



Edge Regular Interval-Valued Pythagorean Fuzzy Graph

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Abstract: In this paper, edge regular interval-valued Pythagorean fuzzy graph and totally edge regular interval-valued Pythagorean fuzzy graphs are introduced. Investigate some of their properties related to edge regular and totally edge regular interval-valued Pythagorean fuzzy graphs.

Keywords: Interval-valued Pythagorean fuzzy graph, Edge regular, totally edge regular.

AMS Subject Classification: 03E05, 05C72, 05C76

I. Introduction

The notion of interval-valued fuzzy sets was introduced by Zadeh [14]. In 1986, Atanassov [2] proposed Intuitionistic Fuzzy Set (IFS) which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After three years Atanassov and Gargov [1] introduced Interval-Valued Intuitionistic Fuzzy set (IVIFS). Ju and Wang [5] gave the definition of interval-valued fuzzy graph. Some works in interval-valued fuzzy graph theory can be found in [3,4]. In 2013, Yager [12, 13] introduced the concept of the Pythagorean fuzzy set (PFS) as a generalization of the intuitionistic fuzzy set (IFS) [2], to manage the complex impreciseness and uncertainty in practical decision-making problems. The important characteristic of the Pythagorean fuzzy model is to relax the condition that the sum of its membership degree and non-membership degree is not greater than one (membership + nonmembership ≤ 1) with the square sum of its membership degree and non-membership degree not greater than one (membership² + nonmembership² ≤ 1). After the inception of Pythagorean fuzzy set by Yager [12], Zhang and Xu [15] presented the mathematical form of the Pythagorean fuzzy set and introduced the concept of the Pythagorean fuzzy number (PFN). Naz et al. [9] proposed the concept of Pythagorean fuzzy graphs (PFGs). Xindong Peng et al. [11] introduced the notion of interval-valued Pythagorean fuzzy sets as a generalization of Pythagorean fuzzy set [12]. Mohamed Y.S and Ali A.M. [6,7] introduced interval-valued Pythagorean fuzzy graph. In this paper, the vertex degree, edge degree, total edge degree of an interval-valued Pythagorean fuzzy graph is defined. Edge regular and totally edge regular interval-valued Pythagorean graphs are introduced. The relation among edge regular and totally edge regular interval-valued Pythagorean fuzzy graph is proved.

II. Preliminaries

Definition 2.1[12]

Let X be a universe of discourse. An Pythagorean Fuzzy Set P in X is given by $P = \{ \langle x, \mu_P(x), \nu_P(x) \rangle / x \in X \}$, where $\mu_P: X \rightarrow [0,1]$ denotes the degree of membership and $\nu_P: X \rightarrow [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set P , respectively, with the condition that $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$. The degree of indeterminacy $\pi_P(x) = \sqrt{1 - \mu_P(x)^2 - \nu_P(x)^2}$.

Let $D[0, 1]$ be the set of all closed subintervals of $[0,1]$ and element of this set are denoted by uppercase letters. If $\tilde{M} \in D[0, 1]$ then it can be represented as $\tilde{M} = [M_L, M_U]$, where M_L and M_U are the lower and upper limits of $\tilde{M}$.

Definition 2.2[11]

An interval-valued Pythagorean fuzzy set A defined in a finite universe of discourse $X = \{x_1, x_2, x_3, \dots, x_n\}$ is given by $A = \{ \langle x, \tilde{M}(x) = [M_{AL}, M_{AU}], \tilde{N}(x) = [N_{AL}, N_{AU}] \rangle / x \in X \}$ where $M_{AL}(x), M_{AU}(x): X \rightarrow [0,1]$ and $N_{AL}(x), N_{AU}(x): X \rightarrow [0,1]$ and $0 \leq M_{AU}^2 + N_{AU}^2 \leq 1$.

Here the numbers $\tilde{M}(x)$ and $\tilde{N}(x)$ denote the degree of membership and degree of non-membership of $x \in X$ in A .

