



On b-Colouring of Line Graph, Middle Graph, Total Graph and Central Graph of Dutch Wind Mill Graph

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Abstract: The b -chromatic number $\phi(G)$, parameter of a graph G is the maximum number of colours for which G has a proper colouring such that every colour class contain a vertex adjacent to every other colour. In this paper, we examine the b -chromatic number of Line graph $L(D_n^m)$, Middle graph $M(D_n^m)$, Total graph $T(D_n^m)$ and Central graph $C(D_3^m)$ of Dutch Wind mill graph.

Keywords: Proper colouring, b -colouring, b -chromatic number, Dutch Wind mill graph, Line graph, Middle graph, Total graph, Central graph.

I. Introduction

A b -coloring by k -colors is a proper coloring of the vertices of a graph G such that in each color class there exists a vertex which has neighbors in all the other $k - 1$ color classes. The b -chromatic number $\phi(G)$ is the largest integer k such that G admits a b -coloring with k -colors. The concept of b -coloring was introduced by Irving and Manlove [3] in 1999 and showed that the problem of determining b -chromatic number is NP-hard for general graphs but it is polynomial for trees. The upper bounds for the b -chromatic number were investigated in the work of Kouider and Maheo [5]. Vivin and Venkatachalam [9] investigated the b -chromatic number of corona graphs. The b -chromatic number of helm and closed helm graph were examined by Vaidya and Shukla [11]. Ansari, Chandel and Jamal [14] find out the b -chromatic number of Prism graph families. In this paper we examine the b -chromatic number of line graph $L(D_n^m)$, Middle graph $M(D_n^m)$, Total Graph $T(D_n^m)$ and Central Graph $C(D_3^m)$ of Dutch Wind mill graph.

II. Preliminaries

2.1 Line Graph

The line graph $L(G)$ [1, 14] of a graph G is the graph whose vertex set is $E(G)$ and two vertices are adjacent in $L(G)$ whenever they are incident in G .

2.2 Middle Graph

The middle graph $M(G)$ [14] of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

2.3 Total Graph

Let G be a graph with two or more vertices then the total graph $T(G)$ [14] of graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G .

2.4 Central graph

The central graph [13] $C(G)$ of a graph G is obtained subdividing each edge of G exactly once and joining all the non adjacent vertices of G .

2.5 Dutch windmill graph

The Dutch windmill graph D_n^m , is the graph obtained by taking m copies of the cycle graph C_n with a vertex in common.

III. Main Results

3.1 Theorem

For the line graph of Dutch windmill graph (D_n^m) , $\phi[L(D_n^m)] = 2m, n > 2$.

Proof:

The Dutch windmill graph D_n^m , is the graph obtained by taking m copies of the cycle graph $C_n, n > 2$, where n denotes the number of vertices of the cycle graph, with a vertex in common. Consider the line graph of Dutch windmill graph $L(D_n^m)$, and the vertex set be $V(L(D_n^m)) = \{u_1, u_2, \dots, u_{nm}\}$. It contains a complete graph with $2m$ vertices and the vertex set be $\{u_1, u_2, \dots, u_{2m}\}$. Assign the colour to $L(D_n^m)$ as follows, first we colour the complete graph K_{2m} using $2m$ colours denoted as $c_1, c_2, \dots, c_{2m}$. Next assign the new colour to the remaining vertices introduced in the edges of m copies of the cycle graph. Suppose if we assign a new colour c_{2m+1} to these vertices, by b-colouring property, the new colour should adjacent with $c_1, c_2, \dots, c_{2m}$ colours. This condition fails, because these vertices having maximum degree 2. It cannot harmonize the new colour c_{2m+1} . Therefore the b-chromatic number of line graph of Dutch windmill graph $\phi[L(D_n^m)]$ is $2m$.

3.2 Theorem

For the middle graph of Dutch windmill graph (D_n^m) , $\phi[M(D_n^m)] = 2m + 1$.

