



Soret and Chemical Reaction Effects on MHD Free Convection Heat and Mass Transfer Flow Past an Inclined Porous Surface in the Presence of Viscous Dissipation and Heat Source

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Abstract: This paper deals with a steady two-dimensional flow of an electrically conducting incompressible dissipating fluid over an inclined semi-infinite porous surface with heat and mass transfer in presence of viscous dissipation, chemical reaction and heat source. The flow is permeated by a uniform transverse magnetic field. A scaling group of transformation is applied to the governing equations. The coupled ordinary differential equations along with the boundary conditions are solved numerically by using Runge-Kutta method along with shooting technique. The effects of various parameters on velocity, temperature and concentration fields as well as skin-friction, Nusselt number and Sherwood number are presented graphically and tabulated form discussed qualitatively.

Keywords: MHD, radiation, viscous dissipation, chemical reaction, Heat source, inclined porous surface

I. Introduction

Heat and mass transfer analysis with chemical reaction has a great interest to engineers and scientist because of its universal applications in many branches of science and engineering. In many chemical processes, a chemical reaction occurs between a foreign mass and a fluid in which a plate is moving. These processes have numerous applications such as polymer production, manufacturing of ceramics or glassware and food processing. Besides, the magneto hydrodynamic laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. This problem has also an important bearing on metallurgy where magneto hydrodynamic (MHD) techniques have recently been used.

The study of free convection flow for an incompressible viscous fluid past an inclined porous surface has attracted the interest of many researchers in view of its important applications to many engineering problems such as cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth, food processing and cooling towers. Gebhart and Mollendorf [1] considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [2] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Chen [3] studied the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. Duwairi [4], who investigated the effect of viscous and Joule heating on forced convection flow from radiative isothermal surfaces found that the heat transfer rate is decreased as the radiation parameter is increased. Kumari *et al.* [5] investigated the mixed convection flow over a vertical wedge embedded in a porous medium. Singh *et al.* [6], investigated the effect of volumetric heat generation/absorption on mixed convection stagnation point flow on an isothermal vertical plate in porous media. Hayat *et al.* [7] studied heat and mass transfer for Soret and Dufour effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid. Ramana Reddy *et al.* [8] studied the problem of MHD flow over a vertical moving porous plate with heat generation by considering double diffusive convection. Heat transfer problems for boundary layer flow concerning a convective boundary condition was investigated by Aziz [9] for the Blasius flow. Makinde [10] studied the heat and mass transfer over a vertical plate with convective boundary conditions. Ishak [11] studied the steady laminar boundary layer flow and heat transfer over a stationary permeable flat plate immersed in an uniform free stream with convective boundary condition. Kishan *et al.* [2] were investigated the effects of viscous dissipation on MHD flow with heat and mass transfer over a stretching surface with heat source, thermal stratification and chemical reaction. Ishak *et al.* [13] studied the problem of steady laminar boundary layer flow and heat transfer over a moving flat surface in a parallel stream with convective boundary condition. Subhashini *et al.* [14] investigated the simultaneous effects of thermal and concentration diffusions on a mixed convection boundary layer flow over a permeable surface under convective surface boundary condition. Karim *et al.* [15] investigated Dufour and Soret effect on steady MHD

flow in presence of heat generation and magnetic field past an inclined stretching sheet. Very recently Bhavana *et al.* [16] analyzed the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source. Shu and Pop [17] numerically studied the natural convection from inclined wall plumes in a porous medium. Parveen *et al.* [18] have studied the Joule heating effect on magneto hydrodynamic natural convection flow along a vertical wavy surface. G. Singh, P.R. Sharma and A.J. Chamkha, [19] Effect of Volumetric Heat Generation/Absorption on Mixed Convection Stagnation Point Flow on an Isothermal Vertical Plate in Porous Media. T. Hayat, M. Mustafa and I. Pop [20], Heat and Mass Transfer for Soret and Dufour Effect on Mixed Convection Boundary Layer Flow over a Stretching Vertical Surface in a Porous Medium filled with a Visco-elastic Fluid. B. Gebhart and J. Mollendorf [21], Viscous dissipation in external natural convection flows. V.M. Soundalgekar [22], Viscous dissipation effects on unsteady free convective flow past an infinite, vertical porous plate with constant suction. M D Enamul Karim, M D Abdus Samad and M D Maruf Hasan [23], "Dufour and Soret effect on steady MHD flow in presence of Heat generation and magnetic field past an inclined stretching sheet. M. Bhavana, D. Chenna Kesaraiah and A. Sudhakarrah [24], "The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source.

The aim of this paper is to investigate the effects of Soret and Chemical reaction effects on inclined porous plate in presence of variable temperature and concentration. The governing equations of flow field are solved numerically using Runge-Kutta fourth order along shooting method. The effect of different physical parameters on velocity, temperature and concentration are discussed and presented graphically.

II. Mathematical Analysis

Consider the heat and mass transfer of a steady two-dimensional hydro magnetic flow of a viscous, incompressible, electrically conducting and dissipating fluid past a semi-infinite inclined plate with an acute angle α to the vertical. The flow is assumed to be in the x -direction, which is taken along the semi-infinite inclined porous plate and y -axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field and Hall effects are negligible. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term.

