



Evaluation of Stress Concentration Factor for Shoulder Fillet on Round Bar under Axial Tension Loading

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Abstract: The aim of this work is to validate the various equations given by researchers for stress concentration factor for shoulder fillet on round bar under axial tension loading conditions. ANSYS workbench was used for finite element analysis of the shoulder filleted round bar. Stress concentration factor is determined based on the equivalent (Von Mises) stresses and the nominal stresses obtained from the finite element of analysis. At last, comparisons of computation results of SCFs obtained by FEA analysis and theoretical results are made in the tabular form and percentage error is estimated. Theoretical SCFs based on modified Pilkey's equations, modified Roark's equations and S. M. Tipton equations and SCFs obtained from FEA results have very good agreement. Graphs are plotted for better understanding of comparison of theoretical results and FEA results and can be useful for design engineers and industrial personnel in design or research.

Keywords: Axial tension; finite element analysis; stress concentration factor; shoulder fillet; equivalent (Von Mises) stress.

I. Introduction

Due to the wide spread use of plates and round bars (or shafts) in various mechanical and other related industries, these elements are most important for the study. Among these, round bars are widely used for making various shafts. These shafts are most vital components for motion and power transmission purpose in all the mechanical system and machineries. These shafts are either made of continuous or it may be of steps. These round bars are designed for various loading conditions for different applications. Owing to its variety of applications, different types of geometric discontinuities with different sizes and shapes are required to perform the specific function. For examples, keyways are required to align the shafts; steps are provided to couple the bearings and pulleys. Discontinuities present on round bars are also called as stress raisers which reduced the strength of round bars and are responsible for prime cause for failure. To avoid abrupt change in cross-section of the round bar, the gradual reduction in diameter (steps on round bar) is employed to keep the stress lines uniform and avoid higher stress concentration. Design of round bar for combined loading is most critical problem in practical applications and in most of the practical applications round bar must be designed for combined loadings. Thus the machine designer is often faced with the task of designing shafts [1]. In the shaft where steps are provided, the abrupt change of this cross section was given a fillet for smooth transition of stress flow and to accommodate the bearings with its housing. To achieve optimal design and to identify the failure causes, it is necessary to understand in details, the stress distribution around the shoulder fillet. Therefore in the design of any mechanical system with round bars, to achieve convenient and efficient design, the studies of the effects of geo-metrical irregularities (discontinuities) are very important. These geometric discontinuities will cause significant stress concentrations. Fillets are widely used in mechanical parts to give smooth transition in regions where there is an abrupt change in cross-section like in shoulders. Figure 1 shows shaft with shoulder fillet. As we know that maximum stress is accumulate in this region. This accumulated stress is known as stress concentration and it is measure by factor known as stress concentration factor (SCF). SCF is the ratio of the maximum stress(σ_{max}) to the nominal stress(σ_0).

$$k_t = \frac{\sigma_{max}}{\sigma_0} \quad (1)$$

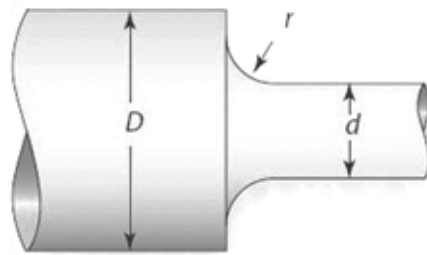


Fig. 1 Geometry of round bar[2]

Figure 1 shows shoulder shaft with fillet radius. In lots of mechanical component design textbooks, the stress concentration factors (SCFs) for the step shaft with different shoulder fillet radius are fully explained and sets of stress concentration factor (SCF) curves were presented by Norton, Shigley and Robert [1-3]. As we know that smaller the fillet radius (r) or larger the D/d ratio, results into high SCF. It can be clearly observed from the Figure 2. Due to mechanical limitations like space limitations or the existence or nearby parts, it is sometimes difficult to take larger fillet radius [5].

