



Magnetohydrodynamic Fluid Flow over an Upper Horizontal Surface of a Paraboloid of Revolution

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Abstract: This paper investigates heat and mass transfer characteristics of an incompressible Magneto hydrodynamic nanofluid flow over an upper horizontal surface of a paraboloid of revolution. The effect of magnetic parameters, velocity power index and thermophoresis parameters were considered. The governing non-linear partial differential equations are transformed and non-dimensionalized to system of first order ordinary differential equation using suitable transformations and the resulting equations are solved using MATLAB bvp4c solver. The influence of various salient physical parameters on velocity, temperature, concentration, Skin friction coefficient, Nusselt and Sherwood numbers are investigated with graphical representation.

Keywords: MHD, Boundary layer, Nano-fluids, paraboloid of revolution.

I. INTRODUCTION

Magneto hydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. The study of magneto hydrodynamics has many important applications, and may be used to deal with series of problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field. Recently, the application of magneto hydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Many applications for the MHD flows of non-Newtonian fluids in a porous medium are encountered in biological systems, heat-storage beds, irrigation problems, process of petroleum, paper, textile and polymer composite industries. Numerous studies have been presented on various aspects of MHD flows of non-Newtonian fluid flows passing through porous medium.

The theory of MHD started when Hartmann created a novel electromagnetic pump in 1918 in connectedness with the jet-wave rectifier [1]. **Roscow** [11] investigated the utilization of magnetic field to regulate the flow of electrically conducting fluids. It was deduced that increment in magnetic field parameter decayed the local skin friction and heat transfer rate. Since then, researches have been conducted on utilization of magnetic field to dispersion of metals, design of MHD pumps, MHD flow meters, etc. The effect of MHD and Hall current has an extraordinary importance for engineers, scientists and mineral scientists. MHD also plays an essential part in the area of medicine that is in the treatment of tumour, diminishing blood hemorrhaging in austere wounds, magnetized plangency imaging and many more [3]. The flow due to one, non-finite plate was initially commenced and discoursed by **Karman and Biot** [12] and **Batchelor** [3]. Studying the impression of viscous dispersal and Joule heating on MHD stagnation motion of Casson fluid using modified homogeneous-heterogeneous chemical reaction, **Khan et al** [7] announced that velocity chart depicts enhancing characteristics for greater homogeneous heat parameter. In another research, **Mosayebidorcheh et al** [9] investigated within two parallel plates, the MHD Couette motion and heat transport of a dusty fluids.

The effect of Soret and Dufour on a steady magneto-hydro-dynamic free convection flow over an inclined porous plate with viscous dissipation was investigated by **Ajala and Onikola** [1]. **O. A Abegunrin and I.L Animasaun** [8] studied the relationship between the thickness of the object and velocity index of the motion of a non-newtonian visco-elastic fluid over an upper object that is neither a cone/wedge nor horizontal/vertical whereby they assumed that

partial slip and buoyancy induces the flow of Williamson fluid over this kind of object which is referred to as an upper horizontal surface of a paraboloid of revolution.

A paraboloid of revolution is constituted by rotating a parabola around its axis. A satellite dish is formed in the similitude of a paraboloid of revolution. A path-breaking research was conducted by **Ivanstov**[8] who studied heat transport rate from needle forged crystals. **Horvay and Chan**[5], then broadened Ivanstov's outcome to elliptic and parabolic cylinders. In a bid to extend Ivanstov Horvay-Chan's ideas, **Dash and Gill**[10] applied this concept to paraboloid of revolution and parabolic cylinders. This work focuses on the flow heat and mass transfer of MHD fluid over an upper horizontal surface of a paraboloid of revolution.