The surface is maintained at a constant temperature T_w , which is higher than the constant temperature T of the surrounding fluid and the concentration C_w is greater than the constant concentration C . Then, under the usual Boussinesq's and boundary layer approximations, the governing equations of continuity, momentum, energy and species are:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad - (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\alpha + g\beta^*(C - C_\infty)\cos\alpha - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{K'}u \quad - (2)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{Q_0}{\rho C_p} (T - T_\infty) \quad - (3)$$

Species equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K'_r (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad - (4)$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\left. \begin{aligned} u = v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad - (5)$$

On introducing the following non-dimensional quantities

$$x^* = \frac{x U_\infty}{\nu}, \quad y^* = \frac{y U_\infty}{\nu}, \quad u^* = \frac{u}{U_\infty}, \quad v^* = \frac{v}{U_\infty}, \quad M = \frac{\sigma B_0^2 \nu}{U_\infty^3}, \quad K = \frac{K' U_\infty^3}{\nu^3}, \quad Gr = \frac{v g \beta (T_w - T_\infty)}{U_\infty^3} \quad - (6)$$

$$Gm = \frac{v g \beta^* (C_w - C_\infty)}{U_\infty^3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad Pr = \frac{\nu}{\alpha}, \quad Ec = \frac{U_\infty^2}{C_p (T_w - T_\infty)}, \quad Sc = \frac{\nu}{D}, \quad Kr = \frac{K'_r \nu}{V_0^2}$$

Substituting Eq. (6) into Eqs. (1)-(4) and dropping bars, we obtain,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad - (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \cos \alpha \theta + Gm \cos \alpha \phi - (M + \frac{1}{K})u \quad - (8)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2 - Q_0^* (T - T_\infty) \quad - (9)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi \quad - (10)$$

The corresponding boundary conditions take the form

$$\left. \begin{aligned} u = v = 0, \theta = 1, \phi = 1 \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad - (11)$$

We find the similarity transformations

$$\eta = x^{\frac{1}{4}} y, \psi^* = x^{\frac{3}{4}} f(\eta), \theta^* = \theta(\eta), \phi^* = \phi(\eta) \quad - (12)$$

Substituting these values in equations (08) to (11), we finally obtain the system of non linear ordinary differential equations

$$f''' + \frac{3}{4} f f'' - \frac{1}{2} f'^2 + Gr \theta \cos \alpha + Gm \phi \cos \alpha + (M + \frac{1}{K}) f' = 0 \quad - (13)$$

$$\theta'' + \frac{3}{4} Pr Ec f'^2 = 0 \quad - (14)$$

$$\phi'' + \frac{3}{4} Sc f \phi' - Kr \phi = Sc So \theta'' \quad - (15)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} f = 0, f' = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \\ f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad - (16)$$

III. Numerical Solution

The set of nonlinear ordinary differential equations (13) – (15) with boundary conditions equation (16) have been solved by using the Runge-Kutta method of fourth order along with shooting technique. First of all higher order non-linear differential equations (13) – (15) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique.

IV. Results and Discussion

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various in governing parameters, namely, thermal Grashof number Gr , modified Grashof number Gm , magnetic field parameter M , permeability parameter K , Prandtl number Pr , Eckert number Ec , inclination angles α , Schmidt number Sc , Soret number So , heat source parameter Q_0 , chemical reaction parameter Kr . In the present study following default parameter values are adopted for computations: $Gr = 1.0$, $Gm = 1.0$, $M = 1.0$, $K = 1.0$, $Pr = 0.71$, $Ec = 0.01$, $\alpha = 1.52$, $Sc = 0.22$, $So = 1.0$, $Kr = 1.0$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graphs.

Fig (1) shows the effects of Eckert number Ec on velocity distribution. It is observed that the velocity increases with increasing values of Eckert number Ec . The effect of inclination of the surface for different parameters is depicted in Fig. 2. For fixed values of the all parameter the velocity is decreased with inclination angles as shown in Fig. 2. The fluid has higher velocity when the surface is vertical than when inclined because the buoyancy effect decreases due to gravity components $\cos \alpha$ as the plate is inclined. The fact is that as the angle of inclination increases the effect of the buoyancy force due to thermal diffusion decreases by a factor of $\cos \alpha$.

Consequently the driving force to the fluid decreases as a result velocity profiles decreases. The influence of the thermal Grashof number Gr on the velocity is presented in Fig. 3. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

Fig. 4 presents typical velocity profiles in the boundary layer for various values of the modified Grashof number Gm , while all other parameters are kept at some fixed values. The modified Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. Fig. 5 shows the effect of porous permeability parameter K on velocity distribution. It is seen that as the porosity parameter values increases the velocity also decreases. Fig. 6 displays the effect of chemical reaction parameter on velocity distribution. It is observed that the velocity decreases with increasing values of chemical reaction. Fig. (7) shows the effect of Magnetic parameter M on velocity distribution. It is seen that the velocity decreases with increasing values of Magnetic parameter. It is known fact that the application transfers Magnetic field which is applied normal to the flow, result in a flow resistive force called the Lorentz force which acts in the opposite direction of the flow. This force has the effect of slowing the motion of the fluid. Fig. 8 illustrate the velocity profile for different values of the Prandtl number Pr . The Prandtl number Pr defines the ratio of momentum Pr diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. Fig. 9 shows the effect of heat source parameter Q_0 on velocity distribution. It is seen that as the heat source parameter Q_0 values increases the velocity is decreases. Fig. 10 shows the effect of Soret number. So on velocity distribution. It is seen that as the Soret number So values increases the velocity also increases. Fig. 11 shows the effect of Schmidt number Sc on velocity distribution. It is seen that as the Schmidt number Sc values increases the velocity is decreases. The effect of the viscous dissipation parameter i.e., the Eckert number E on the velocity and temperature are shown in Figs 1 and 12 respectively. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figs 1 and 12. Figs. 13, 14, 15 and 16 shows the effect of inclination angle α , thermal Grashof number Gr , modified Grashof number Gm , porous permeability parameter K on temperature distribution. It is seen that as the inclination angle α , thermal Grashof number Gr , modified Grashof number Gm , porous permeability parameter K values increases the temperature is decreases. Fig. 17 shows the effect of Chemical reaction parameter K_r on temperature distribution. It is seen that as the Chemical reaction parameter K_r values increases the temperature also increases. Fig. 18 shows the effect of Magnetic parameter M on temperature distribution. It is seen that as the Magnetic parameter M values increases the temperature is decreases. Fig. 19 shows the effect of Prandtl number Pr on temperature distribution. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Prandtl number. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced. Fig. 20 and 21 shows the effect of heat source parameter Q_0 and Soret number So on temperature distribution. It is seen that as the heat source parameter Q_0 and Soret number So values increases the temperature is decreases. Fig. 22 shows the effect of Schmidt number Sc on temperature distribution. It is seen that as the Schmidt number Sc values increases the temperature also increases. Fig. 24, 28, 29, 30 and 32 shows the effect of inclination angle α , Magnetic Parameter M , Prandtl number Pr , heat source parameter Q_0 and Soret number So on concentration distribution. It is seen that as the inclination angle α , Magnetic Parameter M , Prandtl number Pr , heat source parameter Q_0 and Soret number So values increases the concentration is also increase. Fig. 23, 25, 26, 27 and 31 shows the effect of Eckert number Ec , modified Grashof number Gm , thermal Grashof number Gr , Chemical reaction parameter K_r and Schmidt number Sc on concentration distribution. It is seen that as the Eckert number Ec , modified Grashof number Gm , thermal Grashof number Gr , Chemical reaction parameter K_r and Schmidt number Sc values increases the concentration is decrease.