Definition 2.3[6]

An interval valued Pythagorean fuzzy graph with underlying set V is defined to be a pair $G = (P, Q)$ where

- 1) the functions $M_{PL}, M_{PU}: V \rightarrow D[0, 1]$ and $N_{PL}, N_{PU}: V \rightarrow D[0, 1]$ denote the degree of membership and non membership of the element $x \in V$, respectively, such that $0 \leq M_{PU}(x) + N_{PU}(x) \leq 1$ for all $x \in V$.
- 2) the functions $M_{QL}, M_{QU}: E \subseteq V \times V \rightarrow D[0, 1]$ and $N_{QL}, N_{QU}: E \subseteq V \times V \rightarrow D[0, 1]$ are defined by $M_{QL}((x, y)) \leq \min(M_{PL}(x), M_{PL}(y))$ and $N_{QL}((x, y)) \geq \max(N_{PL}(x), N_{PL}(y))$ $M_{QU}((x, y)) \leq$

$$\min(M_{PU}(x), M_{PU}(y)) \text{ and } N_{QU}((x, y)) \geq \max(N_{PU}(x), N_{PU}(y)) \text{ such that } 0 \leq M_{QU}^2((x, y)) + N_{QU}^2((x, y)) \leq 1, \forall (x, y) \in E.$$

Here after, we use the notation xy for (x, y) an element of E .

Definition 2.4 [10] Let $G^* = (V, E)$ be a graph and let $e = uv$ be an edge in G^* . Then the degree of an edge $e = uv \in E$ is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$.

III. Edge regular interval-valued Pythagorean fuzzy graph

Definition 3.1

The degree of a vertex $x \in V$ in an interval-valued Pythagorean fuzzy graph $G = (P, Q)$ is defined as $d(x) = \langle [d_{M_{PL}}(x), d_{M_{PU}}(x)], [d_{N_{PL}}(x), d_{N_{PU}}(x)] \rangle$ where

$$d_{M_{PL}}(x) = \sum_{x,y \neq x \in V} M_{QL}(xy); \quad d_{M_{PU}}(x) = \sum_{x,y \neq x \in V} M_{QU}(xy) \text{ and } \\ d_{N_{PL}}(x) = \sum_{x,y \neq x \in V} N_{QL}(xy); \quad d_{N_{PU}}(x) = \sum_{x,y \neq x \in V} N_{QU}(xy).$$

Definition 3.2

An interval-valued fuzzy graph $G = (P, Q)$ is said to be a *regular* if all vertices have same membership degree.

Definition 3.3

The degree of an edge $uv \in E$ in an interval-valued Pythagorean fuzzy graph $G = (P, Q)$ is a pair $\langle [d_{M_{QL}}(uv), d_{M_{QU}}(uv)], [d_{N_{QL}}(uv), d_{N_{QU}}(uv)] \rangle$, where $d_{M_{QL}}(uv) = d_{M_{PL}}(u) + d_{M_{PL}}(v) - 2M_{QL}(uv)$, $d_{M_{QU}}(uv) = d_{M_{PU}}(u) + d_{M_{PU}}(v) - 2M_{QU}(uv)$, $d_{N_{QL}}(uv) = d_{N_{PL}}(u) + d_{N_{PL}}(v) - 2N_{QL}(uv)$, $d_{N_{QU}}(uv) = d_{N_{PU}}(u) + d_{N_{PU}}(v) - 2N_{QU}(uv)$.

Definition 3.4

The total degree of an edge in an interval-valued Pythagorean fuzzy graph is a pair $\langle [td_{M_{QL}}(uv), td_{M_{QU}}(uv)], [td_{N_{QL}}(uv), td_{N_{QU}}(uv)] \rangle$ where $td_{M_{QL}}(uv) = d_{M_{PL}}(u) + d_{M_{PL}}(v) - M_{QL}(uv)$, $td_{M_{QU}}(uv) = d_{M_{PU}}(u) + d_{M_{PU}}(v) - M_{QU}(uv)$, $td_{N_{QL}}(uv) = d_{N_{PL}}(u) + d_{N_{PL}}(v) - N_{QL}(uv)$ and $td_{N_{QU}}(uv) = d_{N_{PU}}(u) + d_{N_{PU}}(v) - N_{QU}(uv)$.

Definition 3.5

Let $G = (P, Q)$ be an interval-valued Pythagorean fuzzy graph on G^* . If each edge in G has same degree $\langle [m_1, m_2], [n_1, n_2] \rangle$, then G said to be an edge regular interval-valued Pythagorean fuzzy graph or $\langle [m_1, m_2], [n_1, n_2] \rangle$ -edge regular interval-valued Pythagorean fuzzy graph.