II. MATHEMATICAL FORMULATION

The representation of governing equation which model a two-dimensional flow of MHD nanofluid over an upper horizontal surface described as paraboloid of revolution is considered. The temperature at the wall and free stream temperature are $T_w(x)$ and T_∞ respectively where $T_w(x) > T_\infty$. The flow is bounded by the region $A(x+b)^{\frac{m-1}{2}} \leq y < \infty$ (where A, b and m are constants and $m \leq 1$) and the paraboloid revolves with a stretching velocity $u_w = u_0(x+b)^m$. The temperature around the surface is assumed to be $T_w(x) = A(x+b)^{\frac{m-1}{2}}$ (i.e 'b' is regarded as the stretching parameter and 'm' represent the velocity power index). The Magnetic field of strength $H(x) = H_0(x+b)^{\frac{m-1}{2}}$ is taken into consideration. Buoyancy effects are also considered in terms of concentration and temperature. Adopting the order of magnitude of Navier-Stokes equation, the boundary layer equation governing the flow, temperature and concentration of an incompressible MHD nanofluid over an upper horizontal surface of a paraboloid of revolution can be described as follows;

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_\infty \frac{\partial u_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma H(x)^2}{\rho} (u - u_\infty) + g \left(\frac{m+1}{2} \right) \{ \beta (T - T_\infty) + \beta^* (C - C_\infty) \} \quad (2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\sigma H(x)^2}{\rho c_p} (u - u_\infty)^2 + \tau D_B \left[\frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

Concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where the magnetic parameter $M = \frac{\sigma H_0^2}{\rho u_0}$ stretching parameter $\lambda = \frac{v_0}{u_0}$ Where τ_w is the Shear stress, q_w is the

Heat flux and q_m is the Mass flux respectively

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad q_w = -k \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad q_m = -D_B \left. \frac{\partial C}{\partial y} \right|_{y=0}$$

Subject to the boundary conditions:

$$u = u_w, v = 0, T = T_w(x), C = C_w(x), y = 0 \quad (5)$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (6)$$

The following similarity transformations are introduced to transform the nonlinear partial differential equations (1)-(4) into the nonlinear ordinary differential equations.

$$\eta = y \sqrt{\frac{m+1}{2} \frac{u_0}{\nu}} (x+b)^{\frac{m-1}{2}}, \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

$$\psi(x, y) = f(\eta) \sqrt{\frac{2}{m+1}} \nu u_0 (x+b)^{\frac{m+1}{2}}, T_w = A(x+b)^{\frac{1-m}{2}}, C_w = A(x+b)^{\frac{1-m}{2}}$$

Therefore, the resulting non-linear ordinary differential equations with boundary conditions are given by

$$f'''' + f f'' - \left(\frac{2m}{m+1} \right) f'^2 + \left(\frac{2m}{m+1} \right) \lambda^2 + \frac{2m}{m+1} M(\lambda - f') + G_t \theta + G_m \phi = 0 \quad (7)$$

$$(1+R)\theta'' + P_r f \theta' - \left(\frac{1-m}{m+1} \right) P_r \theta f' + P_r E_c f''^2 + \frac{2}{m+1} M P_r (\lambda - f')^2 + P_r N_b \theta' \phi' + P_r N_t \theta'^2 = 0 \quad (8)$$

$$\phi'' + S_c \left[f \phi' + \left(\frac{m-1}{m+1} \right) f' \phi \right] + \frac{N_t}{N_b} \theta'' = 0 \quad (9)$$

Where $P_r = \frac{\nu}{\alpha}$ (Prandtl number), $E_c = \frac{u_0 (x+b)^{2m} f''^2}{C_p (T_w - T_\infty)}$ (Eckert number), $R = \frac{16\sigma^* T^3}{3\alpha \rho C_p k^*}$ (Radiation

parameter), $N_b = \frac{\tau D_B (C_w - C_\infty)}{\nu}$ (Brownian constant), $N_t = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$ (Thermophoretic constant) and

$$S_c = \frac{\nu}{D_B} \text{ (Schmidt number).}$$

III. NUMERICAL SOLUTION

By noting the fact that due to the non-linearity of the differential equations, the analytic solution of equations (2), (3) and (4) subject to (5) and (6) may not exist. The set of ordinary differential equations (2), (3) and (4) together with boundary conditions (5) and (6) are considered and solved by adopting the MATLAB bvp4c in-built solver.

