



Mathematical Model of Turbulent Boundary Layer (TBL) Flow Characteristics with Suspended Particulate Matter (SPM)

Pradeep Kumar Tripathy^{1*}, Ashok Kumar Das² and Saroj Kumar Mishra³

¹Lecturer (Mathematics), U.G.M.I.T., Rayagada, Odisha 765001, INDIA.

²Sr. Lecturer (Mathematics), U.C.P.E.S., Berhampur, Odisha 760010, INDIA.

³Reader Retd. (Mathematics), R.C.M.S.C., Ganjam, Odisha, INDIA.

Abstract: A dynamical mathematical model for studying turbulent boundary layer (TBL) characteristics of a flow embedded with suspended particulate matter (SPM) has been developed. The model prediction envisages the interdependence of SPM and mixing length. Crank-Nicholson implicit finite difference scheme has been employed for model validation. It has been observed that the present model is in good agreement with the experimental result of the 1/7 th power law for fluid velocity.

Keywords: Turbulent Flow, Suspended particulate matter, Flat Plate, Crank-Nicholson Method.

I. Introduction

The dynamical system with suspended particulate matter (SPM) developed by Kliegel(1961), Soo(1961) excluded turbulence whereas the model studied by Rose(1957),Soo(1960),Wen et al(1959) is empirical. Soo(1961) has studied the fully developed turbulent flow by extending the 1/7 th power law of Schlichting(1968)

to the case of gas solid suspension postulating $u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{1}{\tau_p} (v_p - v) > 1$,

$u_p = (u_{p0} - u_{pw}) \left(\frac{y}{R} \right)^{1/m} + u_{pw}$ respectively as density distribution and mean velocity distribution of solid

particles with 'm' and 'α' (constants) to be determined. Assuming the density and velocity profiles postulated by Soo(1961) and incorporating Prandtl's mixing length theory and Bousinesq approximation for closing the system, the integration is performed in the down stream direction.

II. Mathematical Formulation

A realistic theory of fluid model with SPM must account for (i) the solid-fluid mixture is heterogeneous (ii) the average velocity of the particles near the wall is not zero and the excess of particle velocity over the fluid velocity is positive near the wall (iii) the concentration of the solid particles is not uniform because the solid particles have greater density than fluid. Extending the dynamics of a single particle as in case of molecular theory modifying the Basically there have been two approaches to the problem of fluid-solid flow, viz. continuum approach to account for the particles in the fluid phase. In the second approach, the dispersed solid phase is viewed as a continuum despite the discreteness of the particles. Following the second approach, various authors Soo(1961), Marble(1963,70), Saffman(1962), Batchelor(1977), Hinze(1959) have presented modified forms of N.S.equations for the fluid and particle phases.

In case of fluid velocity ranging from 50 to 100ft per second, diameter of solid particles ranging from 100 to 200 microns(for glass beads) and solid to air mass ratios of 0.05 to 0.15, compressibility of fluid phase and the effect of gravity on the density distribution of solid particles are negligible and velocity distribution of fluid stream is not significantly affected by solid particles.

The equation of continuity for the pseudo fluid of SPM can be written as

$$\frac{\partial}{\partial x} (\phi \rho_p u_p) + \frac{\partial}{\partial y} (\phi \rho_p v_p) = 0$$

$$\text{or } \frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0$$

since ρ_p is constant, the equation of continuity reduces to

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0$$

The diffusion of the species of pseudo fluid of SPM will be governed by law of conservation of species and the Fick's Law can be used to determine the species diffusion rate at any point in the medium and the resulting equation could be solved for species concentration distribution. The species conservation equation is given by

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D_p \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)$$

where $c = \frac{\rho_p}{\rho_m}$, ρ_p -density of particle phase, ρ_m -density of the mixture of fluid and particles

$$\text{so } \rho_m = \rho_p + \rho \left(1 - \frac{\rho_p}{\rho_s} \right)$$

where $\frac{\rho_p}{\rho_s} = \phi$ is the volume fraction.

The governing equations for turbulent boundary layer over a flat plate can be written as in deriving the Reynolds equations the following facts have been taken in to consideration.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\rho_p}{\rho \tau_p} (u_p - u) + \frac{\partial}{\partial y} \left(-\overline{u'v'} \right) \quad (2)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{1}{\tau_p} (u_p - u) + \frac{\partial}{\partial y} \left(-\overline{u'_p v'_p} \right) \quad (3)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = -\frac{1}{\tau_p} (v_p - v) + \frac{\partial}{\partial y} \left(-\overline{v_p'^2} \right) \quad (4)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{\partial}{\partial y} \left(D_p \frac{\partial c}{\partial y} - \overline{v'c'} \right) \quad (5)$$

The terms $(u'_p - u')$ and $(v'_p - v')$ are negligible in comparison with $(\overline{u'_p} - \overline{u})$ and $(\overline{v'_p} - \overline{v})$.

