



## Revised extent analysis method through resolving the occurrence of irrational zero weights in fuzzy AHP

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**Abstract:** Analytical hierarchy process (AHP) is one of the most scientific multiple attribute decision making methods and Chang's extent analysis method (EAM) is a highly acclaimed approach for solving AHP in fuzzy settings. The major defect of the EAM is that in some cases the relative crisp weights of criteria or sub-criteria estimated by this method may be irrational zero weights which make the corresponding criteria or sub-criteria redundant in the decision analysis. The purpose of this paper is to resolve the problem suitably so that the revised EAM could be widely acceptable. The advanced approach will make the revised EAM more acceptable to the real-life decision problems in different areas like psychology, management, engineering, education and health. The validity and effectiveness of the presented approach has been demonstrated through a numerical example.

**Keywords:** Fuzzy sets; Triangular fuzzy number; AHP; Extent analysis method; Multiple attribute decision making

### I. Introduction

Human preferences arising out of uncertain human behaviour are intrinsically intransitive which lead us to unstructured problems in social sciences, political sciences, management, economics and psychology. Analytical hierarchy process (AHP) [7] is one of the most scientific and valued multiple attribute decision making methods used as a methodological tool for modelling such complex real-world problems by structuring a hierarchy. This method involves evaluation, ranking and selection of criteria, sub-criteria and alternatives of one level of the hierarchy with respect to an element of next higher level. All the elements of a level are compared pairwise relative to an element of immediate higher level on a 9-point ratio scale by taking integral values 1, 2, 3,...,9 and its reciprocals. A pairwise comparison matrix  $A = (a_{ij})_{n \times n}$ ,  $n$  being the number of elements in the respective level, is constructed. Priority vectors of elements of each level downward starting with 2<sup>nd</sup> level of the hierarchy are evaluated and synthesized appropriately to get a vector of priorities of alternatives with respect to the overall objective.

In classical AHP, the elements of a pairwise comparison matrix are taken to be crisp values as fixed ratios. However, in practical situation the criteria weights or performance ratings of the decision alternatives cannot be assessed absolutely on any numerical scale. As a consequence, they have to be evaluated subjectively by the decision makers using linguistic variables giving rise to unstructured problems. Thus, the real-world decision making problems involve incomplete, vague, imprecise nature of data that can be effectively dealt with fuzzy set theory [11] by replacing crisp ratio by fuzzy ratio in the decision matrix. Many decisions in real life situations are made in an imprecise environment where goals and constraints are not properly defined. Thus, Bellman and Zadeh [2] outlined that the decision making problems cannot be represented in crisp values. Various works have been done towards extension of AHP in fuzzy settings. Laarhoven and Pedrycz [10] extended the classical AHP method by using triangular fuzzy numbers (TFNs) as fuzzy ratio and logarithmic least squares method to determine fuzzy priority vector. Buckley [3] used trapezoidal fuzzy numbers as fuzzy ratios and geometric mean method to compute fuzzy weights. Chen [5] studied the evaluation of performance of weapon systems using fuzzy arithmetic operations. Chang [4] advanced extent analysis method (EAM) to solve fuzzy AHP which resulted in crisp priority vector from a triangular fuzzy pairwise comparison matrix. Later researchers employed EAM at numerous occasions due to its computational simplicity [6, 8]. However, there is a shortcoming crept in the EAM [4] in cases where the relative weights of some criteria or sub-criteria in the estimated weight vector may become zero, making the corresponding criteria or sub-criteria redundant in the decision analysis, and will have no impact on the final ranking if those are discarded altogether at the beginning. Wang et al. [9] underlined the possibility of misapplication of EAM due to occurrence of irrational zero weights to some criteria making their presence redundant in the whole evaluation process of alternatives.

In this paper, an approach has been proposed revising the EAM by using the arithmetic averaging operator at the place of minimum aggregation operator and eliminating the chance of occurrences of irrational zero weights in the EAM enabling the presence of respective criteria or sub-criteria in the evaluation process commensurate with their revised relative weights.

The rest of the paper is structured as follows: In Section II, the basic concepts related to TFNs and geometric mean method for estimating priority vector from triangular fuzzy decision matrix are recapitulated. Section III discusses existing methodology regarding the EAM on fuzzy AHP. Section IV identifies the problem in Chang's EAM and presents the resolution of irrational zero weights. Section V demonstrates the validity and effectiveness of the proposed approach through a numerical example. Conclusions of this research work and future directions are given in Section VI.