The influences of the Schmidt number Sc on the velocity and concentration profiles are plotted in Figs 11 and 31 respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figs 11 and 31.

From Table 1 it is noticed that an increasing in thermal Grashaf number Gr , modified Grashof number Gm and Soret number So results on increasing skin friction, while it decreases with an increase in Prandtl number (Pr), magnetic parameter (M), Schmidt number (Sc), heat source parameter Q_0 , chemical reaction parameter Kr and inclination angle α respectively. Table 2 shows the effects of Prandtl number (Pr), thermal Grashaf number Gr , modified Grashof number Gm , Soret number So and heat source parameter Q_0 results on increasing Nusselt number while it decreases with an increase in magnetic parameter M , Schmidt number Sc and chemical reaction parameter Kr and inclination angle α respectively. Table 3 shows the effects of Prandtl number Pr , thermal Grashaf number Gr , modified Grashof number Gm , magnetic parameter M , Schmidt number Sc , Soret number So , heat source parameter Q_0 , chemical reaction parameter Kr and inclination angle α on rate of mass transfer Sh numerically. It is observed that the rate of mass transfer increases with increasing values of thermal Grashof number Gr , modified Grashof number Gm , Schmidt number Sc and chemical reaction parameter Kr while it decreases in the case of Prandtl number Pr , magnetic parameter M , Soret number So , heat source parameter Q_0 and inclination angle α respectively.