Since by the time the flow becomes turbulent the SPM have had enough time to cover many times of the particle velocity relaxation length so that the slip velocity $(u_p - u)$ and $(v_p - v)$ become very small and the perturbation quantities u', u'_p, v', v'_p are such that we can assume

$$u'_p = o(u'), v'_p = o(v')$$

As has been done in deriving boundary layer equations, where

$$\frac{\partial}{\partial x} \approx O\left(\frac{1}{L}\right) \ll \frac{\partial}{\partial y} \approx O\left(\frac{1}{\delta}\right)$$

The term $\frac{\partial}{\partial x} (\overline{v'^2} - \overline{u'^2})$ can also be neglected.

$$\text{and } -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial y} \approx O(\delta)$$

$$\frac{1}{\rho} \frac{\partial \overline{p}}{\partial y} \approx \frac{\partial}{\partial y} \left(-\overline{v'^2} \right) \Rightarrow \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x} \approx \frac{\partial}{\partial x} \left(-\overline{v'^2} \right)$$

If we assume that the SPM does not affect the mixing length as the particle concentration is very lean, terms

$$\frac{\partial}{\partial y} \left(-\overline{u_p' v_p'} \right) \text{ and } \frac{\partial}{\partial y} \left(-\overline{v_p'^2} \right) \text{ can be neglected.}$$

Using Prandtl's mixing length theory the Reynolds stress $-\overline{u'v'}$ is replaced by $l^2 \left(\frac{\partial u}{\partial y} \right)^2$, where 'l' is known

as the mixing length which is the distance up to which migration of fluid elements is assumed to take place.

By analogy with Fick's Law of diffusion, we write

$$\overline{v'c'} = -K_c \frac{\partial c}{\partial y}$$

in which K_c is the turbulent diffusivity or 'eddy diffusivity'.

After making the above mentioned replacements for the turbulent momentum flux and turbulent mass flux, the equations (1)-(5) reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\rho_p}{\rho \tau_p} (u_p - u) + \frac{\partial}{\partial y} \left(l^2 \left(\frac{\partial u}{\partial y} \right)^2 \right) \quad (7)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{1}{\tau_p} (u_p - u) \quad (8)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = -\frac{1}{\tau_p} (v_p - v) \quad (9)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{\partial}{\partial y} \left(D_p \frac{\partial c}{\partial y} + K_c \frac{\partial c}{\partial y} \right) = (D_p + K_c) \frac{\partial^2 c}{\partial y^2} \quad (10)$$

Now, introducing non dimensional variables

$$x^* = \frac{x}{\lambda}, y^* = \frac{y}{(\nu \tau_p)^{1/2}}, \lambda = \tau_p U$$

$$u^* = \frac{u}{U}, v^* = v \left(\frac{\tau_p}{\nu} \right)^{1/2}, u_p^* = \frac{u_p}{U}, v_p^* = v_p \left(\frac{\tau_p}{\nu} \right)^{1/2}, p^* = \frac{p}{\rho U^2}, \rho_p^* = \frac{\rho_p}{\rho_{p\infty}}$$

the equations (6)-(10) can be written as (after dropping the stars)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (11)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \alpha \rho_p (u_p - u) + \frac{\partial^2 u}{\partial y^2} + \text{Re} l^2 \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^2 \quad (12)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -(u_p - u) \quad (13)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = -(v_p - v) \quad (14)$$

$$u \frac{\partial \rho_p}{\partial x} + v \frac{\partial \rho_p}{\partial y} = \frac{(D_p + K_c)}{U\lambda} \text{Re} \frac{\partial^2 \rho_p}{\partial y^2} \quad (15)$$

The boundary conditions are

$$u(x, 0) = 0, u_p(x, 0) = 0$$

$$v(x, 0) = 0, v_p(x, 0) = 0$$

$$u(x, \infty) = 1, u_p(x, \infty) = 1$$

$$\rho_p(x, \infty) = 1$$

The compatibility conditions are

$$\left[\frac{\partial u_p}{\partial x} \right]_{y=0} = 0, \left[\frac{\partial \rho_p}{\partial x} \right]_{y=0} = 0$$

III. Solutions

Using Crank-Nicholson finite difference scheme, the governing equations (11)-(15) are replaced by the difference equations as given below