## II. Preliminaries

Here, we present a brief review of some existing results to be used for future development.

**A. Definition: Fuzzy Sets ([11]).** A fuzzy set  $\tilde{A}$  in a universe of discourse  $X$  is characterized by a membership function  $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ . The functional value  $\mu_{\tilde{A}}(x)$  represents the membership grade of  $x$  in  $\tilde{A}$ .

**B. Definition: Triangular Fuzzy Number ([10]).** A triangular fuzzy number (TFN),  $\tilde{a}$ , is a fuzzy set where the universe of discourse is the set of real numbers  $\mathbb{R}$ . It is denoted by a triple  $(l, m, u)$  and is defined by

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \leq x \leq m \\ \frac{u-x}{u-m}, & m \leq x \leq u \\ 0, & \text{otherwise} \end{cases}$$

where  $l, m$  and  $u$  are real numbers with  $l \leq m \leq u$  and known as lower value, modal value and upper value, respectively.

If  $l = m = u$  then the TFN reduces to a crisp number.

### C. Algebraic operations on TFNs [10]

Assume that  $\tilde{a}_1 = (l_1, m_1, u_1)$ ,  $\tilde{a}_2 = (l_2, m_2, u_2)$  are two TFNs and  $k > 0$ . Then,

- 1) Addition:  $\tilde{a}_1 \oplus \tilde{a}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$ , where  $l_1 \geq 0, l_2 \geq 0$
- 2) Multiplication:  $\tilde{a}_1 \otimes \tilde{a}_2 = (l_1 l_2, m_1 m_2, u_1 u_2)$ , where  $l_1 \geq 0, l_2 \geq 0$
- 3) Multiplication by a scalar:  $k\tilde{a}_1 = (kl_1, km_1, ku_1)$ , where  $k > 0, l_1 \geq 0$
- 4) Inverse:  $\tilde{a}_1^{-1} = \left( \frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1} \right)$ , where  $l_1 > 0$

### D. Estimation of crisp priority vector from a triangular fuzzy comparison matrix using geometric mean method [3]

A triangular fuzzy comparison decision matrix is considered as

$$\tilde{A} = (\tilde{a}_{ij})_{n \times n} \quad (1)$$

$$\text{where } \tilde{a}_{ij} = \begin{cases} (l_{ij}, m_{ij}, u_{ij}), & i < j \\ (1, 1, 1), & i = j \\ \tilde{a}_{ji}^{-1}, & i > j \end{cases} \text{ for } i, j = 1, 2, \dots, n$$

A  $TFN(l, m, u)$  can be considered as a trapezoidal fuzzy number ( $TrFN$ ) expressed by  $(l, m, m, u)$  [5] and hence  $(l, m, u) = (l, m, m, u)$ . Thus, the results of theories based on  $TrFN$  can be applied to those of  $TFNs$ .

The triangular fuzzy priority vector of fuzzy decision matrix (1) is estimated using the geometric mean method (GMM) [1, 3] as follows:

**Step 1.** The fuzzy geometric mean  $\tilde{a}_i$  of  $i$ th row of the comparison matrix (1) is defined by

$$\tilde{a}_i = (l_i, m_i, u_i) = \left( \sqrt[n]{\prod_{j=1}^n l_{ij}}, \sqrt[n]{\prod_{j=1}^n m_{ij}}, \sqrt[n]{\prod_{j=1}^n u_{ij}} \right), \text{ for } i = 1, 2, \dots, n$$

**Step 2.** The  $n$ -component triangular fuzzy priority vector  $(\tilde{a}_i)_{n \times 1}$  is normalized using algebraic operations as follows:

$$\sigma_i = (p_i, q_i, r_i) = \left( \frac{l_i}{\sum_{j=1}^n u_j}, \frac{m_i}{\sum_{j=1}^n m_j}, \frac{u_i}{\sum_{j=1}^n l_j} \right), \text{ for } i = 1, 2, \dots, n$$

**Step 3.** The triangular fuzzy priority weights  $\sigma_i$  are defuzzified to get crisp priority weights using the formula [1] given by

$$\omega_i = \frac{p_i + 4q_i + r_i}{6}, \text{ for } i = 1, 2, \dots, n$$

**Step 4.** The crisp priority vector  $(\omega_i)_{n \times 1}$  is normalized to get the required crisp priority vector  $(w_i)_{n \times 1}$  where

$$w_i = \frac{\omega_i}{\sum_{i=1}^n \omega_i}, \text{ for } i = 1, 2, \dots, n$$