Setting

$$f = y_1(\eta) \quad (10)$$

Then

$$\frac{df}{d\eta} = \frac{dy_1}{d\eta} = y_2 \quad (11)$$

$$\frac{d^2 f}{d\eta^2} = \frac{d}{d\eta} \left(\frac{dy_1}{d\eta} \right) = \frac{dy_2}{d\eta} = y_3 \quad (12)$$

Let

$$\theta = y_4 \quad (13)$$

$$\frac{d\theta}{d\eta} = \frac{dy_4}{d\eta} = y_5 \quad (14)$$

$$\phi = y_6 \quad (15)$$

$$\frac{d\phi}{d\eta} = \frac{dy_6}{d\eta} = y_7 \quad (16)$$

To transform equations (7)-(9) to first order ordinary differential equation we then substitute equations (10) to (16) into equations (7)-(9) to obtain

$$\frac{dy_3}{d\eta} = -y_1 y_3 + \frac{2m}{m+1} (y_2)^2 - \frac{2M}{m+1} (\lambda - y_2) - \frac{2m}{m+1} \lambda^2 \quad (17)$$

$$(1+R)\theta'' + P_r f \theta' - \left(\frac{1-m}{m+1} \right) P_r \theta f' + P_r E_c f'^2 + \frac{2}{m+1} M P_r (\lambda - f')^2 + P_r N_b \theta' \phi' + P_r N_t \theta'^2 = 0 \quad (18)$$

$$\frac{dy_7}{d\eta} = -S_c \left[y_1 y_7 + \left(\frac{m-1}{m+1} \right) y_2 y_6 \right] - \frac{N_t}{N_b} y_5 \quad (19)$$

MATLAB bvp4c ODE solver

The bvp4c is a collection of MATLAB in-built functions for solving boundary value problems. The problem is first transformed into an ordinary differential equation with initial guess of zeros. The boundary conditions at the wall ($\eta = a$) can be employed as initial condition for the system of initial value ODE. However, the boundary conditions at the free stream ($\eta = b$) cannot be used directly as $\eta \rightarrow \infty$ but must be assumed initial conditions (zero vector) are employed iteratively and compared with the boundary conditions until tolerance value is reached. In this research work, the boundary value problem cannot be solved on an infinite domain and it would be impractical to solve it on a very large finite interval, so the infinite boundary condition at a finite point of $\eta_\infty = 5$ is considered. To integrate the corresponding IVP $f''(0), \theta'(0)$ and $\phi'(0)$ are needed but no such value exist after the non-dimensionalization of the boundary conditions (5) and (6). The appropriate guess value for $f''(0), \theta'(0)$ and $\phi'(0)$ are chosen and the integration is carried out using bvp4c solver. The calculated values for $f'(\eta = 5), \theta(\eta = 5)$ and $\phi(\eta = 5)$ are compared with that of the boundary condition (5) and (6). Adjustments of $f''(0), \theta'(0)$ and $\phi'(0)$ are carried out for better approximation for the solution. Better estimates of $f''(0), \theta'(0)$ and $\phi'(0)$ together with the boundary conditions (5) and (6) are solved using bvp4c solver with $h=0.01$

IV. RESULTS AND DISCUSSION

The dimensionless non-linear coupled differential equations together with boundary conditions are solved using MATLAB bvp4c ODE solver as discussed. Numerical computations have been carried out for various values of all parameters in order to study its effect on MHD fluid flow over an upper horizontal surface of a paraboloid of revolution.

It could be seen from figure 1 and figure 2 which shows the effect of magnetic parameter M on vertical velocity $f(\eta)$ and Horizontal velocity $f'(\eta)$. Result shows that as the values of the magnetic parameter increases, the vertical velocity $f(\eta)$ decrease while there is a corresponding increase in Horizontal velocity $f'(\eta)$. Improving the magnetic field, develops the resistive force called the Lorentz force which works reverse to the flow field as a results depreciates the vertical velocity $f(\eta)$ and enhanced the Horizontal velocity $f'(\eta)$

Figure 3 shows the effect of velocity power index m on Temperature $\theta(\eta)$. It is clear that as the value of velocity power index m increases, the Temperature profile $\theta(\eta)$ increases

Figure 4 shows the influence of Thermophoresis parameter N_t on concentration gradient $\phi'(\eta)$. It is evident that as the values of Thermophoresis parameter N_t increases, there is a declination effect on concentration gradient $\phi'(\eta)$.