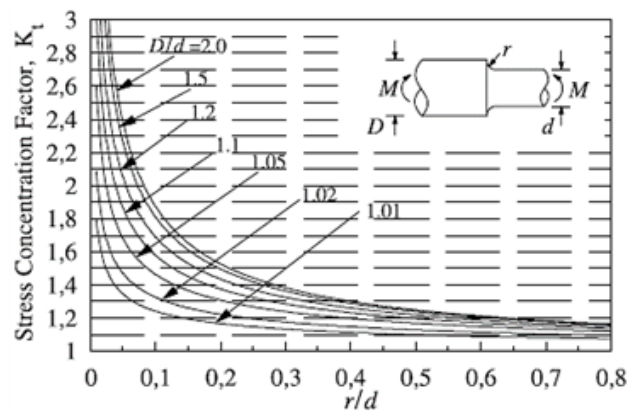


Fig. 2 Stress concentration factor for fillets in round bars [2]

Augusto et al. presented number of techniques to determine the SCF i.e. analytical methods, experimental methods, and computational methods [3]. Here with the use of computational method, stress concentration factor (SCF) which was derived by analytical solutions by Bhavesh Patel et al. [4] are validated. As mentioned in the paper [4], number of researchers has carried out various methodologies to investigate stress concentration factor for axial tension, bending and torsion loading. For axial loading, Pilkey et al., Young et al. and Tipton et al. have given equations for different range of h/r or r/d . All these equations are agreed to the Peterson's results/curves and provided wide range of h/r and r/d and D/d ratios. Various modified equations presented by Bhavesh Patel et al. [4] for axial loading for range from $0.1 \leq h/r \leq 2.0$ and $2.0 \leq h/r \leq 20$ were studied and the results are validated by Finite Element Analysis.

II. Methodology

In the paper of Bhavesh Patel et al. [4] various equations given by Pilkey [5], Roark [6] and Tipton [7] were modified into fillet radius equations for axial loading conditions by simple mathematical simplification for the same range of h/r as mentioned in the Petersons SCF handbook. Also in that paper [4] range of SCF was given for different modified equations. In those equations of axial loading, to get the fillet radius it is essential to put SCF from the range of SCF given by them and that gives an optimum value of SCF. Now in this paper effort was made to validate the results of SCFs which were given by Bhavesh Patel et al. [4] by computational method (Finite Element Analysis). Equivalent (Von Mises) stress has been calculated from the FEA results. Von Mises stress is a stress value used to determine the whether the material will yield or fracture. It mostly used for ductile materials. It states that if the Von Mises stress of a material under load is equal or greater than the yield limit of the same material under tension, then the material will yield. Von Mises theory is based on distortion energy. This theory is more popular due to its approach of treating failure criteria from the energy perspective.

III. Finite Element Analysis

Numerical analysis of shoulder filleted shaft geometries subjected to axial tension were carried out on ANSYS 19.0. Total eight different 3D models of filleted shaft with different D/d ratios were modelled in Solid works 16.0. EN31 was modeled as a linear elastic material in ANSYS. Table 1 shows the material properties of EN31.

Table 1 Material properties of EN31

Property	Value	Unit
Density	7600	Kg/m ³
Isotropic properties		
Young's modulus	207	MPa
Poisson's ratio	0.3	--
Bulk modulus	172.5	GPa
Shear modulus	80	GPa
Tensile Yield strength	460	MPa
Tensile Ultimate strength	560	MPa

The filleted shaft was meshed with tetrahedron elements. The average value of the mesh size is 0.01 mm (fine size), resulting in 105460 nodes and 24461 elements. The meshed shoulder shaft with dimensions of D=40 mm, d=33.33 mm (D/d= 1.20) and fillet radius as 3.34 mm is shown in the figure 3. For loading and boundary condition for axial tension as shown in figure 4, load was applied to the one face of the model and other face was fixed.