### III. Extent Analysis Method (EAM)[4]

The following steps are performed while using EAM for estimating priority vectors:

**Step-1:** For each row  $i$ , row sum  $RS_i$  of the fuzzy comparison matrix  $\tilde{A}$  is calculated by fuzzy arithmetic operations:

$$RS_i = \sum_{j=1}^n \tilde{a}_{ij} = \left( \sum_{j=1}^n l_{ij}, \sum_{j=1}^n m_{ij}, \sum_{j=1}^n u_{ij} \right), \quad i = 1, 2, \dots, n \quad (2)$$

**Step-2:** Above row sums are normalized as

$$\tilde{S}_i = (RS_i) \otimes \left( \sum_{j=1}^n RS_j \right)^{-1}, \quad i = 1, 2, \dots, n \quad (3)$$

**Step-3:** The degree of possibility  $p_{ij}$  of  $\tilde{S}_i \geq \tilde{S}_j$  is computed as

$$p_{ij} = V(\tilde{S}_i \geq \tilde{S}_j) = \begin{cases} 1 & , \text{ if } m_i \geq m_j \\ \frac{u_i - l_j}{(u_i - m_i) + (m_j - l_j)}, & \text{ if } l_j \leq u_i \\ 0 & , \text{ otherwise} \end{cases} \quad (4)$$

for  $i, j = 1, 2, \dots, n; i \neq j$

where  $\tilde{S}_i = (l_i, m_i, u_i)$  and  $\tilde{S}_j = (l_j, m_j, u_j)$ .

The definition of the degree of possibility  $V(\tilde{S}_i \geq \tilde{S}_j)$  of  $\tilde{S}_i \geq \tilde{S}_j$  is shown in Figure 1.

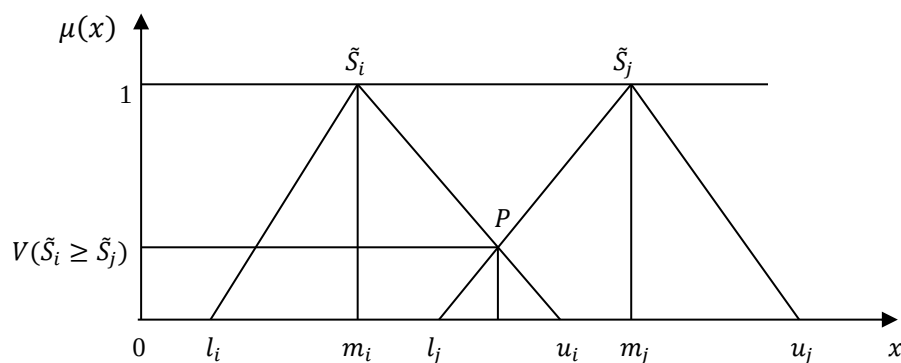
**Step-4:** The degree of possibility  $p_i$  of  $\tilde{S}_i$  over other  $(n-1)$  TFNs  $\tilde{S}_j$  ( $j = 1, 2, \dots, n; j \neq i$ ) is estimated using

$$p_i = V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n, j \neq i) = \min \{ p_{ij} : j = 1, 2, \dots, n, j \neq i \} \quad (5)$$

for  $i = 1, 2, \dots, n$

The possibility degree vector of  $n$  criteria or alternatives thus obtained is  $p = (p_1, p_2, \dots, p_n)^T$ .

**Figure 1** Degree of possibility  $V(\tilde{S}_i \geq \tilde{S}_j)$  of  $\tilde{S}_i \geq \tilde{S}_j$  for TFNs  $\tilde{S}_i$  and  $\tilde{S}_j$



**Step-5:** The  $n$  – component possibility degree vector  $p = (p_1, p_2, \dots, p_n)^T$  is normalized to get the crisp priority vector  $d = (d_1, d_2, \dots, d_n)^T$  of the fuzzy pairwise comparison matrix  $\tilde{A}$  for  $n$  criteria or alternatives.