V. CONCLUSION

Increase in magnetic parameter develops a Lorentz force which decreases the vertical velocity and increase the Horizontal velocity. Temperature profile increases due to the influence of the velocity power index. Thermophoresis parameter increases and yields a decrease in concentration gradient.

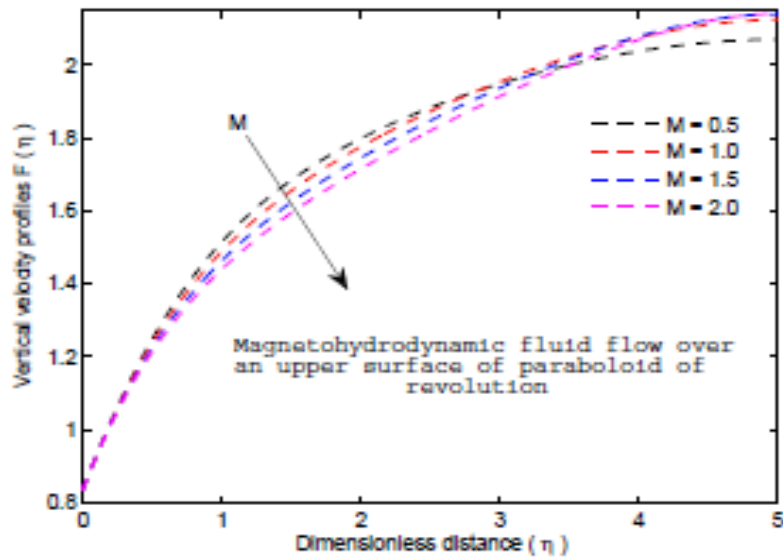


Fig 1: effect of M on vertical velocity

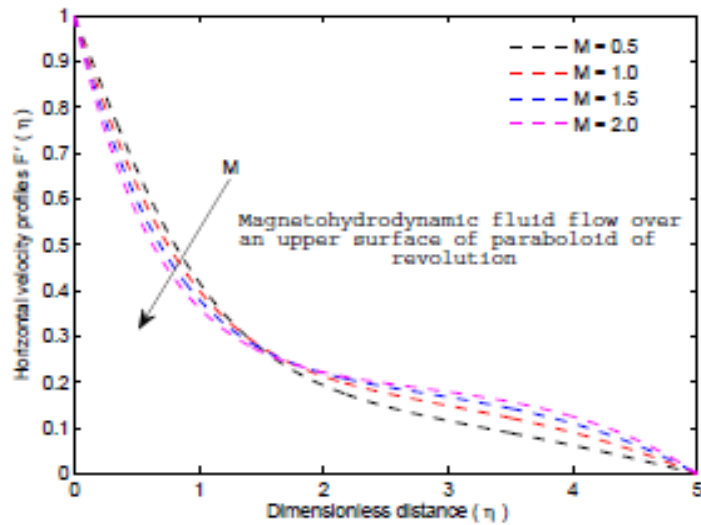


Fig 2: effect of M on horizontal velocity

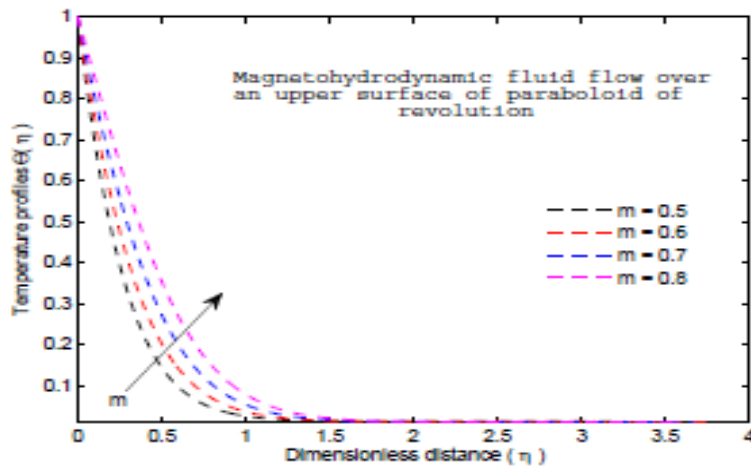


Fig 3: effect of m on Temperature

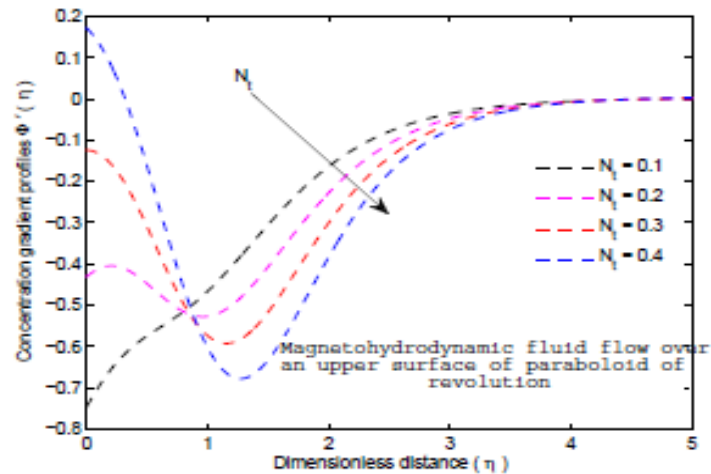


Fig 4: effect of N_t on concentration gradient

REFERENCES

- [1] Ajala O.A and Onikola I.O (2016): Effect of Soret and Dufour on a steady magneto-hydro-dynamic free convection flow over an inclined porous plate with viscous dissipation; *Journal of Advances in Mathematics*, Vol. 11, No 9, pp. 5645-5652, www.cirjam.com.
- [2] A. Salman, C. Farrukh, M. Asad (2017): Analytical technique for magnetohydrodynamic (MHD) fluid flow of a periodically accelerated plate with slippage, *McGraw-Hill, New York* 65, pp 192-198.
- [3] G. Batchelor (1967): An introduction to fluid dynamics, *Results in Physics*.
- [4] G. Ivanstov, *Dokl. Akad. Nauk ussr* 58, 561 (1947). Translation by g. Horvay in general electric report, *Journal of Molecular Liquids* 60-RL-2511M
- [5] G. Horvay and J.W. Chan (1961): Dendritic and spheroidal growth, *Journal of Molecular Liquids* 9, pp 695-705