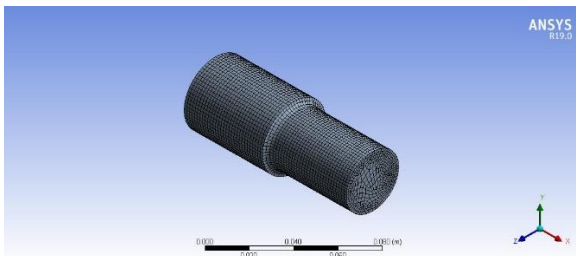


Fig. 3 Meshed model of shoulder shaft

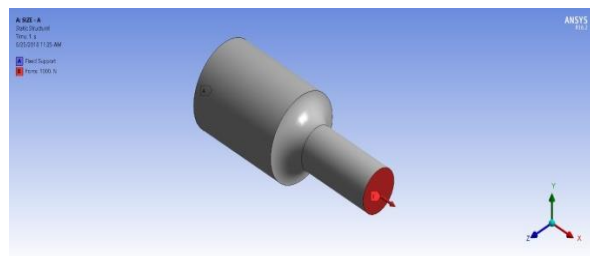


Fig. 4 Loading and boundary conditions

The results of the finite elements analysis simulations using 3D elements are presented and compared for shoulder filleted shaft. Total sixteen different models were prepared and were analyze and simulated. Figure 5 to figure 12 shows various models for axial tension loading. Here for all the variations, we kept load value fixed as 1000 N.

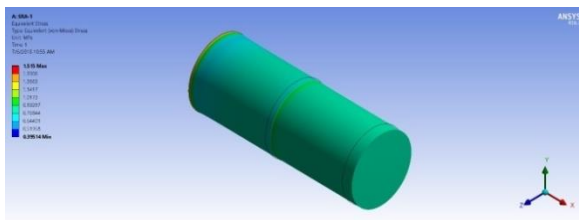


Fig. 5 Equivalent (Von Mises) stress for D/d=1.01 (D= 40 mm and d= 39.60 mm)

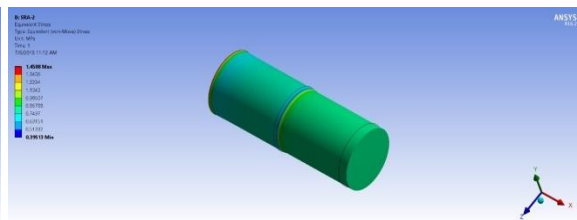


Fig. 6 Equivalent (Von Mises) stress for D/d=1.02 (D= 40 mm and d= 39.21 mm)

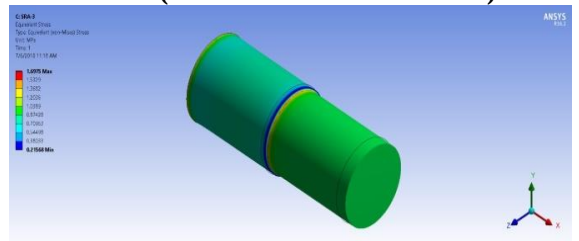


Fig. 7 Equivalent (Von Mises) stress for D/d=1.05 (D= 40 mm and d= 38.095 mm)

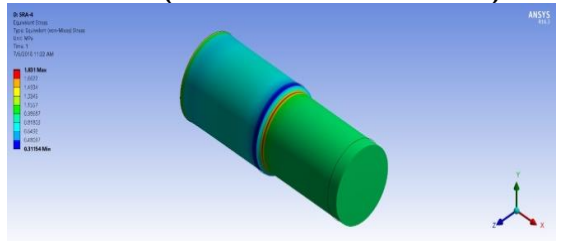


Fig. 8 Equivalent (Von Mises) stress for D/d=1.1 (D= 40 mm and d= 36.36 mm)

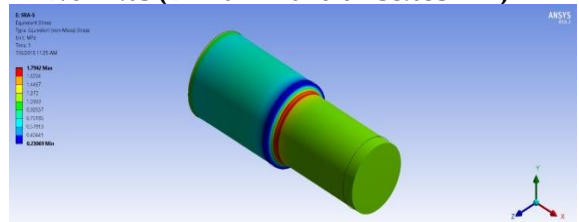


Fig. 9 Equivalent (Von Mises) stress for D/d=1.2 (D= 40 mm and d= 33.33 mm)

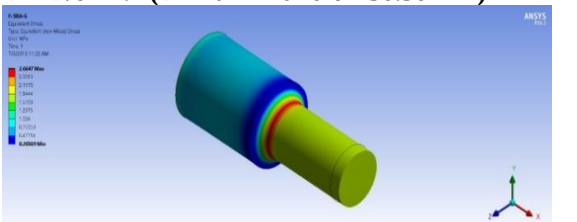


Fig. 10 Equivalent (Von Mises) stress for D/d=1.5 (D= 40 mm and d= 26.67 mm)