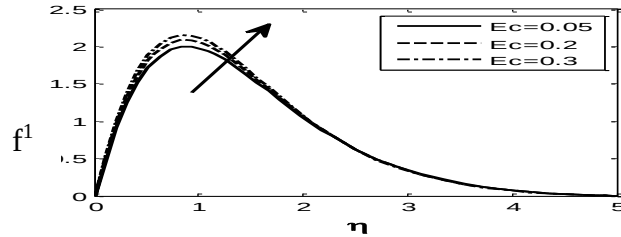


Fig. 1: Velocity Profile for Ec

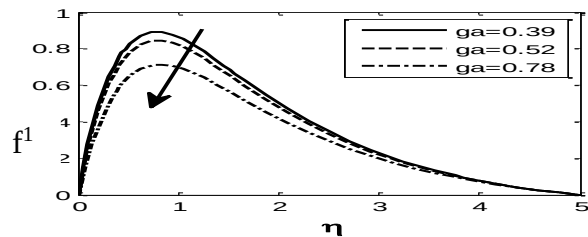


Fig. 2: Velocity Profile for ga

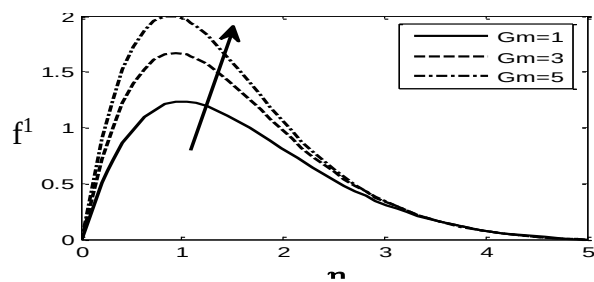


Fig. 3: Velocity Profile for Gm

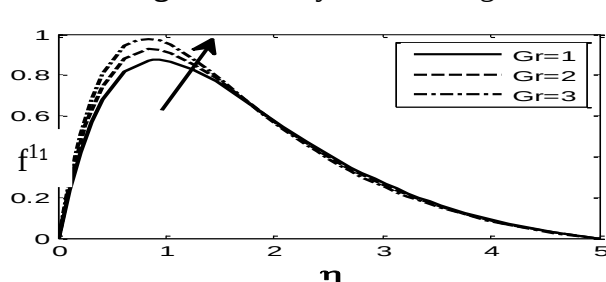


Fig. 4: Velocity Profile for Gr

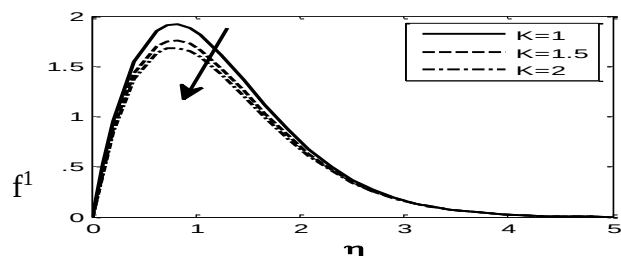


Fig. 5: Velocity Profile for K

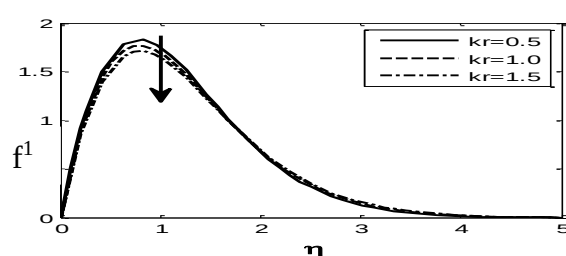


Fig. 6: Velocity Profile for Kr

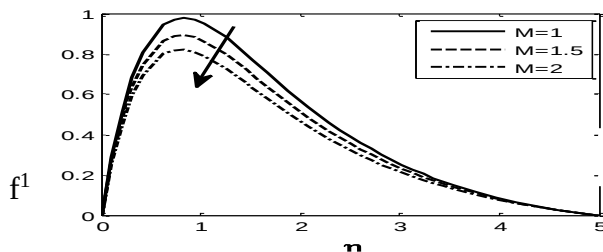


Fig. 7: Velocity Profile for M

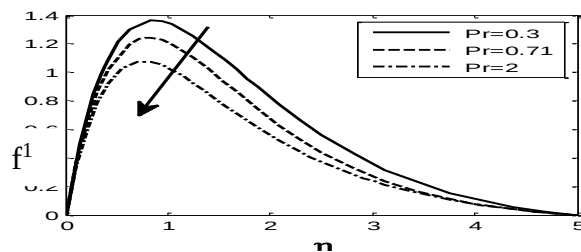


Fig. 8: Velocity Profile for Pr

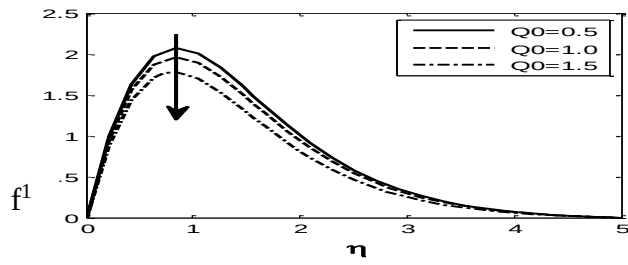


Fig. 9: Velocity Profile for Q_0

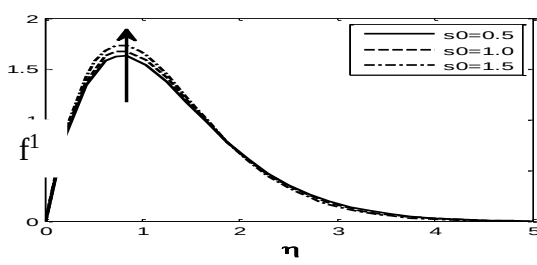


Fig. 10: Velocity Profile for S_o

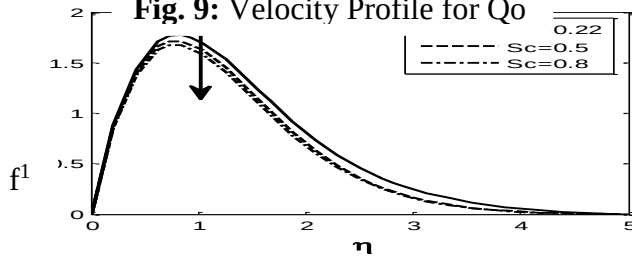


Fig. 11: Velocity Profile for Sc

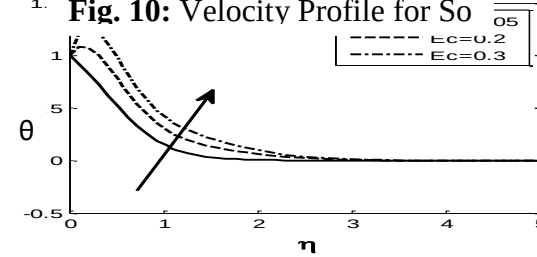


Fig. 12: Temperature Profile for Ec

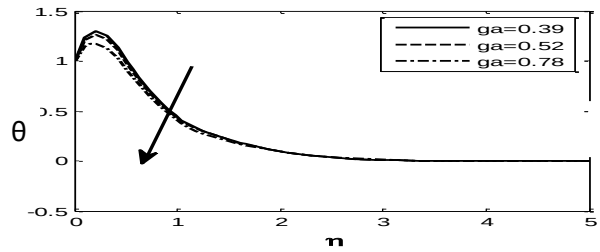


Fig. 13: Temperature Profile for ga

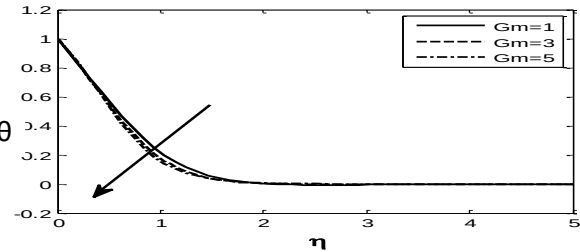