- [6] J. Hartmann (1937): Hg-dynamics theory of the laminar flow of an electrically conductive liquid in a homogeneous magnetic field, *Det Kgl. Danske Videnskaber Selskab, Matematisk-fysiske Meddelelser* 6 (1937) pages 1-28
- [7] M. Khan, T. Hayat, M.I. Khan, A. Alsaedi (2017): A modified homogeneous-heterogeneous reactions for MHD stagnation flow with viscous dissipation and joule heating, *international journal of heat and mass Trans.* 113, 310-317
- [8] O. A. Abegunrin and I. L. Animasaun (2017): Motion of Williamson fluid over an upper horizontal surface of a paraboloid of revolution due to partial slip and buoyancy; *Defect and Diffusion Forum*, Vol. 378, pp. 16-27.
- [9] S. Mosayebidorcheh, O.D. Makinde, D.D Ganji, M. Abedian, Dtm-fdm (2017): hybrid approach to unsteady MHD couette flow and heat transfer of dusty fluid with variable, pp 1-30
- [10] S. Dash, W.N. Gill (1984): Forced convection heat and momentum transfer to dendritic structures (parabolic cylinders and paraboloid of revolution), *International Journal of heat and mass Transfer* 27, pp 1345-1356
- [11] V. Rossow (1957): On flow of electrically conducting fluid over a flat plate in the presence of transverse magnetic field. *NACA Tech* 3971, pages 1-20
- [12] V. Karman, M.A. Biot (1940): Mathematical methods in engineering: An introduction to the mathematical treatment of engineering problems, *McGraw-Hill, New York*.