Example 3.1 Consider an interval-valued Pythagorean fuzzy graph given in fig.1

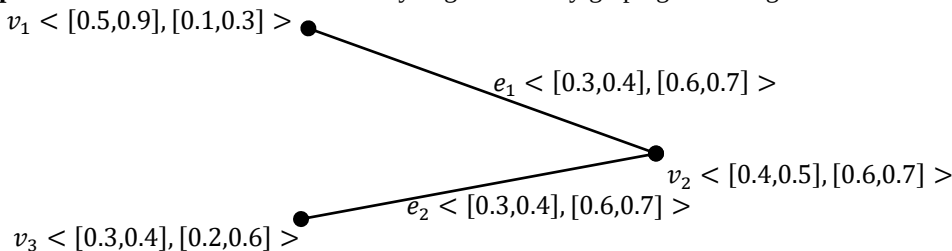


Figure – 1 Edge regular interval-valued Pythagorean fuzzy graph

Here, $[0.3, 0.4], [0.6, 0.7]$ –edge regular interval-valued Pythagorean fuzzy graph.

Definition 3.2

Let $G = (P, Q)$ be an interval-valued Pythagorean fuzzy graph on G^* . If each edge in G has same total degree $\langle [m_1, m_2], [n_1, n_2] \rangle$, then G said to be an *antotally edge regular interval-valued Pythagorean fuzzy graph* or $\langle [m_1, m_2], [n_1, n_2] \rangle$ -totally edge regular interval-valued Pythagorean fuzzy graph.

Example 3.2 Consider an interval-valued Pythagorean fuzzy graph given in Fig.2

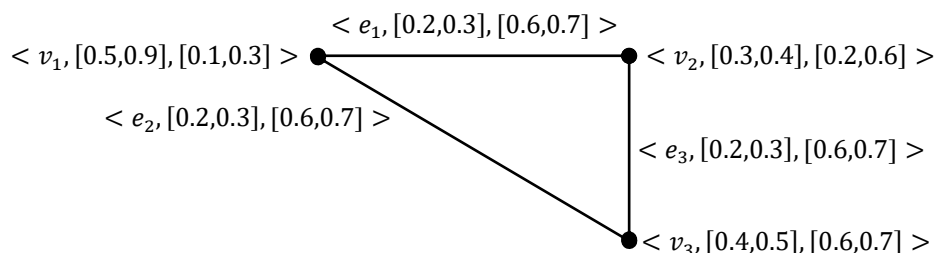


Figure: 2 Totally edge regular interval-valued Pythagorean fuzzy graph

Here, all the edges have same total degree. Hence G is $\langle [0.6, 0.9], [1.8, 2.1] \rangle$ -totally edge regular interval-valued Pythagorean fuzzy graph.

Theorem 3.1

Let G be an interval-valued Pythagorean fuzzy graph on G^* . Then $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function if and only if the following are equivalent

1. G is an edge regular interval-valued Pythagorean fuzzy graph
2. G is a totally edge regular interval-valued Pythagorean fuzzy graph

Proof:

Suppose that $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Let $[M_{QL}(uv), M_{QU}(uv)] = [c_1, c_2]$, $[N_{QL}(uv), N_{QU}(uv)] = [c_3, c_4]$ for all $uv \in E$, $c_1, c_2, c_3, c_4 \in [0, 1]$. Assume that G is $([k_1, k_2], [k_3, k_4])$ -edge regular interval-valued Pythagorean fuzzy graph. Then $d_{M_{QL}}(uv) = k_1$ and $d_{M_{QU}}(uv) = k_2$, $d_{N_{QL}}(uv) = k_3$ and $d_{N_{QU}}(uv) = k_4$ for all $uv \in E$.

Therefore $td_{M_{QL}}(uv) = d_{M_{QL}}(uv) + M_{QL}(uv)$, $td_{M_{QU}}(uv) = d_{M_{QU}}(uv) + M_{QU}(uv)$, $td_{N_{QL}}(uv) = d_{N_{QL}}(uv) + N_{QL}(uv)$,

$td_{N_{QU}}(uv) = d_{N_{QU}}(uv) + N_{QU}(uv)$.

$\Rightarrow td_{M_{QL}}(uv) = k_1 + c_1$, $td_{M_{QU}}(uv) = k_2 + c_2$,

$td_{N_{QL}}(uv) = k_3 + c_3$ and $td_{N_{QU}}(uv) = k_4 + c_4$.

Hence G is $([k_1 + c_1, k_2 + c_2], [k_3 + c_3, k_4 + c_4])$ -totally interval-valued Pythagorean fuzzy graph. Thus (1) $\Rightarrow$ (2)

Now, suppose that G is a $([m_1, m_2], [n_1, n_2])$ -totally edge regular interval-valued Pythagorean fuzzy graph, then $td_{M_{QL}}(uv) = m_1$, $td_{M_{QU}}(uv) = m_2$, $td_{N_{QL}}(uv) = m_3$ and $td_{N_{QU}}(uv) = m_4$, $\forall uv \in E$.