Eqn.-(11)

$$v_{m+\frac{1}{2},n} = v_{m+\frac{1}{2},n-1} - \frac{\Delta y}{2\Delta x} (u_{m+1,n-1} - u_{m,n-1} + u_{m+1,n} - u_{m,n}) \quad (16)$$

Eqn.-(12)

$$A_n u_{m+1,n-1} + B_n u_{m+1,n} + C_n u_{m+1,n+1} = D_n \quad (17)$$

where

$$\begin{aligned} A_n &= \frac{-v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{\left(1 + \frac{\text{Re}.l^2}{2\Delta y} (u_{m+1,n+1} - u_{m+1,n-1} + u_{m,n+1} - u_{m,n+1})\right)}{2(\Delta y)^2} \\ B_n &= \frac{u_{m+1,n}}{2\Delta x} + \frac{\left(1 + \frac{\text{Re}.l^2}{2\Delta y} (u_{m+1,n+1} - u_{m+1,n-1} + u_{m,n+1} - u_{m,n+1})\right)}{(\Delta y)^2} + \frac{\alpha}{4} (\rho_{p_{m+1,n}} + \rho_{p_{m,n}}) \\ C_n &= \frac{v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{\left(1 + \frac{\text{Re}.l^2}{2\Delta y} (u_{m+1,n+1} - u_{m+1,n-1} + u_{m,n+1} - u_{m,n+1})\right)}{2(\Delta y)^2} \\ D_n &= \frac{(u_{m+1,n})^2}{2\Delta x} - \frac{v_{m+\frac{1}{2},n} (u_{m,n+1} - u_{m,n-1})}{4\Delta y} + \\ &\quad \frac{\left(1 + \frac{\text{Re}.l^2}{2\Delta y} (u_{m+1,n+1} - u_{m+1,n-1} + u_{m,n+1} - u_{m,n+1})\right) (u_{m,n+1} - 2u_{m,n} + u_{m,n-1})}{2(\Delta y)^2} + \\ &\quad \frac{\alpha}{4} (\rho_{p_{m+1,n}} + \rho_{p_{m,n}}) (u_{p_{m,n}} + u_{p_{m+1,n}} - u_{m,n}) \end{aligned}$$

Eqn.-13

$$A_n^* u_{p_{m+1,n-1}} + B_n^* u_{p_{m+1,n}} + C_n^* u_{p_{m+1,n+1}} = D_n^* \quad (18)$$

where

$$\begin{aligned}
 A_n^* &= \frac{-(v_{p_{m+1},n} + v_{p_{m,n}})}{8\Delta y} \\
 B_n^* &= \frac{u_{p_{m+1},n}}{2\Delta x} + 0.5 \\
 C_n^* &= \frac{(v_{p_{m+1},n} + v_{p_{m,n}})}{8\Delta y} \\
 D_n^* &= \frac{(u_{p_{m,n}})^2}{2\Delta x} - \frac{(v_{p_{m+1},n} + v_{p_{m,n}})(u_{p_{m,n+1}} - u_{p_{m,n-1}})}{8\Delta y} + \frac{1}{2}(u_{m+1,n} + u_{m,n} - u_{p_{m,n}})
 \end{aligned}$$

Eqn.-14

$$A_n^{**} v_{p_{m+1},n-1} + B_n^{**} v_{p_{m+1},n} + C_n^{**} v_{p_{m+1},n+1} = D_n^{**} \quad (19)$$

where

$$\begin{aligned}
 A_n^{**} &= \frac{-(v_{p_{m+1},n} + v_{p_{m,n}})}{8\Delta y} \\
 B_n^{**} &= \frac{u_{p_{m+1},n} + u_{p_{m,n}}}{2\Delta x} + 0.5 \\
 C_n^{**} &= \frac{(v_{p_{m+1},n} + v_{p_{m,n}})}{8\Delta y} \\
 D_n^{**} &= \frac{(u_{p_{m+1},n} + u_{p_{m,n}})v_{p_{m,n}}}{2\Delta x} - \frac{v_{p_{m,n}}}{2} + v_{m+\frac{1}{2},n} - \frac{(v_{p_{m+1},n} + v_{p_{m,n}})(v_{p_{m,n+1}} - v_{p_{m,n-1}})}{8\Delta y}
 \end{aligned}$$