#### IV. Resolution of irrational zero weights in Chang's EAM [4]

##### A. Analysis of the Problem in EAM

In step-5 of Section III, if it is found that  $d_k = 0$ , for some  $k \in \{1, 2, \dots, n\}$  then the relative weight of the corresponding criterion  $C_k$  is zero. This implies that the final ranking of alternatives will be neutral to the criterion  $C_k$  and hence the criterion  $C_k$  becomes redundant and is not worthy for consideration in the decision analysis. In this scenario, the criterion  $C_k$  could be discarded altogether at the beginning itself from the scheme of things and all the related fuzzy comparison matrices information with respect to the sub-criteria and alternatives in the hierarchy are wasted. But, in practical situation, this is an embarrassing case as the criterion having some actual weight, however small, deserves to be in the group of criteria subjected for evaluation of alternatives through some more intermediate levels of sub-criteria, sub-sub-criteria etc. As described by Wang *et al.* [9], it is observed that the irrationality occurs due to the possible occurrence of zero weight to some criteria in the EAM and hence the zero weight is termed as irrational zero weight. Thus, the EAM cannot make full use of all the fuzzy pairwise comparison matrices information when an irrational zero weight is assigned to some useful decision criteria or sub-criteria and may lead to wrong decision.

For effective use of EAM this problem needs to be resolved by rationalizing the irrational zero weights of criteria or sub-criteria through certain rules resulting in improvement of solution by effecting honourable presence of them in the evaluation process commensurate with their revised relative weights.

##### B. Resolution of irrational zero weight ( $d_k = 0$ ) in the EAM (I): the revised EAM

In Step-4 of Section III, the estimation of the degree of possibility  $p_i$  of  $\tilde{S}_i$  over other  $(n-1)$  TFNs  $\tilde{S}_j$  ( $j = 1, 2, \dots, n; j \neq i$ ) is given by

$$\begin{aligned} p_i &= \min \{1, p_i\} \quad (\because 0 \leq p_i \leq 1) \\ &= \min \{1, V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n, j \neq i)\} \quad (\text{from Eqn. (5)}) \\ &= \min \{V(\tilde{S}_i \geq \tilde{S}_i), V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n, j \neq i)\} \quad [\because V(\tilde{S}_i \geq \tilde{S}_i) = 1] \\ &= \min \{V(\tilde{S}_i \geq \tilde{S}_i), \min \{V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n, j \neq i)\}\} \quad (\text{from Eqn. (5)}) \\ &= \min \{V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n)\} \\ &= \min \{p_{ij} : j = 1, 2, \dots, n\} \end{aligned}$$

Thus, the degree of possibility  $p_i$  may be equivalently obtained by using minimum aggregation operator on the set of  $n$  arguments  $\{V(\tilde{S}_i \geq \tilde{S}_j : j = 1, 2, \dots, n)\}$  and  $d_k = 0$  implies that  $p_k = 0$ .

As  $p_k = \min \{p_{kj} : j = 1, 2, \dots, n\} = p_{kl}$ , for some  $j = l$ , therefore  $p_k = p_{kl} = V(\tilde{S}_k \geq \tilde{S}_l) = 0$ .

It follows that the inherent defect of the EAM due to the occurrence of irrational zero weight to some criteria or alternatives may be attributed to the use of minimum aggregation operator and hence the lacuna may be resolved by replacing this operator by any other suitable aggregation operator. For resolution of the problem, we propose the replacement of minimum aggregation operator with the arithmetic aggregation operator.

Hence, the Step-4 of Section III may follow as:

The degree of possibility of  $p_i$  of  $\tilde{S}_i$  over all  $n$  TFNs  $\tilde{S}_j$  ( $j = 1, 2, \dots, n$ ) is estimated using

$$p_i = \frac{1}{n} \sum_{j=1}^n p_{ij} = \frac{1}{n} \sum_{j=1}^n V(\tilde{S}_i \geq \tilde{S}_j), \text{ for } i = 1, 2, \dots, n \quad (6)$$

In the light of the current discussion, the Chang's EAM can be simplified as:

Through Step-1 to Step-3 of the EAM in Section III, the triangular fuzzy pairwise comparison matrix in Eqn. (1) generates the  $n \times n$  order possibility degree matrix

$$P = (p_{ij})_{n \times n}, \text{ where } p_{ij} = V(\tilde{S}_i \geq \tilde{S}_j) \quad (7)$$

Thus, Eqn. (6) represents the arithmetic aggregation operator applied to the  $i^{th}$  row of the possibility degree matrix (7) and constitutes Step-4 in the revised EAM yielding the possibility vector  $p = (p_1, p_2, \dots, p_n)^T$ . Each

component of this vector thus becomes non-zero. The normalization of the revised possibility vector constitutes Step-5 in the revised EAM giving the crisp weight vector  $w = (w_1, w_2, \dots, w_n)^T$ , none of whose components is zero.