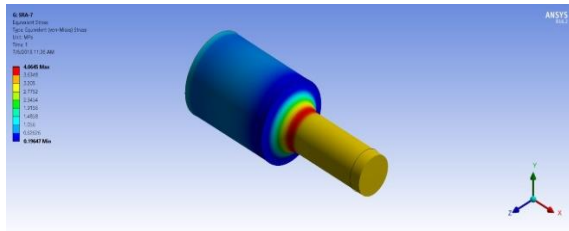


Fig. 11 Equivalent (Von Mises) stress for D/d =2 (D= 40 mm and d= 20 mm)

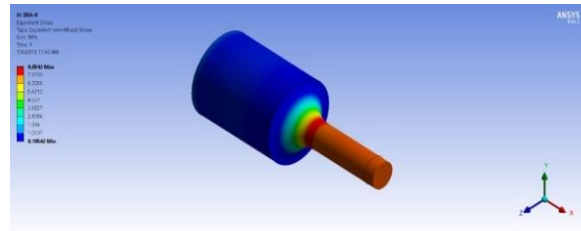


Fig. 12 Equivalent (Von Mises) stress for D/d =3 (D= 40 mm and d= 13.33 mm)

From the above simulated results, equivalent (Von-Mises) stress of shoulder filleted shaft was identified and used it for calculation of stress concentration factor (SCF). Equivalent (Von-Mises) stress, SCF based on Equivalent (Von-Mises) stress and theoretical SCF and their difference in terms of percentage error were presented in the Table 2, Table 3 and Table 4 for all the modified equations presented by Bhavesh Patel *et. al.* [4].

Table 2 Theoretical SCF and its comparison with SCF based on equivalent (Von Mises) stress (as per modified Pilkey's equation [4])

D (mm)	d (mm)	D/d	Fillet height h (mm)	Fillet radius r (mm)	Axial Load P (N)	C/S area A (mm ²)	Nominal stress (MPa)	Equivalent (Von Mises) stress (MPa)	SCF(K) based on equivalent (Von Mises) stress	Theoretical SCF (Kt)	% Error
40	39.6	1.01	0.20	0.20	1000	1231.01	0.812	1.515	1.8650	1.962	4.95
40	39.21	1.02	0.40	0.40	1000	1206.88	0.829	1.4588	1.7606	1.938	9.15
40	38.09	1.05	0.95	0.95	1000	1139.21	0.878	1.6975	1.9338	1.891	2.26
40	36.36	1.10	1.82	1.82	1000	1037.81	0.964	1.831	1.9002	1.811	4.93
40	33.33	1.20	3.34	3.34	1000	872.05	1.147	1.7942	1.5646	1.69	7.42
40	26.67	1.50	6.67	6.67	1000	558.36	1.791	2.6647	1.4879	1.479	0.60
40	20	2.00	10.00	10.00	1000	314.00	3.185	4.0645	1.2763	1.328	3.90
40	13.33	3.00	13.34	13.34	1000	139.49	7.169	8.0542	1.1234	1.213	7.38

Table 3 Theoretical SCF and its comparison with SCF based on equivalent (Von Mises) stress (as per modified Roark's equation [4])

D (mm)	d (mm)	D/d	Fillet height h (mm)	Fillet radius r (mm)	Axial load P (N)	C/S area A (mm ²)	Nominal stress (MPa)	Equivalent (Von Mises) Stress (MPa)	SCF(K) based on equivalent (Von Mises) stress	Theoretical SCF (Kt)	% Error
40	39.6	1.01	0.20	0.20	1000	1231.01	0.812	1.515	1.8650	1.969	5.28
40	39.21	1.02	0.40	0.40	1000	1206.88	0.829	1.4588	1.7606	1.953	9.85
40	38.09	1.05	0.95	0.95	1000	1139.21	0.878	1.6975	1.9338	1.896	1.99
40	36.36	1.10	1.82	1.82	1000	1037.81	0.964	1.831	1.9002	1.817	4.58
40	33.33	1.20	3.34	3.34	1000	872.05	1.147	1.7942	1.5646	1.695	7.69
40	26.67	1.50	6.67	6.67	1000	558.36	1.791	2.6647	1.4879	1.483	0.33
40	20	2.00	10.00	10.00	1000	314.00	3.185	4.0645	1.2763	1.333	4.26
40	13.33	3.00	13.34	13.34	1000	139.49	7.169	8.0542	1.1234	1.217	7.69