Proof:

The Dutch windmill graph D_n^m , is the graph obtained by taking m copies of the cycle graph $C_n, n > 2$, where n denotes the number of vertices of the cycle graph, with a vertex in common. Consider the middle graph of Dutch windmill graph $M(D_n^m)$, and the cardinality of vertex set is $|V(M(D_n^m))| = mn + (n-1)m + 1$. It contains a complete graph with $2m+1$ vertices and the vertex set is $\{u_1, u_2, \dots, u_{2m+1}\}$. Assign the colour to $M(D_n^m)$ as follows, first we colour the complete graph K_{2m+1} using $2m+1$ colours denoted as $c_1, c_2, \dots, c_{2m+1}$. Next assign the new colour to the remaining vertices introduced in the edges of m copies of the cycle graph and the vertices of D_n^m . Suppose if we assign a new colour c_{2m+2} to these vertices, by b-colouring property, the new colour should adjacent with $c_1, c_2, \dots, c_{2m+1}$ colours. This condition fails, because uncoloured vertices of D_n^m having maximum degree 2 and uncoloured vertices introduced in the edges of m copies of the cycle graph having maximum degree 4. It cannot harmonize the new colour c_{2m+2} . Therefore the b-chromatic number of middle graph of Dutch windmill graph $\phi[M(D_n^m)]$ is $2m + 1$.

3.3 Theorem

For the total graph of Dutch windmill graph D_n^m , $\phi[T(D_n^m)] = 2m + 1$.

Proof:

The Dutch windmill graph D_n^m , is the graph obtained by taking m copies of the cycle graph $C_n, n > 2$, where n denotes the number of vertices of the cycle graph, with a vertex in common. Consider the middle graph of Dutch windmill graph $M(D_n^m)$, and the cardinality of vertex set is $|V(M(D_n^m))| = mn + (n-1)m + 1$. It contains a complete graph with $2m+1$ vertices and the vertex set is $\{u_1, u_2, \dots, u_{2m+1}\}$. Assign the colour to $M(D_n^m)$ as follows, first we colour the complete graph K_{2m+1} using $2m+1$ colours denoted as $c_1, c_2, \dots, c_{2m+1}$. Next assign the new colour to the remaining vertices introduced in the edges of m copies of the cycle graph and the vertices of D_n^m . Suppose if we assign a new colour c_{2m+2} to these vertices, by b-colouring property, the new colour should adjacent with $c_1, c_2, \dots, c_{2m+1}$ colours. This condition fails, because uncoloured vertices of D_n^m and uncoloured vertices introduced in the edges of m copies of the cycle graph having maximum degree 4. It cannot harmonize the new colour c_{2m+2} . Therefore the b-chromatic number of total graph of Dutch windmill graph $\phi[T(D_n^m)]$ is $2m + 1$.

3.4 Theorem

For the central graph of Dutch windmill graph D_3^m , $\phi[C(D_3^m)] = 2m + 1$.

Proof:

The Dutch windmill graph D_3^m , is the graph obtained by taking m copies of the cycle graph C_3 with vertices u_1^i, u_2^i, u_3^i , for $i = 1, 2, 3, \dots, m$, labelled in clockwise direction in which $u_1^1 = u_1^2 = u_1^3 = \dots = u_1^m = u$ (say) is the common vertex. In $C(D_3^m)$, the newly introduced vertex in the edge connecting u_x^i and u_y^i is represented by u_{xy}^i for $i = 1, 2, 3, \dots, m$, and for $xy = 1, 2, 3$. Each u_r^i for $i = 1, 2, 3, \dots, m$ and $r = 1, 2, 3$ will act as a root vertex of a star of degree $2m$. Assign the colours $c_1, c_2, \dots, c_{2m}$ to the vertices u_2^i, u_3^i in a clockwise direction for $i = 1, 2, 3, \dots, m$, and assign the colour c_{2m+1} to the vertex u . In order to obtain b-colouring for $C(D_3^m)$, colour the vertices u_{23}^i as c_{2m+1} , u_{12}^i as c_{2i} and u_{13}^i as c_{2i-1} , for $i = 1, 2, 3, \dots, m$. Clearly it will form the b-colouring of the $C(D_3^m)$. Hence the b-chromatic number of the central graph of Dutch windmill graph is $2m + 1$.

IV. Conclusion

In this paper, we examine the b-chromatic number of

- Line graph $\phi[L(D_n^m)] = 2m, n > 2$.
- Middle graph $\phi[M(D_n^m)] = 2m + 1$.
- Total graph $\phi[T(D_n^m)] = 2m + 1$.
- Central graph $\phi[C(D_3^m)] = 2m + 1$.

V. References

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