### C. Remarks

The revised extent analysis method is denoted by EAM (I).

## V. A Numerical Example

In this example, a person wants to admit his child in one of the three schools, viz., S1, S2, and S3. The goal of the problem is to select the best school. The schools are evaluated with respect to the three criteria - infrastructure (In), faculty (Ft) and examination results (Er). The respective hierarchical structure is shown in Fig. 2. The fuzzy pairwise comparison matrices of criteria and alternatives with respect to goal and criteria respectively are entered in the Table-1. The weights and priorities or performance values of criteria and alternatives respectively are computed using three approaches, viz., EAM, GMM and EAM (I). The priority vectors are synthesized to obtain global performance vector for final ranking of alternatives as reflected in Table-2.

Figure 2 Hierarchical structure

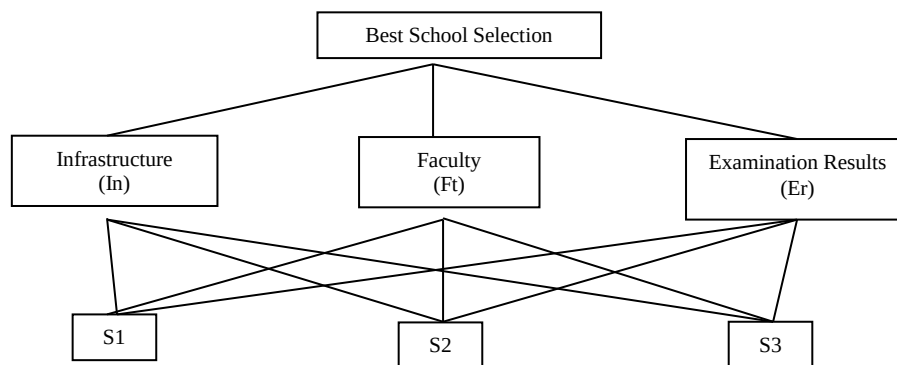


Table 1 Fuzzy pairwise comparison matrices of criteria and alternatives (schools) with respect to goal and criteria respectively and estimation of their priority vectors using EAM, GMM and EAM (I)

| Goal<br>(Best School<br>Selection) | In              | Ft              | Er              | Priority vector |       |         |
|------------------------------------|-----------------|-----------------|-----------------|-----------------|-------|---------|
|                                    |                 |                 |                 | EAM             | GMM   | EAM (I) |
| In                                 | (1, 1, 1)       | (2/3, 1, 3/2)   | (2/5, 1/2, 2/3) | 0               | 0.243 | 0.293   |
| Ft                                 | (2/3, 1, 3/2)   | (1, 1, 1)       | (2/7, 1/3, 2/5) | 0               | 0.211 | 0.268   |
| Er                                 | (3/2, 2, 5/2)   | (5/2, 3, 7/2)   | (1, 1, 1)       | 1               | 0.546 | 0.439   |
| Criterion<br>(In)                  | S1              | S2              | S3              | Priority vector |       |         |
|                                    |                 |                 |                 | EAM             | GMM   | EAM (I) |
| S1                                 | (1, 1, 1)       | (5/2, 3, 7/2)   | (3/2, 2, 5/2)   | 0.909           | 0.526 | 0.493   |
| S2                                 | (2/7, 1/3, 2/5) | (1, 1, 1)       | (2/7, 1/3, 2/5) | 0               | 0.140 | 0.163   |
| S3                                 | (2/5, 1/2, 2/3) | (5/2, 3, 7/2)   | (1, 1, 1)       | 0.091           | 0.334 | 0.345   |
| Criterion<br>(Ft)                  | S1              | S2              | S3              | Priority vector |       |         |
|                                    |                 |                 |                 | EAM             | GMM   | EAM (I) |
| S1                                 | (1, 1, 1)       | (2/3, 1, 3/2)   | (2/9, 1/4, 2/7) | 0               | 0.185 | 0.258   |
| S2                                 | (2/3, 1, 3/2)   | (1, 1, 1)       | (2/5, 1/2, 2/3) | 0               | 0.235 | 0.297   |
| S3                                 | (7/2, 4, 9/2)   | (3/2, 2, 5/2)   | (1, 1, 1)       | 1               | 0.580 | 0.445   |
| Criterion<br>(Er)                  | S1              | S2              | S3              | Priority vector |       |         |
|                                    |                 |                 |                 | EAM             | GMM   | EAM (I) |
| S1                                 | (1, 1, 1)       | (2/7, 1/3, 2/5) | (2/3, 1, 3/2)   | 0               | 0.223 | 0.201   |
| S2                                 | (5/2, 3, 7/2)   | (1, 1, 1)       | (1, 1, 1)       | 1               | 0.457 | 0.479   |
| S3                                 | (2/3, 1, 3/2)   | (1, 1, 1)       | (1, 1, 1)       | 0               | 0.319 | 0.320   |