Eqn.-15

$$A_n^{***} \rho_{p_{m+1},n-1} + B_n^{***} \rho_{p_{m+1},n} + C_n^{***} \rho_{p_{m+1},n+1} = D_n^{***} \quad (20)$$

where

$$\begin{aligned}
 A_n^{***} &= \frac{-v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{(D_p + K_c)}{U\lambda} \frac{\text{Re}}{2(\Delta y)^2} \\
 B_n^{***} &= \frac{u_{m+1,n} + u_{m,n}}{2\Delta x} + \frac{(D_p + K_c)}{U\lambda} \frac{\text{Re}}{(\Delta y)^2} \\
 C_n^{***} &= \frac{v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{(D_p + K_c)}{U\lambda} \frac{\text{Re}}{2(\Delta y)^2} \\
 D_n^{***} &= \frac{(u_{m+1,n} + u_{m,n})\rho_{p_{m,n}}}{2\Delta x} - \frac{(\rho_{p_{m,n+1}} - \rho_{p_{m,n-1}})v_{m+\frac{1}{2},n}}{4\Delta y} + \frac{(D_p + K_c)}{U\lambda} \frac{\text{Re}}{2(\Delta y)^2} (\rho_{p_{m,n+1}} - 2\rho_{p_{m,n}} + \rho_{p_{m,n-1}})
 \end{aligned}$$

However, if we assume that the SPM does affect the mixing length, the 2nd terms of right hand side of equation

(3) and (4) i.e. $\frac{\partial}{\partial y}(-u'_p v'_p)$, $\frac{\partial}{\partial y}(-v_p'^2)$ can not be neglected.

Boussnesq, in 1877, had suggested that the turbulent stress in analogy to viscous stress can be related to the rate of strain, in case of incompressible flow, as

$$-\overline{u'_i u'_j} = K_m \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right)$$

where K_m is the eddy diffusivity for momentum transfer for fluid phase. Boussnesq approximations for particle phase may be stated as follows

$$-\overline{u'_{p_i} u'_{p_j}} = K_{mp} \left(\frac{\partial \overline{u_{p_i}}}{\partial x_j} + \frac{\partial \overline{u_{p_j}}}{\partial x_i} \right)$$

where K_{mp} is the eddy diffusivity for momentum transfer for particle phase

$$\frac{\partial \overline{c_i}}{\partial t} + \overline{u} \frac{\partial \overline{c_i}}{\partial x} + \overline{v} \frac{\partial \overline{c_i}}{\partial y} + \overline{w} \frac{\partial \overline{c_i}}{\partial z} = \frac{\partial}{\partial x} \left(K_x \frac{\partial \overline{c_i}}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial \overline{c_i}}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial \overline{c_i}}{\partial z} \right) + \overline{R_i} + S_i$$

where K_y, K_z designate horizontal and vertical components of turbulent diffusivity.

This equation, or its minor variants, are often referred to as the atmospheric dispersion equation, or K-theory. This is an approximate, semi-empirical expression. The equation can be simplified for certain cases of practical interest. For steady unidirectional flow in combination with steady pollutant emission rates, the atmospheric dispersion equation reduces to

$$\overline{u} \frac{\partial \overline{c}}{\partial x} = \frac{\partial}{\partial y} \left(K_y \frac{\partial \overline{c}}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial \overline{c}}{\partial z} \right)$$

some solutions are available for the case in which $K_y = K_z = \text{constant}$.

The governing equations then become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (21)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\rho_p}{\rho \tau_p} (u_p - u) + K_m \left(\frac{\partial^2 u}{\partial y^2} \right) \quad (22)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{1}{\tau_p} (u_p - u) + K_{mp} \frac{\partial^2 u_p}{\partial y^2} \quad (23)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = -\frac{1}{\tau_p} (v_p - v) + 2K_{mp} \frac{\partial^2 v_p}{\partial y^2} \quad (24)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{(D_p + K_c)}{U \lambda} \text{Re} \frac{\partial^2 c}{\partial y^2} \quad (25)$$

Introducing non dimensional variables, as earlier, the equations can be written, after dropping the stars, as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (26)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + \varepsilon_m \text{Re}) \frac{\partial^2 u}{\partial y^2} + \alpha \rho_p (u_p - u) \quad (27)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -(u_p - u) + (\varepsilon_{mp} \text{Re}) \frac{\partial^2 u_p}{\partial y^2} \quad (28)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = -(v_p - v) + 2\varepsilon_{mp} \text{Re} \frac{\partial^2 v_p}{\partial y^2} \quad (29)$$

$$u \frac{\partial \rho_p}{\partial x} + v \frac{\partial \rho_p}{\partial y} = \varepsilon'_c \text{Re} \frac{\partial^2 \rho_p}{\partial y^2} \quad (30)$$

$$\text{where } \varepsilon_m = \frac{K_m}{U\lambda}, \varepsilon_{mp} = \frac{K_{mp}}{U\lambda}, \varepsilon'_c = \frac{(D_p + K_c)}{U\lambda}$$