$td_{M_{QL}}(uv) = d_{M_{QL}}(uv) + M_{QL}(uv) = m_1$, $td_{M_{QU}}(uv) = d_{M_{QU}}(uv) + M_{QU}(uv) = m_2$, $td_{N_{QL}}(uv) = d_{N_{QL}}(uv) + N_{QL}(uv) = n_1$, $td_{N_{QU}}(uv) = d_{N_{QU}}(uv) + N_{QU}(uv) = n_2$, for all $uv \in E$.

$\Rightarrow d_{M_{QL}}(uv) = m_1 - k_1$, $d_{M_{QU}}(uv) = m_2 - k_2$, $d_{N_{QL}}(uv) = n_1 - k_3$ and $d_{N_{QU}}(uv) = n_2 - k_4$ for all $uv \in E$.

Hence, G is a $([m_1 - k_1, m_2 - k_2], [n_1 - k_3, n_2 - k_4])$ -edge regular interval-valued fuzzy graph.

Thus (2) $\Rightarrow$ (1)

Conversely,

Let us assume that (1) and (2) are equivalent. To prove that $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Suppose $[\tilde{M}_Q, \tilde{N}_Q]$ is not constant.

Then $M_{QL}(uv) \neq M_{QL}(xy)$, $M_{QU}(uv) \neq M_{QU}(xy)$ and $N_{QL}(uv) \neq N_{QL}(xy)$, $N_{QU}(uv) \neq N_{QU}(xy)$ for at-least one pair of edges $uv, xy \in E$.

Let G be a $([k_1, k_2], [k_3, k_4])$ -edge regular interval-valued Pythagorean fuzzy graph.

Then $M_{QL}(uv) \neq M_{QL}(xy) = k_1$, $M_{QU}(uv) \neq M_{QU}(xy) = k_2$ and $N_{QL}(uv) \neq N_{QL}(xy) = k_3$, $N_{QU}(uv) \neq N_{QU}(xy) = k_4$.

$\Rightarrow td_{M_{QL}}(uv) = d_{M_{QL}}(uv) + M_{QL}(uv) = k_1 + M_{QL}(uv)$, $td_{M_{QU}}(uv) = d_{M_{QU}}(uv) + M_{QU}(uv) = k_2 + M_{QU}(uv)$, $td_{N_{QL}}(uv) = d_{N_{QL}}(uv) + N_{QL}(uv) = k_3 + N_{QL}(uv)$, $td_{N_{QU}}(uv) = d_{N_{QU}}(uv) + N_{QU}(uv) = k_4 + N_{QU}(uv)$.

Since $M_{QL}(uv) \neq M_{QL}(xy)$, $M_{QU}(uv) \neq M_{QU}(xy)$ and $N_{QL}(uv) \neq N_{QL}(xy)$,

$N_{QU}(uv) \neq N_{QU}(xy)$, we have $td_{M_{QL}}(uv) \neq td_{M_{QL}}(xy)$, $td_{M_{QU}}(uv) \neq td_{M_{QU}}(xy)$, $td_{N_{QL}}(uv) \neq td_{N_{QL}}(xy)$

and $td_{N_{QU}}(uv) \neq td_{N_{QU}}(xy)$. Hence G is not a totally edge regular interval-valued Pythagorean fuzzy graph, which is a contradiction to our assumption.

Further, let G be a totally edge regular interval-valued Pythagorean fuzzy graph.

$\Rightarrow td_{M_{QL}}(uv) = td_{M_{QL}}(xy)$, $td_{M_{QU}}(uv) = td_{M_{QU}}(xy)$, $td_{N_{QL}}(uv) = td_{N_{QL}}(xy)$ and $td_{N_{QU}}(uv) = td_{N_{QU}}(xy)$.

$\Rightarrow d_{M_{QL}}(uv) + M_{QL}(uv) = d_{M_{QL}}(xy) + M_{QL}(xy)$, $d_{M_{QU}}(uv) + M_{QU}(uv) = d_{M_{QU}}(xy) + M_{QU}(xy)$,

$d_{N_{QL}}(uv) + N_{QL}(uv) = d_{N_{QL}}(xy) + N_{QL}(xy)$, $d_{N_{QU}}(uv) + N_{QU}(uv) = d_{N_{QU}}(xy) + N_{QU}(xy)$.