Table 2 Synthesis of the weight vector and local priority vectors of criteria and alternatives (schools) respectively into global performance scores of Schools and deriving their ranking

| Synthesis<br>[EAM] | 0     | 0  | 1  | Global performance scores<br>(Schools) | Ranking |
|--------------------|-------|----|----|----------------------------------------|---------|
|                    | In    | Ft | Er |                                        |         |
| S1                 | 0.909 | 0  | 0  | 0                                      | III/II  |
| S2                 | 0     | 0  | 1  | 1                                      | I       |
| S3                 | 0.091 | 1  | 0  | 0                                      | II/III  |

| Synthesis<br>[GMM] | 0.243 | 0.211 | 0.546 | Global performance scores<br>(Schools) | Ranking |
|--------------------|-------|-------|-------|----------------------------------------|---------|
|                    | In    | Ft    | Er    |                                        |         |
| S1                 | 0.526 | 0.185 | 0.223 | 0.289                                  | III     |
| S2                 | 0.140 | 0.235 | 0.457 | 0.333                                  | II      |
| S3                 | 0.334 | 0.580 | 0.319 | 0.378                                  | I       |

| Synthesis<br>[EAM (I)] | 0.293 | 0.268 | 0.439 | Global performance scores<br>(Schools) | Ranking |
|------------------------|-------|-------|-------|----------------------------------------|---------|
|                        | In    | Ft    | Er    |                                        |         |
| S1                     | 0.493 | 0.258 | 0.201 | 0.302                                  | III     |
| S2                     | 0.163 | 0.297 | 0.479 | 0.338                                  | II      |
| S3                     | 0.345 | 0.445 | 0.320 | 0.361                                  | I       |

#### A. Remark

From the Table 2, it is found that the global performance vector of schools in EAM coincides with the local priority vector of the criterion examination results (Er) [weight = 1] forming the basis of ranking of schools disregarding the presence of other criteria infrastructure (In) [weight = 0] and faculty (Ft) [weight = 0] which sounds absurd as the person would like to assess the schools involving all the criteria weighing differently. Further, in both the approaches GMM and EAM (I), all the criteria take part in evaluation process of schools generating identical ranking of schools validating the effectiveness of the improved extent analysis method EAM (I).

#### VI. Conclusion

As an effort to sort out discrepancies in the Chang's EAM, the improved extent analysis method EAM (I) is presented. It was proved that Chang's EAM used minimum aggregation operator that remained the main reason for the occurrence of irrational zero weights of criteria or sub-criteria and hence the problem got resolved by replacing it with arithmetic averaging operator. Wang et al. [9] observed that the EAM fails to make complete use of all information provided by fuzzy pairwise comparison matrices which may result in wastage of some significant information related to fuzzy pairwise comparison matrices when it assigns an irrational zero weight to some important decision criteria or sub-criteria. Thus, the proposed approach effectively makes the honourable presence of the apparently obsolete criteria or sub-criteria associated with irrational zero weights occurred in the EAM in the decision analysis. No information of any criteria, sub-criteria and alternatives is wasted and all information is processed in a better way and the EAM is improved towards more acceptable solution in the developed approach. The computational complexity of the revised extent analysis method EAM (I) almost gets unaffected as arithmetic aggregation involves quite simple arithmetic calculations. The validity and effectiveness of the advanced approach is demonstrated through a numerical example. Thus, the proposed approach is an effort towards making decisive presence of the prior obsolete criteria or sub-criteria by rationalizing the irrational zero weights in the EAM enabling their presence in the decision analysis commensurate with their revised relative weights due to integration of aggregation operator in the extent analysis method. The EAM can further be improved by employing more suitable weighted aggregation operators.

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