using the Crank-Nicholson scheme for the above equations
eqn.-27

$$A_n u_{m+1,n-1} + B_n u_{m+1,n} + C_n u_{m+1,n+1} = D_n \quad (31)$$

where

$$A_n = \frac{-v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{(1 + \varepsilon_m \text{Re})}{2(\Delta y)^2}$$

$$B_n = \frac{u_{m+1,n}}{2\Delta x} + \frac{(1 + \varepsilon_m \text{Re})}{(\Delta y)^2} + \frac{\alpha}{4} (\rho_{p_{m+1,n}} + \rho_{p_{m,n}})$$

$$C_n = \frac{v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{(1 + \varepsilon_m \text{Re})}{2(\Delta y)^2}$$

$$D_n = \frac{(u_{m+1,n})^2}{2\Delta x} - \frac{v_{m+\frac{1}{2},n}(u_{m,n+1} - u_{m,n-1})}{4\Delta y} + \frac{(1 + \varepsilon_m \text{Re})(u_{m,n+1} - 2u_{m,n} + u_{m,n-1})}{2(\Delta y)^2} +$$

$$\frac{\alpha}{4} (\rho_{p_{m+1,n}} + \rho_{p_{m,n}}) (u_{p_{m,n}} + u_{p_{m+1,n}} - u_{m,n})$$

eqn.-28

$$A_n^* u_{p_{m+1,n-1}} + B_n^* u_{p_{m+1,n}} + C_n^* u_{p_{m+1,n+1}} = D_n^* \quad (32)$$

where

$$A_n^* = \frac{-(v_{p_{m+1,n}} + v_{p_{m,n}})}{8\Delta y} - \frac{(\varepsilon_{mp} \text{Re})}{2(\Delta y)^2}$$

$$B_n^* = \frac{u_{p_{m+1,n}}}{2\Delta x} + 0.5 + \frac{(\varepsilon_{mp} \text{Re})}{(\Delta y)^2}$$

$$C_n^* = \frac{(v_{p_{m+1,n}} + v_{p_{m,n}})}{8\Delta y} - \frac{(\varepsilon_{mp} \text{Re})}{2(\Delta y)^2}$$

$$D_n^* = \frac{(u_{p_{m,n}})^2}{2\Delta x} - \frac{(v_{p_{m+1,n}} + v_{p_{m,n}})(u_{p_{m,n+1}} - u_{p_{m,n-1}})}{8\Delta y} + \frac{1}{2} (u_{m+1,n} + u_{m,n} - u_{p_{m,n}}) +$$

$$\frac{(\varepsilon_{mp} \text{Re})}{2(\Delta y)^2} (u_{p_{m,n+1}} - 2u_{p_{m,n}} + u_{p_{m,n-1}})$$

eqn.-29

$$A_n^{**} v_{p_{m+1,n-1}} + B_n^{**} v_{p_{m+1,n}} + C_n^{**} v_{p_{m+1,n+1}} = D_n^{**} \quad (33)$$

where

$$\begin{aligned}
 A_n^{**} &= \frac{-(v_{p_{m+1,n}} + v_{p_{m,n}})}{8\Delta y} - \frac{(2\varepsilon_{mp} \text{Re})}{2(\Delta y)^2} \\
 B_n^{**} &= \frac{u_{p_{m+1,n}} + u_{p_{m,n}}}{2\Delta x} + 0.5 + \frac{(2\varepsilon_{mp} \text{Re})}{(\Delta y)^2} \\
 C_n^{**} &= \frac{(v_{p_{m+1,n}} + v_{p_{m,n}})}{8\Delta y} - \frac{(2\varepsilon_{mp} \text{Re})}{2(\Delta y)^2} \\
 D_n^{**} &= \frac{(u_{p_{m+1,n}} + u_{p_{m,n}})v_{p_{m,n}}}{2\Delta x} - \frac{v_{p_{m,n}}}{2} + v_{m+\frac{1}{2},n} - \frac{(v_{p_{m+1,n}} + v_{p_{m,n}})(v_{p_{m,n+1}} - v_{p_{m,n-1}})}{8\Delta y} \\
 &+ \frac{(2\varepsilon_{mp} \text{Re})}{2(\Delta y)^2} (v_{p_{m,n+1}} - 2v_{p_{m,n}} + v_{p_{m,n-1}})
 \end{aligned}$$

IV. Discussion of results

The numerical results obtained suggest some of the characteristic features of gas particulate flow in the boundary layer. The values used for numerical computations are presented in Table-1 which have been modified from the data used by to suit our problem.