$d_{M_{QL}}(uv) - d_{M_{QL}}(xy) = M_{QL}(xy) - M_{QL}(uv)$, $d_{M_{QU}}(uv) - d_{M_{QU}}(xy) = M_{QU}(xy) - M_{QU}(uv)$, $d_{N_{QL}}(uv) - d_{N_{QL}}(xy) = N_{QL}(xy) - N_{QL}(uv)$ and $d_{N_{QU}}(uv) - d_{N_{QU}}(xy) = N_{QU}(xy) - N_{QU}(uv)$.

$\Rightarrow d_{M_{QL}}(uv) - d_{M_{QL}}(xy) \neq 0$, $d_{M_{QU}}(uv) - d_{M_{QU}}(xy) \neq 0$, $d_{N_{QL}}(uv) - d_{N_{QL}}(xy) \neq 0$ and $d_{N_{QU}}(uv) - d_{N_{QU}}(xy) \neq 0$.

$\Rightarrow d_{M_{QL}}(uv) \neq d_{M_{QL}}(xy)$, $d_{M_{QU}}(uv) \neq d_{M_{QU}}(xy)$, $d_{N_{QL}}(uv) \neq d_{N_{QL}}(xy)$ and $d_{N_{QU}}(uv) \neq d_{N_{QU}}(xy)$.

Thus G is not an edge regular interval-valued Pythagorean fuzzy graph.

This is a contradiction to our assumption. Hence $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Theorem 3.2

If an interval-valued fuzzy graph G is both edge regular and totally edge regular, then $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Proof:

Let G be a $([k_1, k_2], [k_3, k_4])$ -edge regular and $([r_1, r_2], [r_3, r_4])$ -totally edge regular interval-valued fuzzy graph.

$\Rightarrow d_{M_{QL}}(uv) = k_1$ and $d_{M_{QU}}(uv) = k_2$, $d_{N_{QL}}(uv) = k_3$ and $d_{N_{QU}}(uv) = k_4$ for all $uv \in E$.

Now, $td_{M_{QL}}(uv) = r_1$ and $td_{M_{QU}}(uv) = r_2$,

$td_{N_{QL}}(uv) = r_3$ and $td_{N_{QU}}(uv) = r_4$.

$\Rightarrow d_{M_{QL}}(uv) + M_{QL}(uv) = r_1$, $d_{M_{QU}}(uv) + M_{QU}(uv) = r_2$,

$d_{N_{QL}}(uv) + N_{QL}(uv) = r_3$, $d_{N_{QU}}(uv) + N_{QU}(uv) = r_4$

$\Rightarrow M_{QL}(uv) = r_1 - k_1$, $M_{QU}(uv) = r_2 - k_2$, $N_{QL}(uv) = r_3 - k_3$ and $N_{QU}(uv) = r_4 - k_4$.

Hence, $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Remark 3.1

The converse of the above theorem 3.2 need not be true. It can be seen from the following example.

Example 3.3 Consider an interval-valued Pythagorean fuzzy graph given in Figure 3,

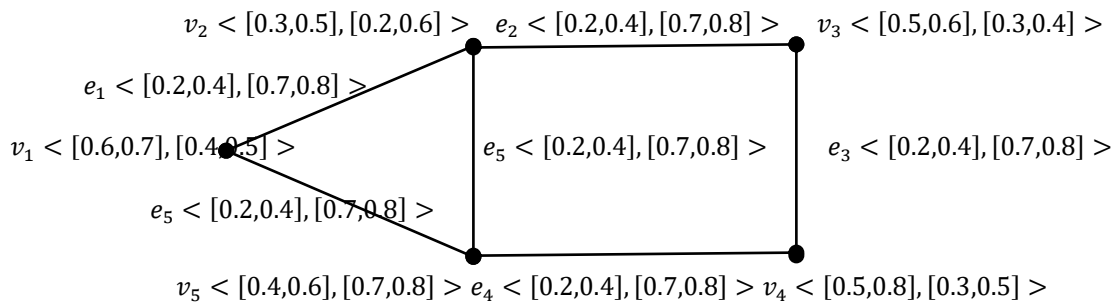


Figure- 3

Here, $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function but the graph is neither edge regular nor totally edge regular interval-valued Pythagorean fuzzy graph.

Theorem 3.3

Let G be a regular interval-valued fuzzy graph on $G^* = (V, E)$. Then G is edge regular if and only if $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Proof:

Let G be an $[k_1, k_2], [k_3, k_4]$ – regular interval-valued Pythagorean fuzzy graph.