Fig. 14: Temperature Profile for G_m

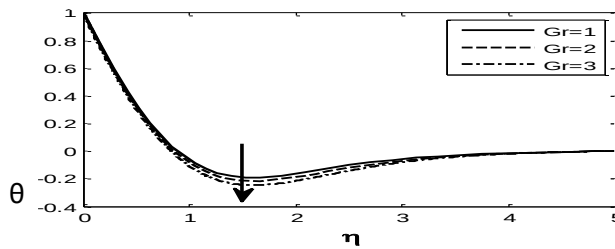


Fig. 15: Temperature Profile for Gr

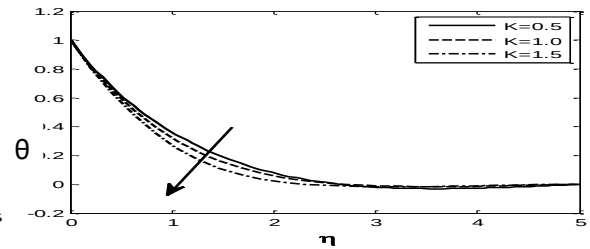


Fig. 16: Temperature Profile for K

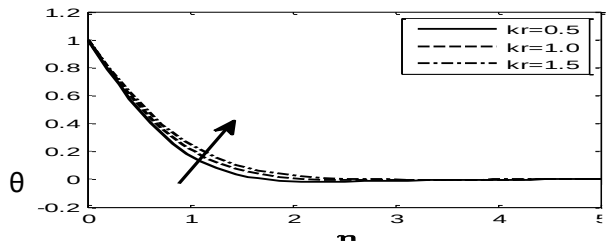


Fig. 17: Temperature Profile for K_r

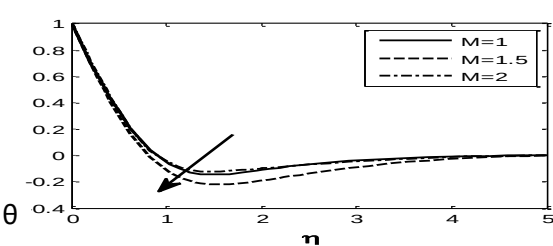


Fig. 18: Temperature Profile for M

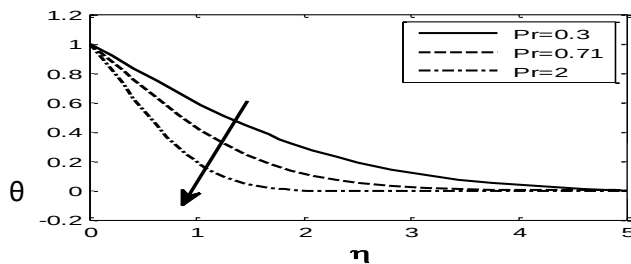


Fig. 19: Temperature Profile for Pr

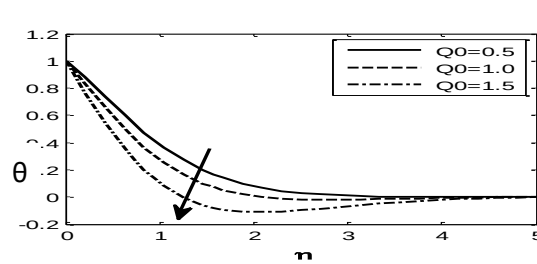


Fig. 20: Temperature Profile for Q_0

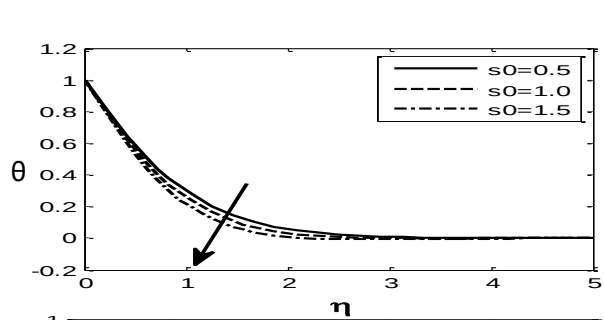


Fig. 21: Temperature Profile for S0

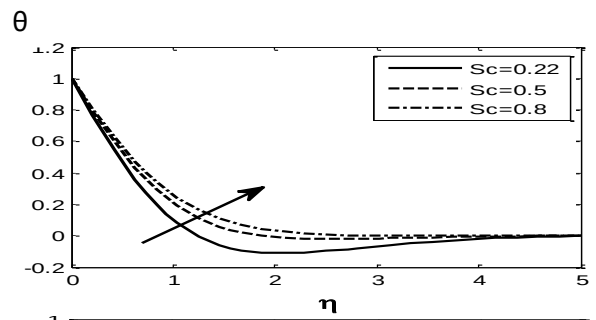


Fig. 22: Temperature Profile for Sc

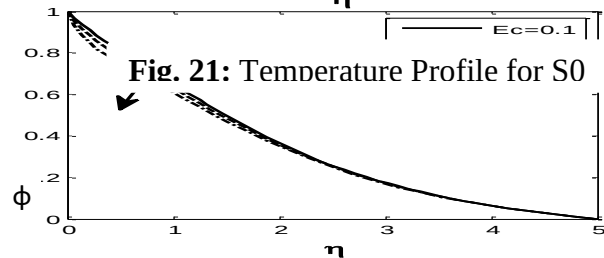


Fig. 23: Concentration Profile for Ec

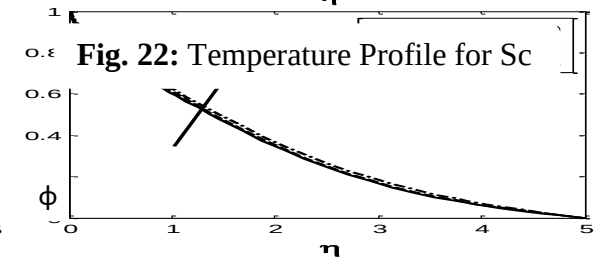


Fig. 24: Concentration Profile for ga

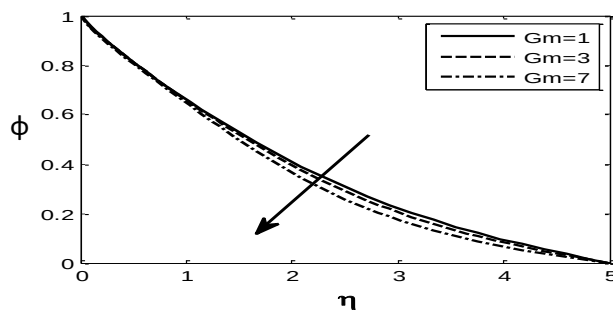


Fig. 25: Concentration Profile for Gm

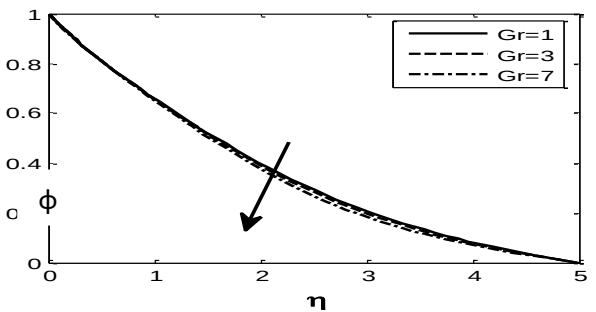


Fig. 26: Concentration Profile for Gr

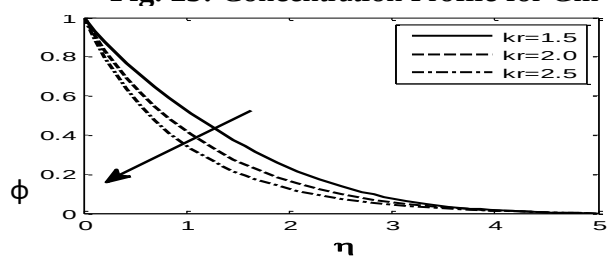