Fig. 1 depicts the velocity profile of carrier fluid which is numerically computed and drawn against the familiar 1/7th power law-velocity profile. The computed profile does differ from the 1/7th power law profile due to the presence of SPM which affects the mixing length.

Fig.2 presents the velocity profile of SPM drawn against the mean velocity distribution of solid particles approximated by an equation analogous to the 1/7th velocity law of fluid as discussed in the introduction chapter. The computed results show that the particle velocity increases monotonically from the value at the wall (0.205-for glass particles) to its asymptotic value unity at the outer edge of the boundary layer.

Fig.-3 shows the density profile of SPM against that as postulated and used by other authors. The computed density profile agrees well with the postulate. The density profile as postulated decreases from its value at the wall (1.2-for glass particles) monotonically to unity outside the boundary layer whereas in the present result the density profile first decreases from its value at the wall and then increases to unity.

Fig.4 presents the velocity profile of carrier fluid in presence of SPM which do not affect the mixing length. The Reynolds stress terms appearing in the governing equations have been approximated by quantities as predicted by Bousennessque. It is observed that when SPM do not affect the mixing length, the velocity profile of fluid deviates slightly from the one which have been computed using mixing length theory (Fig.1).

Fig.5 makes a comparison of the velocity distribution of SPM and that of the postulated distribution. In this case also, the velocity distribution differs from the one that have been computed using mixing length theory.

Fig.6 depicts the density profile of SPM which when compared with that obtained in the 1st approach (Fig.3) resembles well. No significant change is observed in the density profile in the two approaches.

V. Conclusions

A comparative study of the results obtained in both the approaches suggests that the first where mixing length theory has been used gives a better structure of the boundary layer flow. The deviations marked in the computed results from the experimental results might have been caused by the following factors:

- Replacement of derivatives by finite differences
- Boundary layer assumptions
- Postulation of velocity and density distribution from experimental results
- Assumptions made to close the system

The shortcomings of the present result may be overcome by considering higher order closure systems.

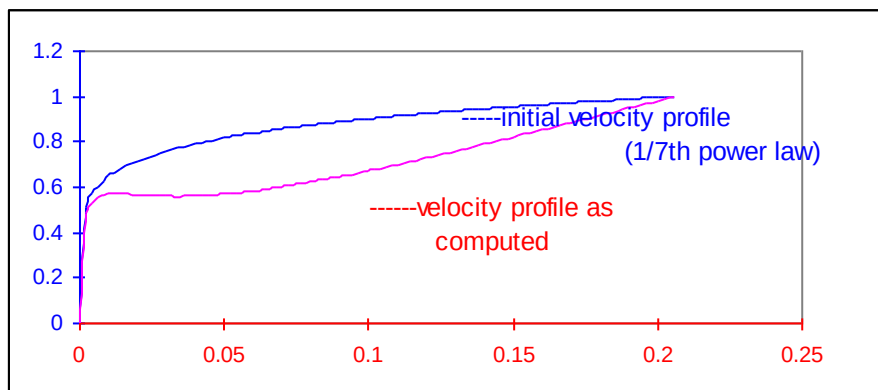


Figure 1: Velocity Profile-'U'

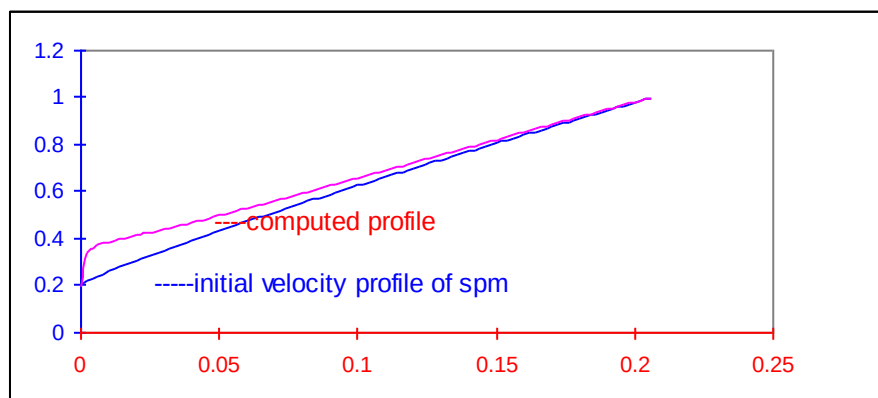


Figure 2: Velocity Profile-'Up'