$d_{M_{QL}}(uv) = k_1$, $d_{M_{QU}}(uv) = k_2$, $d_{N_{QL}}(uv) = k_3$, $d_{N_{QU}}(uv) = k_4$

Assume that $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Then to prove that G is edge regular interval-valued Pythagorean fuzzy graph.

Let $M_{QL}(uv) = c_1$, $M_{QU}(uv) = c_2$, $N_{QL}(uv) = c_3$, $N_{QU}(uv) = c_4$

By the definition of edge degree,

$d_{M_{QL}}(uv) = d_{M_{PL}}(u) + d_{M_{PL}}(v) - 2M_{QL}(uv) = k_1 + k_1 - 2c_1 = d_1(\text{say}),$

$d_{M_{QU}}(uv) = d_{M_{PU}}(u) + d_{M_{PU}}(v) - 2M_{QU}(uv) = k_2 + k_2 - 2c_2 = d_2(\text{say})$

$d_{N_{QL}}(uv) = d_{N_{PL}}(u) + d_{N_{PL}}(v) - 2N_{QL}(uv) = k_3 + k_3 - 2c_3 = d_3(\text{say})$

$d_{N_{QU}}(uv) = d_{N_{PU}}(u) + d_{N_{PU}}(v) - 2N_{QU}(uv) = k_4 + k_4 - 2c_4 = d_4(\text{say})$

$\Rightarrow d_{M_{QL}}(uv) = d_1$, $d_{M_{QU}}(uv) = d_2$, $d_{N_{QL}}(uv) = d_3$ and $d_{N_{QU}}(uv) = d_4$

Hence G is $([d_1, d_2], [d_3, d_4])$ – regular interval-valued Pythagorean fuzzy graph.

Conversely, assume that G is edge regular interval-valued Pythagorean fuzzy graph.

Then to prove that $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

Let $d_{M_{QL}}(uv) = d_1$, $d_{M_{QU}}(uv) = d_2$, $d_{N_{QL}}(uv) = d_3$ and $d_{N_{QU}}(uv) = d_4$ for all $uv \in E$.

By the definition of edge degree,

$d_1 = k_1 + k_1 - 2M_{QL}(uv)$, $d_2 = k_2 + k_2 - 2M_{QU}(uv)$, $d_3 = k_3 + k_3 - 2N_{QL}(uv)$

$d_4 = k_4 + k_4 - 2N_{QU}(uv)$

$d_{M_{QL}}(uv) = d_{M_{PL}}(u) + d_{M_{PL}}(v) - 2M_{QL}(uv)$

$d_1 = k_1 + k_1 - 2M_{QL}(uv)$

$M_{QL}(uv) = \frac{2k_1 - d_1}{2}$

$d_{M_{QU}}(uv) = d_{M_{PU}}(u) + d_{M_{PU}}(v) - 2M_{QU}(uv)$

$$\begin{aligned}
 d_2 &= k_2 + k_2 - 2M_{QU}(uv) \\
 M_{QU}(uv) &= \frac{2k_2 - d_2}{2} \\
 d_{N_{QL}}(uv) &= d_{N_{PL}}(u) + d_{N_{PL}}(v) - 2N_{QL}(uv) \\
 d_3 &= k_3 + k_3 - 2N_{QL}(uv) \\
 N_{QL}(uv) &= \frac{2k_3 - d_3}{2} \\
 d_{N_{QU}}(uv) &= d_{N_{PU}}(u) + d_{N_{PU}}(v) - 2N_{QU}(uv) \\
 d_2 &= k_4 + k_4 - 2N_{QU}(uv)
 \end{aligned}$$

$$N_{QU}(uv) = \frac{2k_4 - d_4}{2} \text{ for all } uv \in E.$$

Hence $[\tilde{M}_Q, \tilde{N}_Q]$ is a constant function.

IV. CONCLUSION

An interval-valued set Pythagorean fuzzy set model is suitable for modeling problems with indeterminacy, uncertainty, and inconsistent information in which human knowledge is necessary and human appraisal needed. Interval-valued Pythagorean fuzzy models give more flexibility and compatibility to the system as compared to the classical, interval-valued fuzzy and interval-valued intuitionistic fuzzy models. In this paper, edge regular and totally edge regular interval-valued Pythagorean fuzzy graphs are defined. A necessary and sufficient condition for edge regular and totally edge regular interval-valued Pythagorean fuzzy graphs are derived.

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