Fig. 27: Concentration Profile for Kr

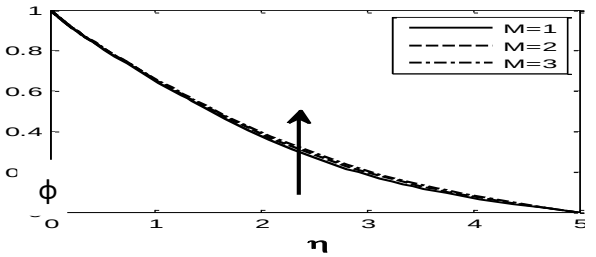


Fig. 28: Concentration Profile for M

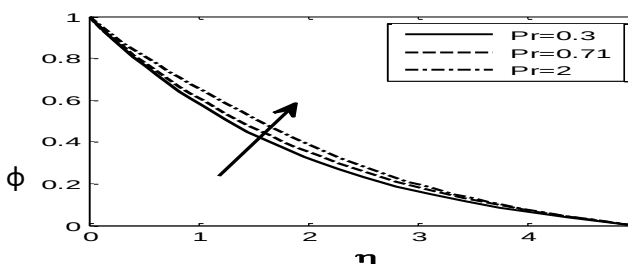


Fig. 29: Concentration Profile for Pr

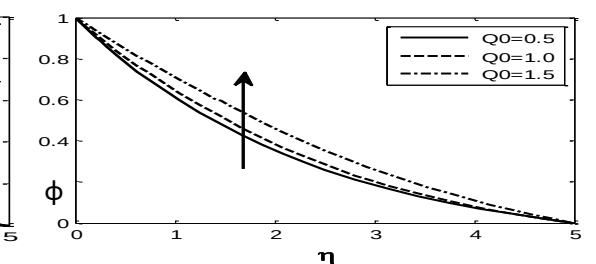


Fig. 30: Concentration Profile for Q0

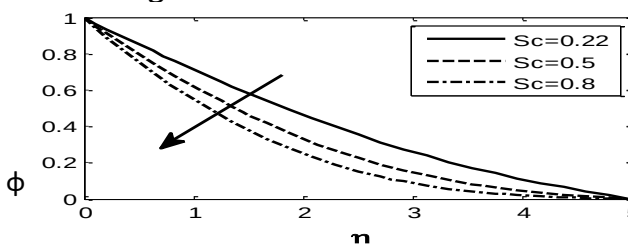


Fig. 31: Concentration Profile for

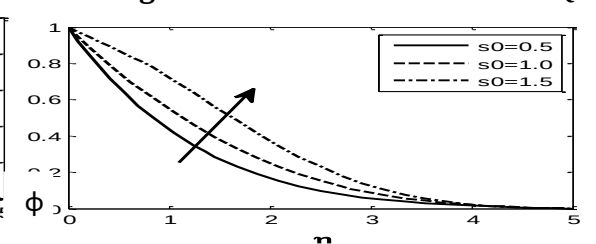


Fig. 32: Concentration Profile for S0

Table 1: Skin friction τ for different values of M, Pr, Sc, So, γ , Gr, Gm, kr and γ

Pr	Gr	Gm	M	Sc	So	Qo	Kr	α	Cf
0.71	1	1	1	0.22	1	0.1	1	0.52	0.90196
5	1	1	1	0.22	1	0.1	1	0.52	0.75724
0.71	3	1	1	0.22	1	0.1	1	0.52	1.71355
0.71	1	3	1	0.22	1	0.1	1	0.52	1.73340
0.71	1	1	2	0.22	1	0.1	1	0.52	0.78773
0.71	1	1	1	1	1	0.1	1	0.52	0.82758
0.71	1	1	1	0.22	1.5	0.1	1	0.52	0.90530
0.71	1	1	1	0.22	1	0.5	1	0.52	0.83548
0.71	1	1	1	0.22	1	0.1	1.5	0.52	0.88498
0.71	1	1	1	0.22	1	0.1	1	0.78	0.74637

Table 2: Nusselt number Nu for different values of Pr, Gr, Gm, M, Sc, S0, Q0, Kr and γ