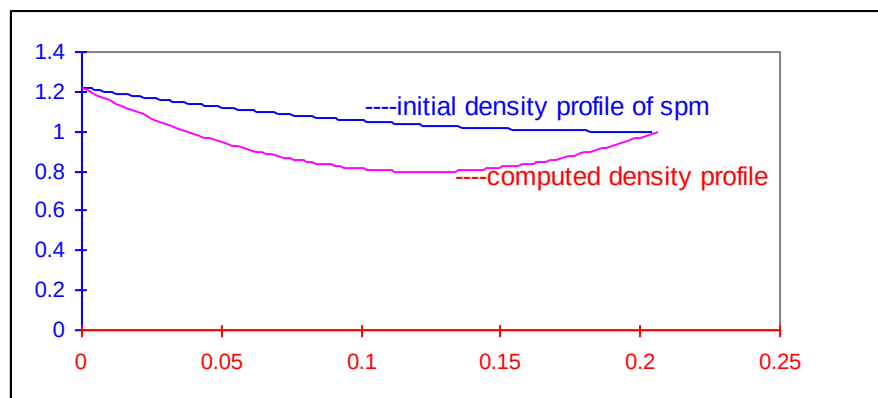


Figure 3: Density Profile

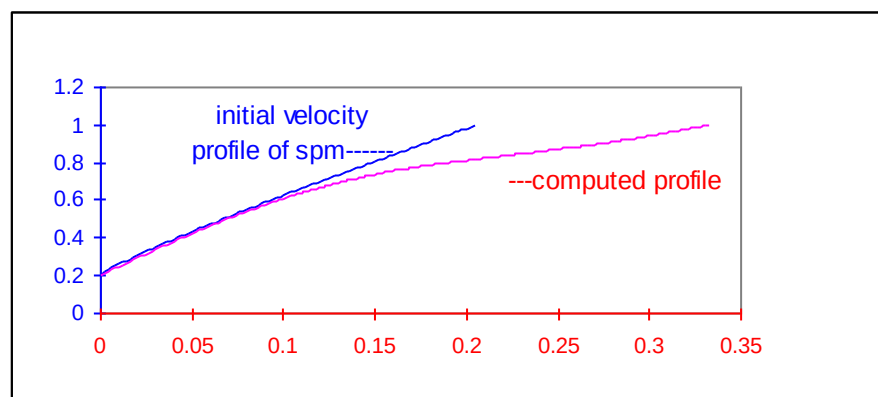


Figure 4: Velocity profile of fluid

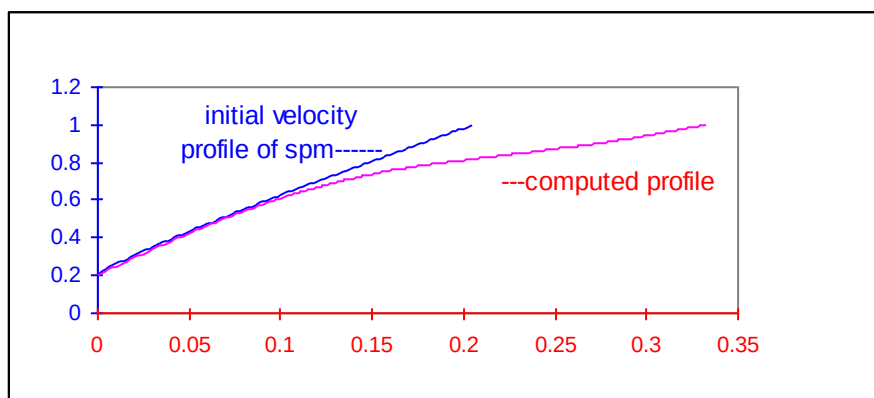


Figure 5: Velocity profile of SPM

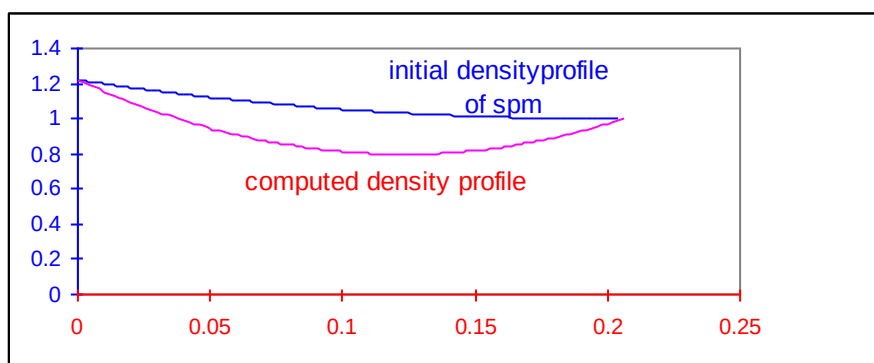


Figure 6: Density profile of SPM

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Table-1(a)

