1 t + h_2)^{\frac{1}{2}}]^{m_1}$$

$$\alpha = (h_1 t + h_2)^{\frac{m_1}{2}} \quad (2.30)$$

using (2.29) and (2.30) the line element (2.1) becomes

$$ds^2 = -dt^2 + (h_1 t + h_2)^{m_1} dx^2 + (h_1 t + h_2) e^{-2x} dy^2 + (h_1 t + h_2) e^{2x} dz^2 \quad (2.31)$$

IV. Physical and Geometrical Behaviour of the model

From this model, we can find other geometrical and physical parameter. The expression for Hubble parameter, Expansion scalar θ , spatial volume V , shear scalar σ^2 and deceleration parameter q respectively given by

$$H = \frac{1}{3} \left[\frac{r_1}{h_1 t + h_2} \right] \quad (2.32)$$

$$\theta = 3H = \frac{r_1}{(h_1 t + h_2)} \quad (2.33)$$

$$\text{Where } r_1 = \left(\frac{m_1}{2} + 1 \right) h_1 = \text{constant}$$

$$\sigma = \frac{1}{\sqrt{3}} \left[\frac{r_2}{(h_1 t + h_2)} \right] \quad (2.34)$$

$$\text{Where } r_2 = \frac{(m_1 - 1)h_1}{2} = \text{Constant}$$

$$V = (h_1 t + h_2)^{\frac{m_1 + 2}{2}} \quad (2.35)$$

$$q = \frac{-(3m_1 + 4)}{3(m_1 + 2)} \quad (2.36)$$

$$\frac{\square}{\square} = \frac{\square_2}{\sqrt{3} \square_1} = \quad (2.37)$$

V. Observation

1- $\frac{\sigma}{\theta} = \text{Constant}$, therefore model dose not approach isotropy for large value of t .

2- When $t \rightarrow \infty$ then scalar expansion θ , Shear scalar σ , Hubble parameter all approaches to Zero.

3- Negative Value of deceleration parameter indicate that model is decelerating. The special volume with time is increasing an it become for sufficiently large value of t .

VI. Conclusion

In this paper we have studied Shishir cosmological model Bianchi type VI with use barotropic in modified theory of general relativity. We investigated that Hubble parameter H , Scalar expansion θ , and shear scalar σ are infinite at $t \rightarrow 0$ and approaches Zero at $t \rightarrow \infty$. Here we see that the spatial Volume of the universe increase with time.

Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \text{constant}$, Which gives us the anisotropy is maintained for all time and it can be see that the model is irrotational. Therefore, the model describes a continuously expanding shearing and non rotating universe with big bang in theory modified theory of general relativity.

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