Pr	Gr	Gm	M	Sc	So	Qo	Kr	α	Nu
0.71	1	1	1	0.22	1	0.1	1	0.52	0.39031
5	1	1	1	0.22	1	0.1	1	0.52	1.15088
0.71	3	1	1	0.22	1	0.1	1	0.52	0.44558
0.71	1	3	1	0.22	1	0.1	1	0.52	0.45240
0.71	1	1	2	0.22	1	0.1	1	0.52	0.37164
0.71	1	1	1	1	1	0.1	1	0.52	0.34691
0.71	1	1	1	0.22	1.5	0.1	1	0.52	0.39348
0.71	1	1	1	0.22	1	0.5	1	0.52	0.74774
0.71	1	1	1	0.22	1	0.1	1.5	0.52	0.37917
0.71	1	1	1	0.22	1	0.1	1	0.78	0.37640

Table 3: Sherwood number Sh for different values of Pr, Gr, Gm, M, Sc, So, Q0, Kr and γ

Pr	Gr	Gm	M	Sc	So	Qo	Kr	α	Sh
0.71	1	1	1	0.22	1	0.1	1	0.52	0.46832
5	1	1	1	0.22	1	0.1	1	0.52	0.32632
0.71	3	1	1	0.22	1	0.1	1	0.52	0.47550
0.71	1	3	1	0.22	1	0.1	1	0.52	0.47624
0.71	1	1	2	0.22	1	0.1	1	0.52	0.46695
0.71	1	1	1	1	1	0.1	1	0.52	0.97692
0.71	1	1	1	0.22	1.5	0.1	1	0.52	0.45321
0.71	1	1	1	0.22	1	0.5	1	0.52	0.40219
0.71	1	1	1	0.22	1	0.1	1.5	0.52	0.56978
0.71	1	1	1	0.22	1	0.1	1	0.78	0.46713

V. Conclusions

The present work helps us understanding numerically as well as physically free convection flow in an inclined porous plate in the presence of MHD and viscous dissipation has been employed. The set of governing equations and the boundary condition are reduced to ordinary differential equations with appropriate boundary conditions. Furthermore the similarity equations are solved numerically by using Runge–Kutta fourth order method along shooting technique. A comparison with previously published work is performed and the results are found to be in good agreement. Based on the obtained results, the following conclusions may be drawn.

From the numerical results, it is seen that the effect of increasing thermal Grashof number or solutal Grashof number is manifested as an increase in flow velocity. It is interesting to note that the temperature decreases much faster than the air temperature. In the presence of a magnetic field parameter, the permeability of porous medium, viscous dissipation is demonstrated to exert a more significant effect on the flow field and, thus, on the heat transfer from the plate to the fluid. The velocity and concentration is found to decrease gradually as the Schmidt number is increased. The velocity and concentration is found to decrease gradually as the chemical reaction is increased. The results of the study are of great interest because flows on a vertical stretching surface play a predominant role in applications of science and engineering, as well as in many transport processes in nature.

- The velocity decreases with an increase in inclination angle, Porosity parameter, Chemical reaction parameter, Magnetic parameter, Prandtl number, Schmidt number. But the velocity increases with an increase in Eckert number, modified Grashof number, thermal Grashof number and solet number.
- The temperature decreases with an increase in inclination angle, modified Grashof number, thermal Grashof number, Porosity parameter, Magnetic parameter, Prandtl number, heat source parameter and solet number. But the temperature increases with an increase in Eckert number, Chemical reaction parameter and Schmidt number.
- The concentration decreases with an increase in Eckert number, modified Grashof number, thermal Grashof number, Chemical reaction parameter and Schmidt number. But the velocity increases with an increase in inclination angle, Magnetic parameter, Prandtl number, heat source parameter and solet number.

- The skin friction increase of thermal Grashof number Gr , modified Grashof number Gm and Soret number So . But the skin friction decreases with an increase in Prandtl number Pr , magnetic parameter M , Schmidt number Sc , heat source parameter Q_0 , chemical reaction parameter K_r and inclination angle α .
- The Nusselt number increase of Prandtl number Pr , thermal Grashof number Gr , modified Grashof number Gm , Soret number So and heat source parameter Q_0 . But the Nusselt number decreases with an increase in magnetic parameter M , Schmidt number Sc , Chemical reaction parameter K_r and inclination angle α .
- The Sherwood number increase of thermal Grashof number Gr , modified Grashof number Gm , Schmidt number (Sc) and chemical reaction parameter K_r . But the Sherwood number decreases with an increase in Prandtl number Pr , magnetic parameter M , Soret number So , heat source parameter Q_0 and inclination angle α .