α	d m	ρ_s Kg m^{-3}	a micron	U ms $^{-1}$	$\frac{K_m}{Ud}$	$\frac{K_{mp}}{Ud}$	$\frac{K_c}{Ud}$
0.0141	0.003175	801	5.00E-05	12.68	0.0481	0.489	0.632
0.0141	0.003175	1602	5.00E-05	12.68	0.0481	0.489	0.632
0.0141	0.003175	2403	5.00E-05	12.68	0.0481	0.489	0.632
0.0141	0.003175	8010	5.00E-05	12.68	0.0481	0.489	0.632
0.00657	0.003175	801	5.00E-05	18.07	0.0481	0.469	0.675
0.00657	0.003175	1602	5.00E-05	18.07	0.0481	0.469	0.675

0.00657	0.003175	2403	5.00E-05	18.07	0.0481	0.469	0.675
0.00657	0.003175	8010	5.00E-05	18.07	0.0481	0.469	0.675
0.0157	0.003175	801	5.00E-05	18.26	0.0443	0.551	0.686
0.0157	0.003175	1602	5.00E-05	18.26	0.0443	0.551	0.686
0.0157	0.003175	2403	5.00E-05	18.26	0.0443	0.551	0.686
0.0157	0.003175	8010	5.00E-05	18.26	0.0443	0.551	0.686
0.0146	0.00635	801	5.00E-05	27	0.0418	0.608	0.804
0.0146	0.00635	1602	5.00E-05	27	0.0418	0.608	0.804
0.0146	0.00635	2403	5.00E-05	27	0.0418	0.608	0.804
0.0146	0.00635	8010	5.00E-05	27	0.0418	0.608	0.804

Where,

$$\lambda = \frac{2}{9} \left(\frac{\rho_s}{\rho} \right) \frac{a^2 U}{\mu}$$

$$\varepsilon_m = \frac{K_m d}{U \lambda}$$

$$\varepsilon_{mp} = \frac{K_{mp}}{K_m} \frac{K_m}{U \lambda} = \frac{K_{mp}}{K_m} \varepsilon_m$$

$$\varepsilon_c = \frac{K_c}{K_m} \varepsilon_m$$

Table-1(b)

ρ Kg m^{-3}	μ Kg m^{-3}	ρ_s Kg m^{-3}	a m	U ms^{-1}	λ m	$\frac{K_m}{U \lambda} = \varepsilon_m$	$\frac{K_{mp}}{U \lambda} = \varepsilon_{mp}$	$\frac{K_c}{U \lambda} = \varepsilon_{cp}$
0.9752	1.54E-05	801	1.00E-04	12.68	1.46E+00	1.04E-04	5.10E-05	6.59E-05
0.9752	1.54E-05	1602	1.00E-04	12.68	2.93E+00	5.22E-05	2.55E-05	3.30E-05
0.9752	1.54E-05	2403	1.00E-04	12.68	4.39E+00	3.48E-05	1.70E-05	2.20E-05
0.9752	1.54E-05	8010	1.00E-04	12.68	1.46E+01	1.04E-05	5.10E-06	6.59E-06
0.9752	1.54E-05	801	1.00E-04	18.07	2.09E+00	7.32E-05	3.43E-05	4.94E-05
0.9752	1.54E-05	1602	1.00E-04	18.07	4.17E+00	3.66E-05	1.72E-05	2.47E-05
0.9752	1.54E-05	2403	1.00E-04	18.07	6.26E+00	2.44E-05	1.14E-05	1.65E-05
0.9752	1.54E-05	8010	1.00E-04	18.07	2.09E+01	7.32E-06	3.43E-06	4.94E-06
0.9752	1.54E-05	801	1.00E-04	18.26	2.11E+00	6.67E-05	3.68E-05	4.58E-05
0.9752	1.54E-05	1602	1.00E-04	18.26	4.22E+00	3.34E-05	1.84E-05	2.29E-05
0.9752	1.54E-05	2403	1.00E-04	18.26	6.33E+00	2.22E-05	1.23E-05	1.53E-05
0.9752	1.54E-05	8010	1.00E-04	18.26	2.11E+01	6.67E-06	3.68E-06	4.58E-06
0.9752	1.54E-05	801	1.00E-04	27	3.12E+00	8.51E-05	5.18E-05	6.85E-05
0.9752	1.54E-05	1602	1.00E-04	27	6.24E+00	4.26E-05	2.59E-05	3.42E-05
0.9752	1.54E-05	2403	1.00E-04	27	9.35E+00	2.84E-05	1.73E-05	2.28E-05
0.9752	1.54E-05	8010	1.00E-04	27	3.12E+01	8.51E-06	5.18E-06	6.85E-06