Table 4 SCF and its comparison with theoretical SCF based on equivalent (VonMises) stress (as per modified S.M. Tipton equation [4])

D (mm)	d (mm)	D/d	Fillet height h (mm)	Fillet radius r (mm)	Axial load P (N)	C/S area A (mm ²)	Nominal stress (MPa)	Equivalent (Von Mises) stress (MPa)	SCF (K) based on equivalent (Von Mises) stress	Theoretical SCF (K _t)	% Error
40	39.6	1.01	0.20	0.20	1000	1231.01	0.812	1.515	1.8650	2.014	7.40
40	39.21	1.02	0.40	0.40	1000	1206.88	0.829	1.4588	1.7606	1.9974	11.86
40	38.09	1.05	0.95	0.95	1000	1139.21	0.878	1.6975	1.9338	1.9493	0.79
40	36.36	1.10	1.82	1.82	1000	1037.81	0.964	1.831	1.9002	1.87	1.62
40	33.33	1.20	3.34	3.34	1000	872.05	1.147	1.7942	1.5646	1.734	9.77
40	26.67	1.50	6.67	6.67	1000	558.36	1.791	2.6647	1.4879	1.4633	1.68
40	20	2.00	10.00	10.00	1000	314.00	3.185	4.0645	1.2763	1.328	3.90
40	13.33	3.00	13.34	13.34	1000	139.49	7.169	8.0542	1.1234	1.213	7.38

IV. Results and Discussion

There are no such criteria available to select optimum fillet radius for shoulder filleted shaft which will give minimum stress concentration factor. Therefore, an attempt was made to establish a criterion for selection of fillet radius for shoulder fillets which will give minimum SCF. It can reduce the iteration for design of round bar with fillet radius. Here, various modified equations presented by researchers [4] under axial tension loading condition were studied and results are compared with FEA results. ANSYS workbench 19.0 was used for FEA analysis. Equivalent (Von Mises) stresses for all the eight geometries of different D/d ratios were calculated. By using the generalized equation of SCF (i.e. SCF is the ratio of maximum stress to the nominal stress), SCF was calculated. The simulated results for SCF based on equivalent (Von Mises) stress are tabulated in the Table 2 (as per modified Pilkey's equation), Table 3 (as per modified Roark's equation) and Table 4 (as per modified S.M. Tipton equation). The percentage error of theoretical results and FEA results based on equivalent (Von-Mises) stresses are almost nearer. So it is concluded that based on equivalent (Von-Mises) stresses, theoretical results obtained from the modified equations are agreed with FEA results. These tabulated results are also presented in the form of chart.

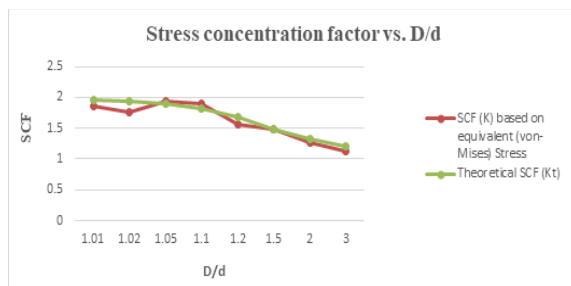


Fig. 13 D/d vs. SCF (From the results of modified Pilkey's equation)

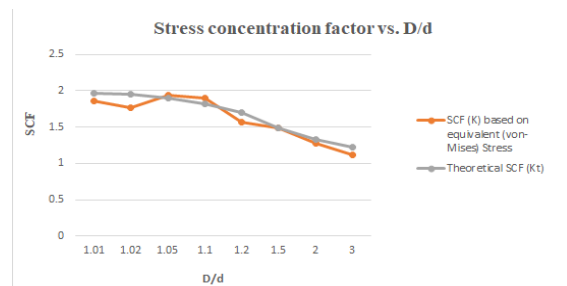


Fig. 14 D/d vs. SCF (From the results of modified Roark's equation)