References

- [1] Gebhart and Mollendorf, considered the effects of viscous dissipation for external natural convection flow over a surface. *Journal of Fluid Mechanics* 38(01):97 - 107 · August 1969, DOI: 10.1017/S0022112069000061, DOI: 10.1007/BF00645477.
- [2] Soundalgekar, analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate, *Astrophysics and Space Science* 62(2):389-396 · January 1979.
- [3] Chen C. H. (2004): Heat and Mass Transfer in MHD Flow by Natural Convection from a Permeable, Inclined Surface with Variable Wall Temperature and Concentration, *ActaMechanica*, Vol.172, pp. 219-235. <http://dx.doi.org/10.1007/s00707-004-0155-5>
- [4] Duwairi, H. M. (2005): Viscous and Joule Heating Effects on Forced Convection Flow from Radiate Isothermal Porous Surfaces, *International Journal of Numerical Methods Heat Fluid Flow*, Vol.15, No.5, pp. 429-440. <http://dx.doi.org/10.1108/09615530510593620>
- [5] Kumari, M., Takhar H. S. and Nath, G. (2001): Mixed Convection Flow over a Vertical Wedge Embedded in a Highly Porous Medium, *Journal of Heat and Mass Transfer*, Vol.37, pp. 139-146. <http://dx.doi.org/10.1007/s002310000154>
- [6] Singh et al., investigated the effect of volumetric heat generation/absorption on mixed convection stagnation point flow on an isothermal vertical plate in porous media, *Int. J. Industrial Mathematics* Vol. 2, No. 2 (2010) 59-71, Available online at <http://ijim.srbiau.ac.ir>.
- [7] Hayat et al., studied heat and mass transfer for Soret and Dufour effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid, *Communications in Nonlinear Science and Numerical Simulation* 15(5):1183-1196 · May 2010, DOI: [10.1016/j.cnsns.2009.05.062](https://doi.org/10.1016/j.cnsns.2009.05.062).
- [8] Ramana Reddy, G. V., Ramana Murthy, and Ch.V. and Bhaskar Reddy, N. (2011): MHD Flow over a Vertical Moving Porous Plate with Heat Generation by Considering Double Diffusive Convection, *International Journal of Applied Mathematics and Mechanics*, Vol.7, No.4, pp. 53-69.
- [9] Aziz, A. (2009): A Similarity Solution for Laminar Thermal Boundary Layer over a Flat Plate with a Convective Surface Boundary Condition, *Communications in Nonlinear Science and Numerical Simulation*, Vol.14, No.4, pp. 1064-1068. <http://dx.doi.org/10.1016/j.cnsns.2008.05.003>
- [10] Makinde, O.D. (2010): On MHD Heat and Mass Transfer over a Moving Vertical Plate with a Convective Surface Boundary Condition, *Canadian Journal of Chemical Engineering*, Vol.88, pp. 983-990. <http://dx.doi.org/10.1002/cjce.20369>
- [11] Ishak, A. (2010): Similarity Solutions for Flow and Heat Transfer over a Permeable Surface with Convective Boundary Condition, *Applied Mathematics and Computation*. Vol. 217, pp. 837-842. <http://dx.doi.org/10.1016/j.amc.2010.06.026>
- [12] Kishan, N. and Amrutha, P. (2011): Effects of Viscous Dissipation on MHD Flow with Heat and Mass Transfer over a Stretching Surface with Heat Source, Thermal Stratification and Chemical Reaction, *Journal Of Naval Architecture And Marine Engineering*, Vol. 7, No. 1, pp. 11-18. <http://dx.doi.org/10.3329/jname.v7i1.3254>
- [13] Ishak, A. Yacob, N.A. and Bachok, N.(2011): Radiation Effects on the Thermal Boundary Layer Flow over a Moving Plate with Convective Boundary Condition, *Meccanica*, Vol.46, pp. 795-801. <http://dx.doi.org/10.1007/s11012-010-9338-4>
- [14] Subhashini, S.V., Samuel, N. and Pop, I. (2011): Double-diffusive Convection from a Permeable Vertical Surface under Convective Boundary Condition, *International Communications in Heat and Mass Transfer*, Vol. 38, pp.1183-1188. <http://dx.doi.org/10.1016/j.icheatmasstransfer.2011.06.006>
- [15] Karim et al. investigated Dufour and Soret effect on steady MHD flow in presence of heat generation and magnetic field past an inclined stretching sheet, *Open Journal of Fluid Dynamics* 2(03):91-100 · January 2012, DOI: 10.4236/ojfd.2012.23009.
- [16] Bhavana et al. analyzed the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source, *International Journal of Innovative Research in Science, Engineering and Technology* Vol. 2, Issue 5, May 2013, ISSN: 2319-8753.
- [17] Shu, J. J. and Pop, I. (1997): Inclined Wall Plumes in Porous Media, *Fluid Dynamics Research*, Vol. 21, pp. 303-317. [http://dx.doi.org/10.1016/S0169-5983\(97\)81115-X](http://dx.doi.org/10.1016/S0169-5983(97)81115-X).
- [18] Parveen, N and Alim, M. A., (2012): Joule Heating Effect on Magnetohydrodynamic Natural Convection Flow along a Vertical Wavy Surface, *Journal of Naval Architecture and Marine Engineering*, Vol.9, No.1, pp.11-24.. <http://dx.doi.org/10.3329/jname.v9i1.5954>
- [19] G. Singh, P.R. Sharma and A.J. Chamkha, Effect of Volumetric Heat Generation/Absorption on Mixed Convection Stagnation Point Flow on an Isothermal Vertical Plate in Porous Media, *Int. J. Industrial Mathematics*, vol.2(2), pp. 59-71, 2010.
- [20] T. Hayat, M. Mustafa and I. Pop, Heat and Mass Transfer for Soret and Dufour Effect on Mixed Convection Boundary Layer Flow over a Stretching Vertical Surface in a Porous Medium filled with a Visco-elastic Fluid, *Commun. Nonlinear SciNumerSimulat*, vol.15, pp.1183-1196, 2010.
- [21] B. Gebhart and J. Mollendorf, Viscous dissipation in external natural convection flows, *J. Fluid. Mech.*, vol. 38, pp. 97-107, 1969.
- [22] V.M. Soundalgekar, Viscous dissipation effects on unsteady free convective flow past an infinite, vertical porous plate with constant suction, *Int. J. Heat Mass Transfer*, vol.15, pp. 1253-1261, 1972.
- [23] M D Enamul Karim, M D AbdusSamad and M D Maruf Hasan, "Dufour and Soret effect on steady MHD flow in presence of Heat generation and magnetic field past an inclined stretching sheet," *Open Journal of Fluid Dynamics*, vol.2, pp. 91-100, 2012.
- [24] M. Bhavana, D. ChennaKesaraiah and A. Sudhakarrah, "The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source," *Int. J. of Innovative R. in Sci. Eng. and Tech.*, vol. 2(5), pp. 1617-1628, 2013.