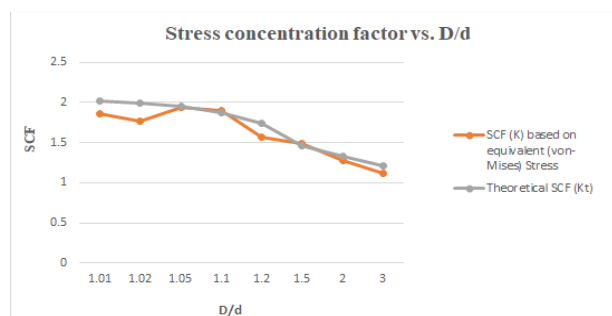


Fig. 15 D/d vs. SCF (From the results of modified S. M. Tipton equation)

From the graphs as shown in the figure 13, figure 14 and figure 15 it is clearly observed that both theoretical and computational results have agreement for all the modified equations.

V. Conclusion

The stress concentration of shoulder filleted round bars (shafts) under axial tension, bending and torsional loading is often encountered. In this presented paper, the modified equations given by Bhavesh Patel et al. [4] for axial tension loading condition were studied and validated by computational technique. The conclusions can be made as follow:

1. Stress concentration factor of shoulder filleted round bar under axial tension loading are analyzed using Finite Element of Analysis (FEA) for modified Pilkey's equations, modified Roark's equations and modified S.M. Tipton equations. They are shown in the table 2-4 for the different geometry variation (D/d ratios) of shoulder filleted shaft.
2. For modified Pilkey's equations, a very good agreement of theoretical SCFs and SCFs obtained by FEA results for all the geometrical variation of D/d ratios. Maximum percentage variation is 9.15 % for the geometry with D/d = 1.02 (see figure 13 and 2nd row of table 2).
3. For modified Roark's equations, a very good match of theoretical SCFs and SCFs obtained by FEA results for all the geometrical variation of D/d ratios. Maximum percentage variation is 9.85 % for the geometry with D/d = 1.02 (see figure 14 and 2nd row of table 3).
4. For modified S. M. Tipton equations, a very good fit of data for theoretical SCFs and SCFs obtained by FEA results for all the geometrical variation of D/d ratios. At D/d = 1.02 the maximum percentage variation is 11.86 % and for the geometry with D/d = 1.2 the percentage variation is 9.77 (see figure 15 and 2nd and 5th row of table 4).
5. At the end, SCFs are provided in a graphical way on the bases of all the modified equations so that they can easy to understand by the design engineers and industrial personnel in design or research.

References

1. Robert L. Norton, "Machine Design: An Integrated Approach", 3rd Edition: Pearson Prentice Hall, Pearson Education. Inc.; New Jersey; 2006.
2. F O Sonmez, "Optimal Shape Design of Shoulder Fillets for Flat and Round Bars under Various Loadings". *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2009; 223: 1741-1754p.
3. A. Ajovalasit, V. Nigrelli, G. Pitarresi, G. Mariotti, "On The History of Torsional Stress Concentrations in Shafts: From Electrical Analogies to Numerical Methods". *The Journal of Strain Analysis for Engineering Design*. 2014; 49: 452-466p.
4. Bhavesh Patel, Hiren Prajapati, "Investigation of Optimum Stress Concentration Factor for Shoulder Fillet in Round Bar under Different Loading Conditions". *Trends in Mechanical Engineering & Technology*. 2018; volume 8, issue 2, 18-26p.
5. W. D. Pilkey, D. F. Pilkey, "Peterson's stress Concentration Factors", 3rd Edition: John Wiley & Sons, Inc., Hoboken, New Jersey; 2008.
6. R. G. Budynas, W. C. Young, A. M. Sadegh, "Roark's Formulas for Stress and Strain. 7th Edition: McGraw-Hill; McGraw-Hill Companies, Inc. USA; 2009.
7. R. Rolovic, J. R. Sorem, S. M. Tipton, "Multi Axial Stress Concentration in Filleted Shafts" *Journal of Mechanical Design*. 2000; 123: 300-303p.

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